SSLP311 - Central crack obliques in a finished rectangular plate, with two materials, subjected to uniform traction

Summary:

This test is resulting from the validation independent of version 3 in breaking process.

It is about a two-dimensional test in statics with bi--material in the presence of an internal crack of interface obliques.

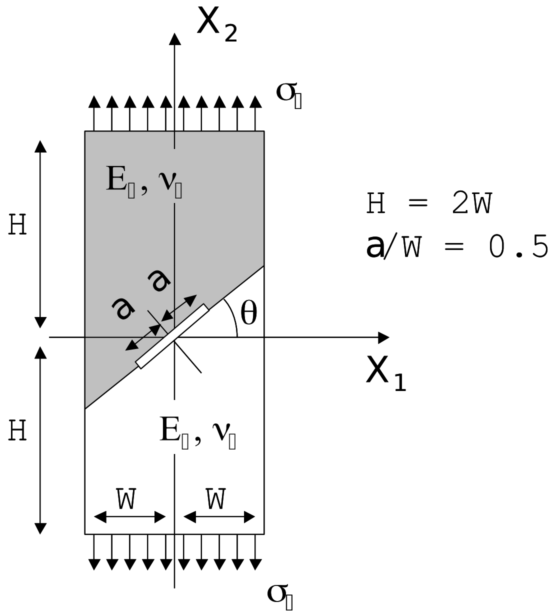
The behavior of the structure (bi--material) is elastic linear isotropic.

The case test understands four modelings in plane constraints in which the influence of the slope of the crack θ is studied (4 cases).

The calculation of the factors of intensity of the constraints is not available for a crack located at the interface of a bi--material; the comparison with the reference solution is thus done on the rate of refund of energy only, calculated with the operator `CALC_G`.

1 Problem of reference

1.1 Geometry



4 values of the angle are considered θ : 15°, 30°, 45° and 60°.
Other dimensions are selected such as $H=2\mathrm{W}=4\mathrm{a}$.
The value of a is worth $1.\mathrm{E}-3\,m$.

1.2 Properties of materials

Material n° 1

Rubber band, linear, isotropic, Young modulus $E_1=2\mathrm{E}{+}12\,Pa$ and Poisson's ratio $\nu_1=0{,}3$.

Material n° 2

Rubber band, linear, isotropic, Young modulus $E_2=2\mathrm{E}{+}11\,Pa$ and Poisson's ratio $\nu_2=0{,}3$.

1.3 Boundary conditions and loading

- The rigid modes are blocked by the boundary conditions following:
  $UX=UY=0$ with the left lower corner of the model.
  $UY=0$ on the lower edge.
- Loading: uniform tension $\sigma_{yy}=\sigma_0$ on the higher edge.

  The value of σ_0 is $100\,MPa$.

2 Reference solution

2.1 Method of calculating used for the reference solution

Method of the elements of border, with quadratic elements [bib1].

The calculation of K_I and K_{II} is carried out by an integral of contour (integral M [bib2]) in which intervene the constraints and displacements calculated in the part, as well as the constraints and displacements deduced from analytically definite solutions asymptotic, in which K_I and K_{II} are alternatively worthless.

As comparison, the calculation of K is also carried out by the method of virtual extension.

2.2 Results of reference

The results of the reference solution are presented in the table below, for the various values of the angle and the two ends of the crack, with $F_j = \dfrac{K_j}{\sigma_0\sqrt{\pi a}} \quad (j=I,II)$.

Method		Left side				Right side			
		$\theta=15°$	$\theta=30°$	$\theta=45°$	$\theta=60°$	$\theta=15°$	$\theta=30°$	$\theta=45°$	$\theta=60°$
integral	F_I	1.0115	0.7868	0.5211	0.2770	1.1266	0.9910	0.7646	0.4919
M	F_{II}	~0.4434	~0.6244	~0.6723	~0.5804	0.0862	0.2961	0.4056	0.4057
extension	F_I	1.0110	0.7864	0.5210	0.2769	1.1260	0.9904	0.7643	0.4919
virtual	F_{II}	~0.4429	~0.6240	~0.6720	~0.5801	0.0865	0.2960	0.4055	0.4056

The relation between the total rate of restitution of energy G and them K_j is written as follows [bib3]:

$$G=\beta(K_I^2+K_{II}^2)$$

with:

$$\beta=\frac{1}{16Ch^2(\alpha\pi)}\left(\frac{1+\kappa_1}{\mu_1}+\frac{1+\kappa_2}{\mu_2}\right) \quad \text{and} \quad \kappa_i=\frac{3-\nu_i}{1+\nu_i}, \quad \mu_i=\frac{E_i}{2(1+\nu_i)}, \quad \alpha=\frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{\mu_1}+\frac{1}{\mu_2}\right)\left(\frac{\kappa_2}{\mu_2}+\frac{1}{\mu_1}\right)^{-1}\right]$$

2.3 Uncertainty on the solution

Estimated at less than 0.1%. It is noted that the difference between the method of the integrals of contour and the method of virtual extension is generally lower than 0,05%.

2.4 Bibliographical references

1) Stress intensity Factor analysis of interface ace using boundary element method. Of application contour-integral method. NR. MIYAZAKI, T. IKEDA, T.SODA and T. MUNAKATA. Engng.Fract.Mechs., 45, n°5, 599-610, 1993.

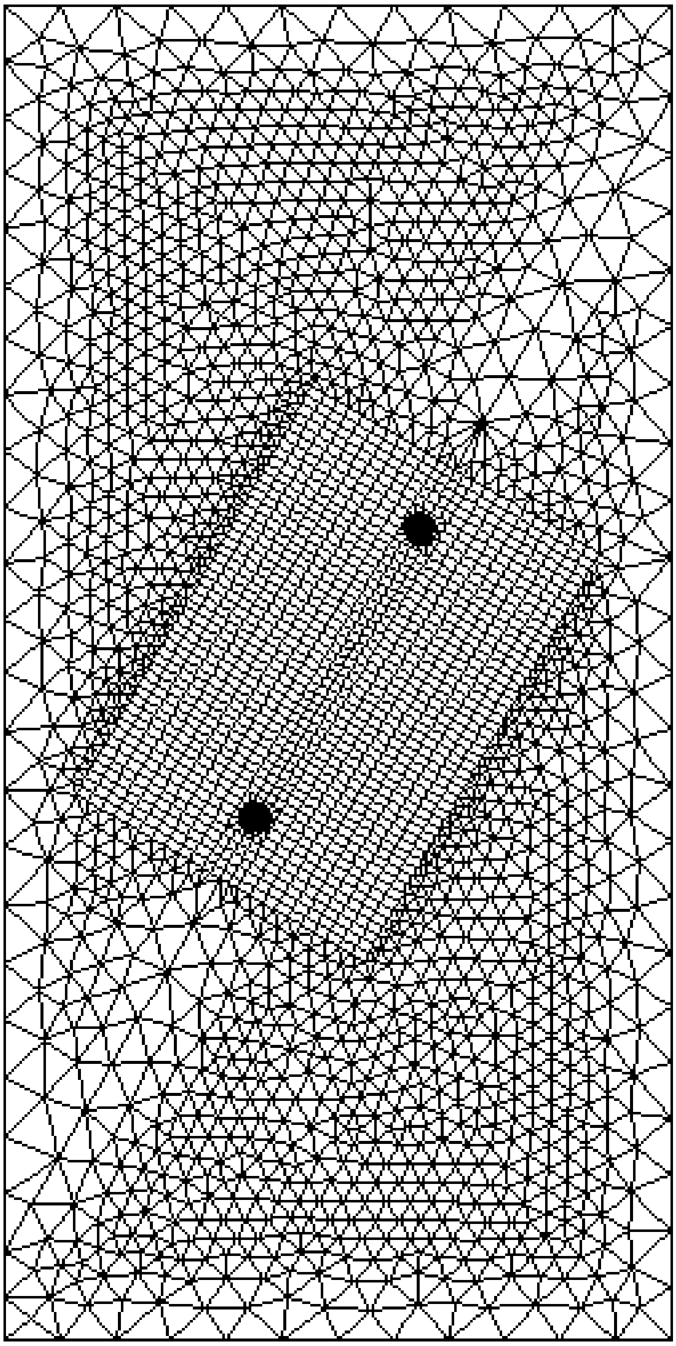
2) Year analysis of interface aces between dissimilar isotropic materials using conservation integrals in elasticity. J.F. YAU and T.C. CHANG. Engng.Fract.Mechs., 20,423-432, 1984.

3) Adhesive The strength of joints using the theory of aces. B. Mr. MALYSHEV and R.L. SALGANIK. Int.J.Fract.Mech., 1,114-128, 1965.

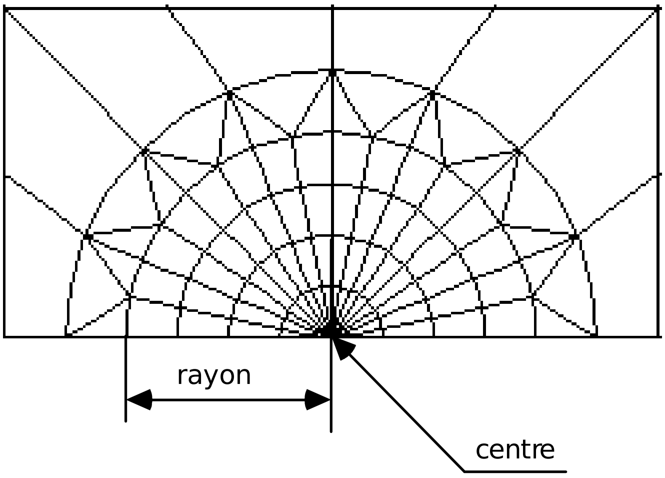
3 Modeling A

3.1 Characteristics of modeling

Various modelings are identical except for the slope of the crack.



Complete grid for an angle $\beta = 60°$



Zoom on the point of crack

The ray is worth $7.5E-5m$.

There are four crowns defined by the order CALC_THETA :

crown 1: $Rinf=0.$ $R\sup=1.875E-5m$
crown 2: $Rinf=1.875E-5m$ $R\sup=3.750E-5m$
crown 3: $Rinf=3.750E-5m$ $R\sup=5.625E-5m$
crown 4: $Rinf=5.625E-5m$ $R\sup=7.500E-5m$

The direction of propagation is defined by: $\cos\theta, \sin\theta$

3.2 Characteristics of the grid

The grid consists of 10676 nodes and 4584 elements, including 1392 elements QUA8 and 3168 elements TRI6.

3.3 Features tested

The calculation of K_I and K_{II} is not valid for a bimatériau: the option CALC_K_G cannot be used and only the calculation of the rate of refund of energy is possible.

3.4 Sizes tested and results

3.4.1 Values tested

Identification	Reference	*Aster*	% difference
Left end, $\theta=15°$			
G , crown 1	9,67362E+1	9,2428E+1	˜ 4.45
G , crown 2	9,67362E+1	9,6392E+1	˜ 0.356
G , crown 3	9,67362E+1	9,6417E+1	˜ 0.330
G , crown 4	9,67362E+1	9,6421E+1	˜ 0.326
K_I	5,6694E+6	-	-
K_{II}	˜ 2,4852E+6	-	-
Right end, $\theta=15°$			

G , crown 1	1,0125E+2	9,6763E+1	˜ 4.33
G , crown 2	1,0125E+2	1,0093E+2	˜ 0.315
G , crown 3	1,0125E+2	1,0095E+2	˜ 0.295
G , crown 4	1,0125E+2	1,0095E+2	˜ 0.291
K_I	6,3145E+6	-	-
K_{II}	4,8309E+5	-	-

3.4.2 Remarks

To obtain it G on the bottom of crack, one calculates the rate of refund of energy using the relation enters G and them K_j [bib3]:

$$\kappa_1=\kappa_2=2.076923$$
$$\mu_1=7.6923\,E+11$$
$$\mu_2=7.6923\,E+10$$
$$\alpha=-9.37742\,E-2$$
$$\beta=2.524488\,E-12$$
$$G=\beta\left(K_I^2+K_{II}^2\right)$$

4 Modeling B

4.1 Values tested

Identification	Reference	*Aster*	% difference
Left end, $\theta=30°$			
G , crown 1	8,0017E+1	7,6431E+1	˜ 4.48
G , crown 2	8,0017E+1	7,9707E+1	˜ 0.387
G , crown 3	8,0017E+1	7,9730E+1	˜ 0.358
G , crown 4	8,0017E+1	7,9734E+1	˜ 0.353
K_I	4,4100E+6	-	-
K_{II}	˜ 3,499E+6	-	-
Right end, $\theta=30°$			
G , crown 1	8,48417E+1	8,1080E+1	˜ 4.433
G , crown 2	8,48417E+1	8,4583E+1	˜ 0.305
G , crown 3	8,48417E+1	8,4602E+1	˜ 0.282
G , crown 4	8,48417E+1	8,4602E+1	˜ 0.282
K_I	5,5545E+6	-	-
K_{II}	1,6596E+6	-	-

5 Modeling C

5.1 Values tested

Identification	Reference	*Aster*	% difference
Left end, $\theta=45°$			
G , crown 1	5,73826E+1	5,48161E+1	˜ 4.473
G , crown 2	5,73826E+1	5,71687E+1	˜ 0.373
G , crown 3	5,73826E+1	5,71865E+1	˜ 0.342
G , crown 4	5,73826E+1	5,7189E+1	˜ 0.337
K_I	2,92076E+6	-	-
K_{II}	˜ 3,7682E+6	-	-
Right end, $\theta=45°$			
G , crown 1	5,94122E+1	5,7039E+1	˜ 3.994
G , crown 2	5,94122E+1	5,9505E+1	0.157
G , crown 3	5,94122E+1	5,9516E+1	0.175
G , crown 4	5,94122E+1	5,9518E+1	0.179
K_I	4,28557E+6	-	-
K_{II}	2,27338E+6	-	-

6 Modeling D

6.1 Values tested

Identification	Reference	*Aster*	% difference
Left end, $\theta=60°$			
G , crown 1	3,28015E+1	3,10680E+1	˜ 5.285
G , crown 2	3,28015E+1	3,24037E+1	˜ 1.213
G , crown 3	3,28015E+1	3,24140E+1	˜ 1.181
G , crown 4	3,28015E+1	3,24156E+1	˜ 1.177
K_I	1,55258E+6	-	-
K_{II}	˜ 3,2531E+6	-	-
Right end, $\theta=60°$			
G , crown 1	3,22436E+1	3,11825E+1	˜ 3.291
G , crown 2	3,22436E+1	3,25321E+1	0.895
G , crown 3	3,22436E+1	3,25383E+1	0.914
G , crown 4	3,22436E+1	3,25398E+1	0.919
K_I	2,75709E+6	-	-
K_{II}	2,27394E+6	-	-

7 Summary of the results

The calculation of K_I and K_{II} is not available for a crack located at the interface of a bimatériau, and the comparison is thus done directly on the rate of refund of energy G .

The calculation of G is not precise on the first crown in all the cases of slope of the crack, which confirms that it is necessary to avoid taking a ray $Rinf$ no one. With regard to the other crowns, the variations are about 0.4%. In the case of slope $\theta=60°$ the variation exceeds 1%. As a whole, the results are satisfactory for G .