SSLP313 - Crack inclined in an unlimited plate, subjected to a uniform traction ad infinitum

Summary:

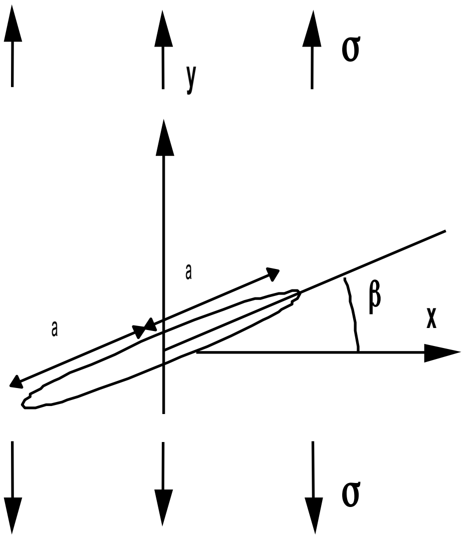
This test is resulting from the validation independent of version 3 of Code_Aster in breaking process.

One calculates K_I, K_{II} and the rate of refund of energy for a right crack, tilted of an angle b, in a large-sized plate subjected to a uniform traction. The model is two-dimensional in plane constraints. The material is elastic linear isotropic. This test of reference in $2D$ allows to check the separability of K_I and K_{II} in a mixed mode.
The reference solution, given for a theoretically unlimited field, is analytical.

Besides the energy method (`CALC_G`), one tests the method of calculating of the factors of intensity of the constraints by extrapolation of displacements (`POST_K1_K2_K3`). Modeling B allows to test this last method with a kind of grid recommended (nodes mediums with the quarter) to obtain a precise solution.

1 Problem of reference

1.1 Geometry



One allots an unspecified value to the slope, $\beta = 37\, degrés$.
One chooses $a = 1.E-3\, m$.

1.2 Properties of material

The material is elastic linear isotropic, of Young modulus $E = 2.E11\, Pa$ and of Poisson's ratio $\nu = 0.3$.
The traction diagram is defined such as:

- the slope is equal to 3.
- the elastic limit is equal to $1.88\, GPa$.

The assumption of the plane constraints is applied.

1.3 Boundary conditions and loadings

•Arbitrary limits of the field with a grid:

$-x_{max} \leq x \leq x_{max}$ with $x_{max} = 10a$

$-y_{max} \leq y \leq y_{max}$ with $y_{max} = 20a$

•Boundary conditions:

In order to block the 3 rigid modes exclusively plans.
$UX = UY = 0$ with the left lower corner of the complete model.
$UY = 0$ with the corner lower right of the complete model.
On the lower edge, we impose $UY = 0$

•Loading: uniform tension $\sigma_{yy} = \sigma_0$ on the higher edge:

The value of σ_0 is worth $100 MPa$, in plane constraints.

2 Reference solution

2.1 Method of calculating used for the reference solution

Function of constraint of Airy.

2.2 Results of reference

$$K_I = \sigma_0 \sqrt{(\pi_0)} \cos^2 \beta$$

$$K_{II} = \sigma_0 \sqrt{(\pi_0)} \sin \beta \cos \beta$$

$$G_{ref} = \frac{1}{E}\left(K_I^2 + K_{II}^2\right) \text{ (in plane constraints)}$$

2.3 Uncertainty on the solution

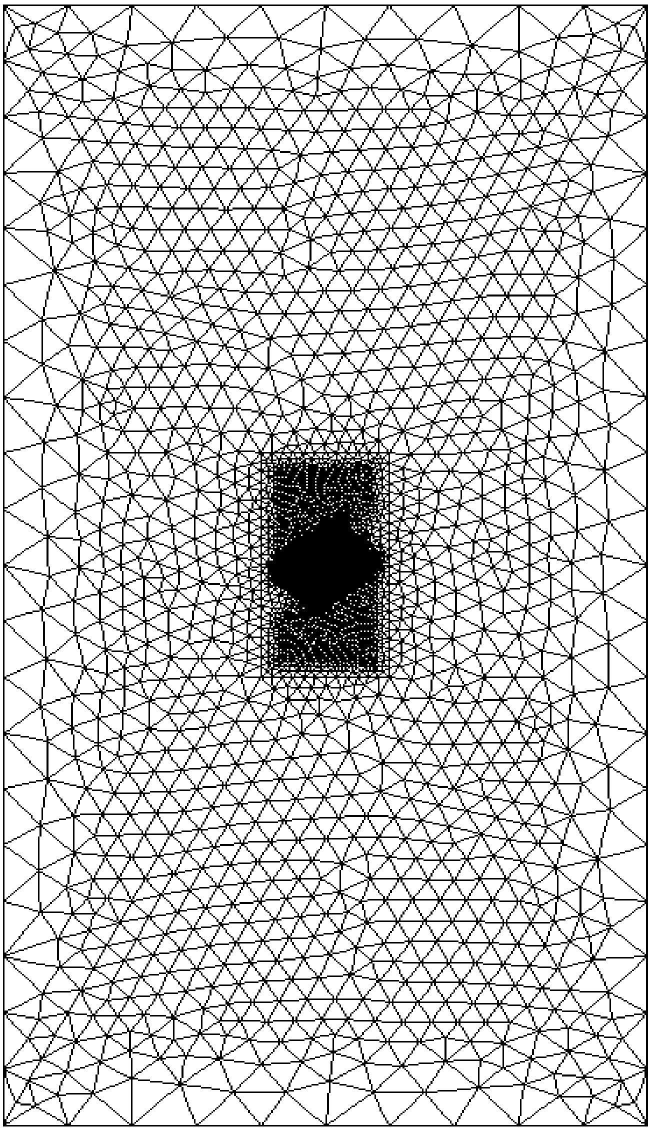
Exact analytical solution (Irwin) in unlimited medium.

2.4 Bibliographical references

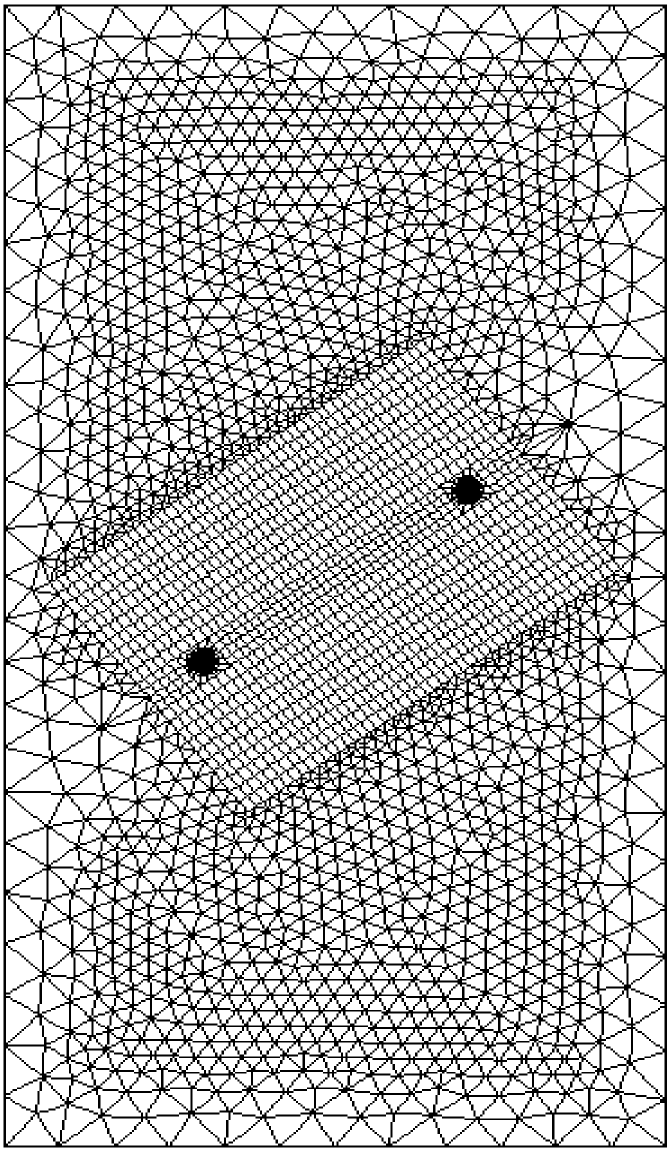
1 Y. MURAKAMI Stress intensity factors handbook, box 4.2, page 188. The Society of Materials Science, Japan, Pergamon Near, 1987.

3 Modeling A

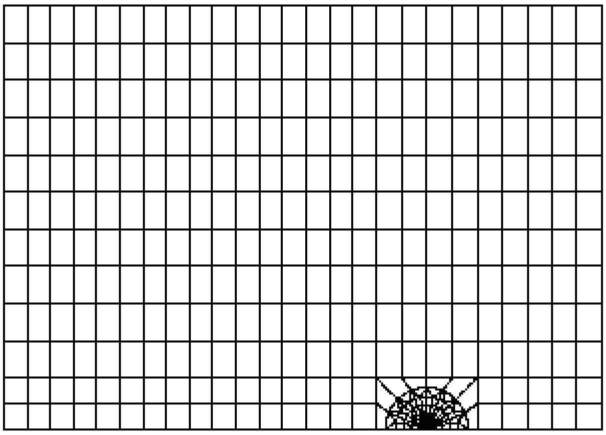
3.1 Characteristics of modeling



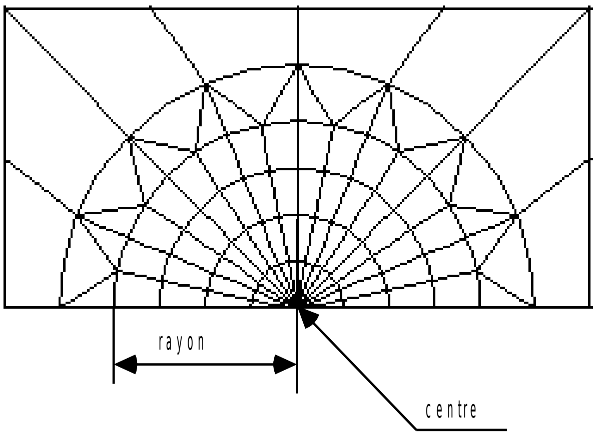
Complete model



After symmetrization and orientation



Initial block 2D



The ray is worth $7{,}5E\text{-}5\,m$

3.2 Characteristics of the grid

The grid consists of 14888 nodes and 6674 elements, including 1392 elements QUAD8 and 5282 elements TRIA6.

3.3 Sizes tested and results

3.3.1 Results got with CALC_G

Identification	Reference (analytical)	% tolerance
G	$1.0019\ 10^{2}$	2.0
K_I	$3.5750\ 10^{6}$	1.0
K_{II}	$2.6939\ 10^{6}$	1.0

Table 3.3.1-1

3.3.2 Results got with POST_K1_K2_K3

Identification	Reference (analytical)	% tolerance
G	$1.0019\ 10^{2}$	4.0
K_I	$3.5750\ 10^{6}$	1.2
K_{II}	$2.6939\ 10^{6}$	3.0

Table 3.3.2-1

Values obtained with POST_K1_K2_K3 are also tested in not-regression:

Identification	Reference (not-regression)	% tolerance
G	96,85	0.1
K_I	$3.5379307159775\ 10^{6}$	0.1
K_{II}	$2.6179089643527\ 10^{6}$	0.1

Table 3.3.2-2

One also tests the possibility of giving to POST_K1_K2_K3 directly tables of the jumps of displacement, by making a test from report with the values previously obtained

Identification	Reference (another aster)	% tolerance
G	96,85	0.1
K_I	$3.5379307159775\ 10^{6}$	0.1
K_{II}	$2.6179089643527\ 10^{6}$	0.1

Table 3.3.2-3

4 Modeling B

4.1 Characteristics of modeling

Even form of grid that previously, but modification of the coordinates of the nodes mediums of the edges touching the bottom of crack, to move them with the quarter of these edges (method of Barsoum).

This modification of the coordinates of the nodes is carried out by an accessible procedure GIBI in card-indexing it data of grid (SSLP313B.datg).

4.2 Characteristics of the grid

The grid consists of 14888 nodes and 6674 elements, including 1392 elements QUAD8 and 5282 elements TRIA6.

4.3 Sizes tested and results

4.3.1 Results got with CALC_G

Identification	Reference (analytical)	% tolerance
G	$1.0019\ 10^{2}$	2.0
K_I	$3.5750\ 10^{6}$	1.0
K_{II}	$2.6939\ 10^{6}$	1.0

Table 4.3.1-1

4.3.2 Results got with POST_K1_K2_K3

Identification	Reference (analytical)	% tolerance
G	$1.0019\ 10^{2}$	4.0
K_I	$3.5750\ 10^{6}$	1.2
K_{II}	$2.6939\ 10^{6}$	3.0

Table 4.3.2-1

5 Summary of the results

With this choice of the limits of the field of calculation, we obtain variations about 1% on the coefficients K_I and K_{II}, and on the rate of refund of energy G.

With regard to the method POST_K1_K2_K3, the results are further away from the reference with a standard grid (of -1% with -30% of variation), on the other hand, with a grid of the type Barsoum (nodes mediums with the quarter on the sides), recommended for this kind of method, the variations are understood enters -3% and $+1.2\%$, which is relatively precise.