SSLP310 - Crack pressurized in unlimited a plan field

Summary:

This test is resulting from the validation independent of version 3 in breaking process.

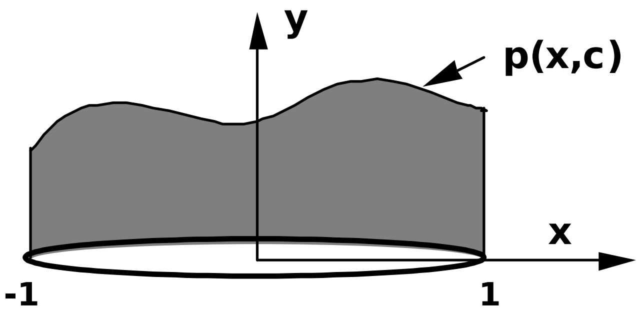
It is about one two-dimensional test in statics (plane strains or stresses) which aims at the checking of G and K_I under loading by pressure distributed not uniform on the lips, in unlimited medium. One also checks the nullity of K_{II} with the option **CALC_K_G** of the operator **CALC_G**.

behavior of the structure is isotropic linear rubber band.

The case test understands only one plane modeling 2D in which one studies the influence of the parameter C intervening in the loading. Mechanical calculation is done by call to the macro-order **MACRO_ELAS_MULT**.

1 Problem of reference

1.1 Geometry



The rectilinear crack is considered $-1 \leq x \leq 1$ in unlimited the plan field.

1.2 Properties of material

The material is elastic linear homogeneous of Young modulus E and of Poisson's ratio ν .
$E=1000\ MPa$, $\nu=0,3$

1.3 Boundary conditions and loadings

Boundary conditions

Linear relation $UX(-1,0)+UX(1,0)=0$

Condition of symmetry $UY=0$ for $x\leq 1$, $x\geq 1$ and $y=0$.

Loading n° 1

$p(x)=1$

Loading n° 2

$p(x,c)=\exp(cx)$ where c is a parameter

Loading n° 3

$p(x,c)=Sh(cx)$ where c is a parameter

Loading n° 4

$p(x,c)=Ch(cx)$ where c is a parameter

Loading n° 5

$p(x,c)=\cos(cx)$ where c is a parameter

2 Reference solution

2.1 Method of calculating used for the reference solution

Exact calculation symbolic system using software MAPLE V [bib1].

2.2 Results of reference

Loading n° 1

$K_I(x=1)=\sqrt{\pi}$

Loading n° 2

$K_I(x=1,c)=\sqrt{\pi}(I_0(c)+I_1(c))$ where I_0 and I_1 are the functions of Bessel modified of first species of indices 0 and 1 [bib2].

Loading n° 3

$K_I(x=1,c)=\sqrt{\pi}\,I_1(c)$

Loading n° 4

$K_I(x=1,c)=\sqrt{\pi}\,I_O(c)$

Loading n° 5

$K_I(x=1,c)=\sqrt{\pi}\,J_0(c)$ where J_0 is the function of Bessel of first species of index 0 [bib2].

In all the cases of loading

$G=\frac{K_I^2}{E}$ in plane constraints

$G=\frac{(1-\nu^2)K_I^2}{E}$ in plane deformations

2.3 Bibliographical references

[1] There the evaluation of stress intensity factors for is simple ace under parametric loading. Technical notes. N.I. IOKADIMIS and G.T. ANASTASSELOS. Computers and Structures, 51, n°6, 791-794, 1994.

[2] Handbook of mathematical functions, Chapter 9. Mr. ABRAMOWITZ and I.A. STEGUN (Editors). United States Dept. of Commerce, National Office of Standards.

3 Modeling A

3.1 Characteristics of modeling

The model is limited to the finished area $-x_{max} \le x \le x_{max}$, $-y_{max} \le y \le y_{max}$ with $x_{max} = y_{max} = 15$.

It consists of 1156 quadrangles with 8 nodes and 3398 triangles with 6 nodes.
It comprises 10372 nodes.

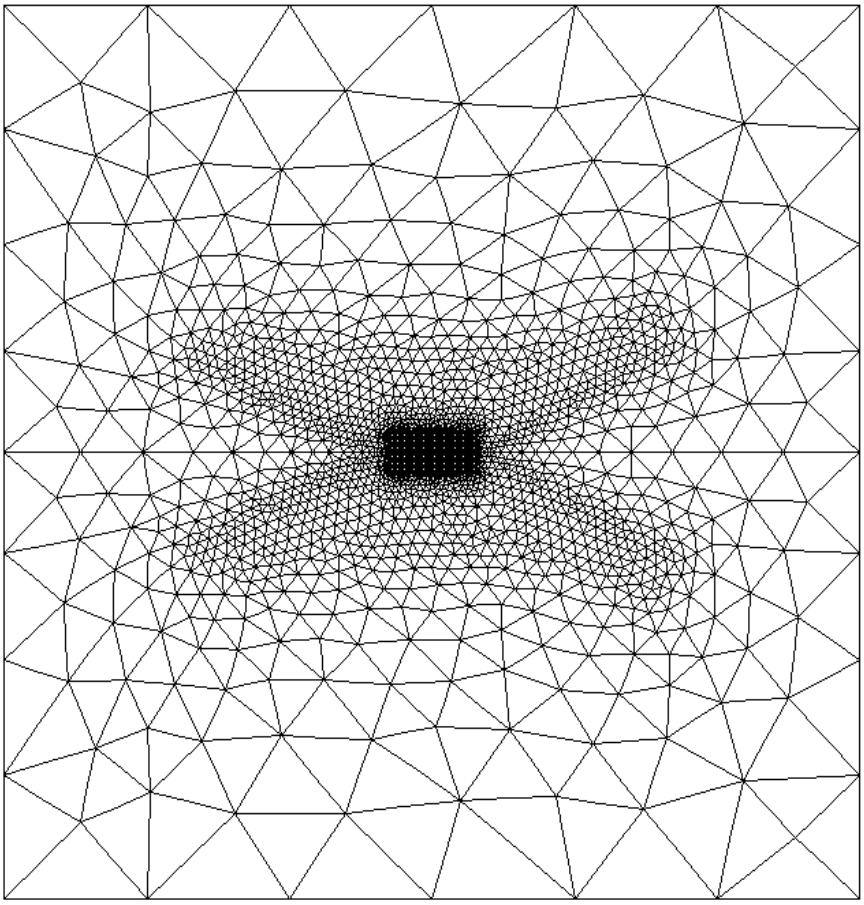
One uses the assumption of the plane constraints.

3.2 Characteristics of the grid

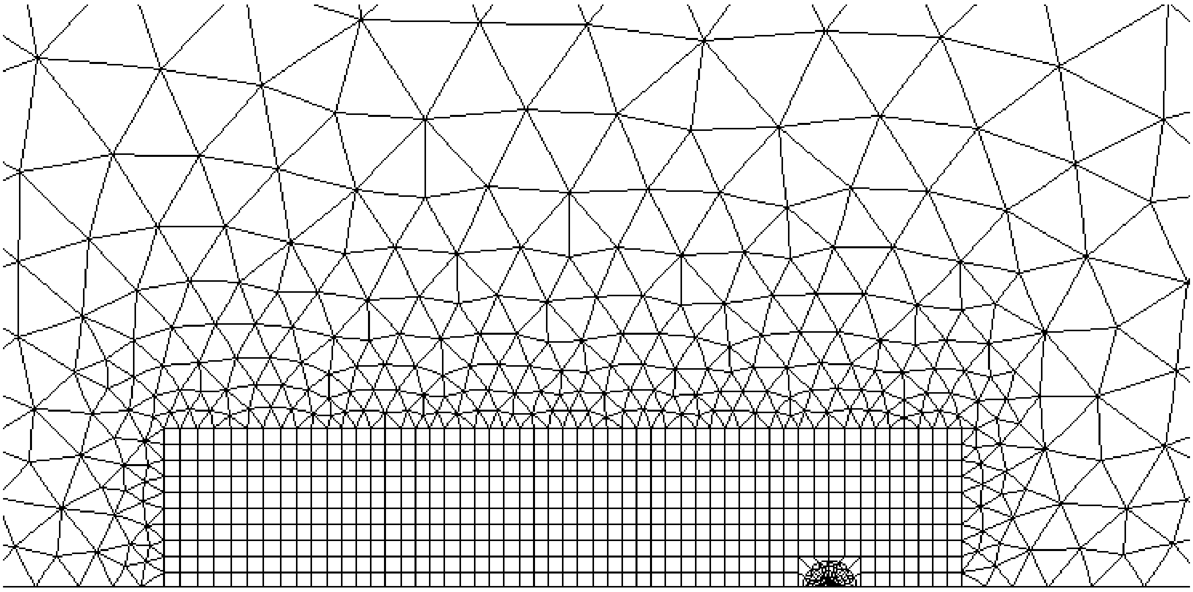
The grid is generated with *gibi* (use of procedure FISS2D). The topological parameters concerning refinement around the bottom of crack (torus) are:

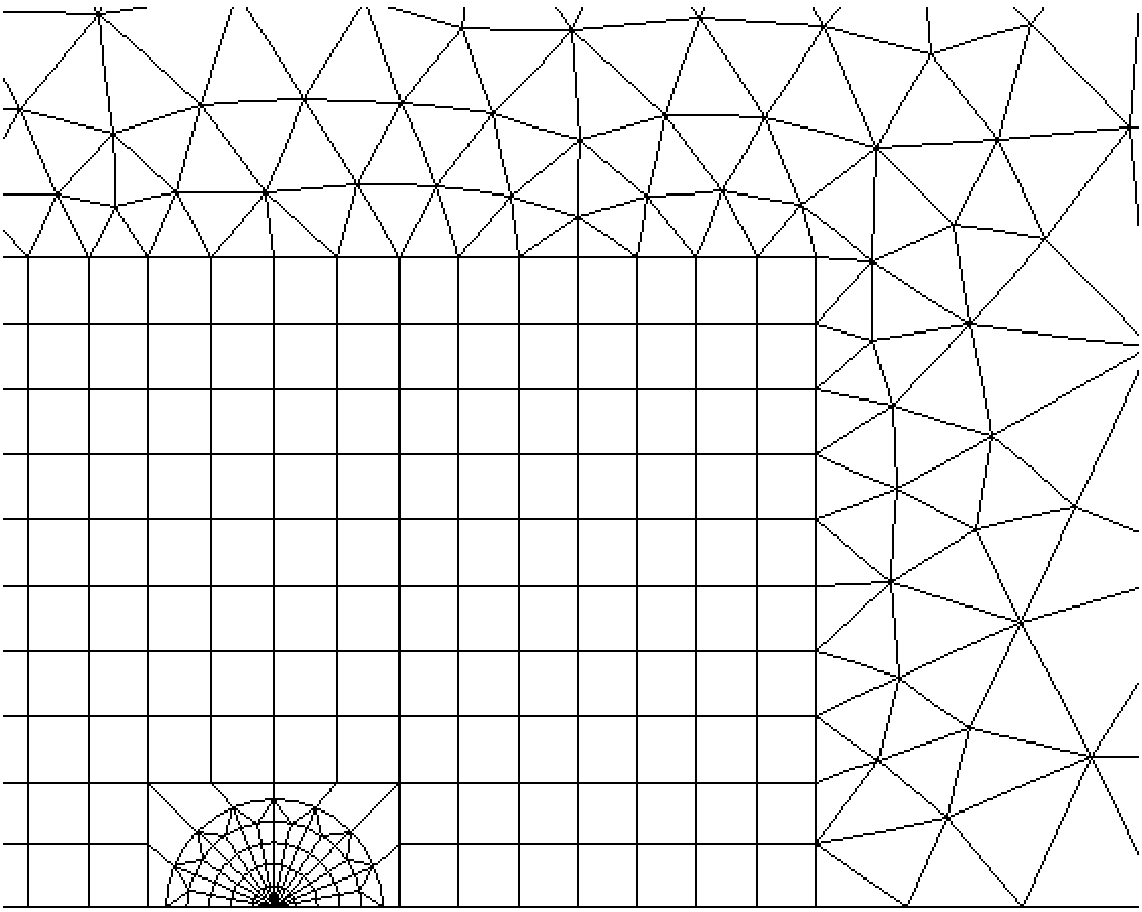
- $nc=4$ (many crowns)
- $ns=8$ (many sectors)
- $nt=1$ (many crowns of déraffinement)

The radiant fine grid is limited at the right end of the crack.
The partly current density of the grid of the crack is selected in order to be able to discretize the loading suitably $p(x,c)$.



Zooms on the grid of the crack:





3.3 Features tested

The whole of the cases of loading is treated simultaneously in the macro-order **MACRO_ELAS_MULT**. The rate of refund of energy G is calculated in postprocessing by the method theta (operator **CALC_G**) for each case of loading successively and various crowns of integration.

3.4 Sizes tested and results

3.5 Values tested

Crown 0 (triangles)

$R_{\text{inf}}=0\,mm$, $R_{\text{sup}}=0{,}02\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,04077E-3	-3.209
K_I , loading n°1	1.77245	1.66	-6.57
G , loading n°2, $c=1$	1,05349E-2	0.01	-3.738
K_I , loading n°2, $c=1$	3.24576	3.03108	-6.61
G , loading n°2, $c=5$	8.356742	7.9065	-5.39
K_I , loading n°2, $c=5$	91.41522	85.4189	-6.56
G , loading n°3, $c=1$	1,00344E-3	9,6006E-4	-4.323
K_I , loading n°3, $c=1$	1.00172	0.93505	-6.66
G , loading n°3, $c=5$	1.86052	1.760148	-5.4
K_I , loading n°3, $c=5$	43.13380	40.33829	-6.48
G , loading n°4, $c=1$	5,03571E-3	4,86064E-3	-3.477
K_I , loading n°4, $c=1$	2.24404	2.09602	-6.6
G , loading n°4, $c=5$	2.331095	2.20566	-5.38
K_I , loading n°4, $c=5$	48.28142	45.08068	-6.63
G , loading n°5, $c=1$	1,839487E-3	1,78707E-3	-2.849
K_I , loading n°5, $c=1$	1.356277	1.267569	-6.54
G , loading n°5, $c=2{,}4048255577$	0	4,1738E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	2,0383E-3	-

Crown 1 (quadrangles)

$R_{\text{inf}}=0{,}02\,mm$, $R_{\text{sup}}=0{,}04\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,1669E-3	0.807
K_I , loading n°1	1.77245	1.78079	0.471
G , loading n°2, $c=1$	1,05349E-2	1,056655E-2	0.30
K_I , loading n°2, $c=1$	3.24576	3.256597	0.334
G , loading n°2, $c=5$	8.356742	8.25545	-1.212
K_I , loading n°2, $c=5$	91.41522	91.528	0.123
G , loading n°3, $c=1$	1,00344E-3	1,000804E-3	- 0.263
K_I , loading n°3, $c=1$	1.00172	1.003475	0.175
G , loading n°3, $c=5$	1.86052	1.83815	-1.202
K_I , loading n°3, $c=5$	43.13380	43.2091	0.175
G , loading n°4, $c=1$	5,03571E-3	5,06348E-3	0.552
K_I , loading n°4, $c=1$	2.24404	2.25312	0.405
G , loading n°4, $c=5$	2.331095	2.302636	-1.221
K_I , loading n°4, $c=5$	48.28142	48.3188	0.078
G , loading n°5, $c=1$	1,839487E-3	1,86066E-3	1.152
K_I , loading n°5, $c=1$	1.356277	1.363914	0.563
G , loading n°5, $c=2{,}4048255577$	0	3,98377E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	4,721938E-3	-

Crown 2 (quadrangles)

$R_{\inf}=0{,}04\,mm$, $R_{\sup}=0{,}06\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,1678E-3	0.835
K_I , loading n°1	1.77245	1.78075	0.468
G , loading n°2, $c=1$	1,05349E-2	1,056949E-2	0.328
K_I , loading n°2, $c=1$	3.24576	3.256529	0.332
G , loading n°2, $c=5$	8.356742	8.257967	-1.182
K_I , loading n°2, $c=5$	91.41522	9,1527E1	0.123
G , loading n°3, $c=1$	1,00344E-3	1,001087E-3	-0.234
K_I , loading n°3, $c=1$	1.00172	1.0034589	0.174
G , loading n°3, $c=5$	1.86052	1.838717	-1.172
K_I , loading n°3, $c=5$	43.13380	43.2088	0.174
G , loading n°4, $c=1$	5,03571E-3	5,064887E-3	0.579
K_I , loading n°4, $c=1$	2.24404	2.25307	0.402
G , loading n°4, $c=5$	2.331095	2.30333	-1.191
K_I , loading n°4, $c=5$	48.28142	48.31838	0.077
G , loading n°5, $c=1$	1,839487E-3	1,86117E-3	1.179
K_I , loading n°5, $c=1$	1.356277	1.363877	0.560
G , loading n°5, $c=2{,}4048255577$	0	4,0008E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	4,711869E-3	-

Crown 3 (quadrangles)

$R_{\text{inf}}=0{,}06\,mm$, $R_{\text{sup}}=0{,}08\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,16794E-3	0.839
K_I , loading n°1	1.77245	1.78078	0.471
G , loading n°2, $c=1$	1,05349E-2	1,05699E-2	0.333
K_I , loading n°2, $c=1$	3.24576	3.2566	0.334
G , loading n°2, $c=5$	8.356742	8.25837	1.177
K_I , loading n°2, $c=5$	91.41522	91.5293	0.125
G , loading n°3, $c=1$	1,00344E-3	1,001132E-3	- 0.230
K_I , loading n°3, $c=1$	1.00172	1.003481	0.176
G , loading n°3, $c=5$	1.86052	1.838809	- 1.167
K_I , loading n°3, $c=5$	43.13380	43.20984	0.176
G , loading n°4, $c=1$	5,03571E-3	5,065103E-3	0.584
K_I , loading n°4, $c=1$	2.24404	2.25312	0.405
G , loading n°4, $c=5$	2.331095	2.303447	- 1.186
K_I , loading n°4, $c=5$	48.28142	48.31948	0.079
G , loading n°5, $c=1$	1,839487E-3	1,86124E-3	1.183
K_I , loading n°5, $c=1$	1.356277	1.363907	0.563
G , loading n°5, $c=2{,}4048255577$	0	4,00631E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	4,71155E-3	-

3.6 Remarks

Crown 0 (surrounding the bottom of crack and made up by triangles) gives poor results compared to the other crowns.

Relative variations maxima between crowns 1.2 and 3 for G and K_I are given below for the various loadings.

	Loading 1	Loading 2	Loading 3	Loading 4	Loading 5
Variation on G	0.03%	0.03%	0.03%	0.03%	0.03%
Variation on K_I	0.002%	0.002%	0.002%	0.002%	0.002%

Variations on G and K_I are negligible.

In all the cases of loading and for all the crowns, one also checked that it K_{II} is null.

4 Summary of the results

Except for the results got on crown 0, calculations of K and G are very close to the exact theoretical solution. Indeed, the variations are always lower than 1,2% for the calculation of G and lower than 0,6% for the calculation of K_I .