SSLP305 - Thin disc in support under concentrated loading

Summary:

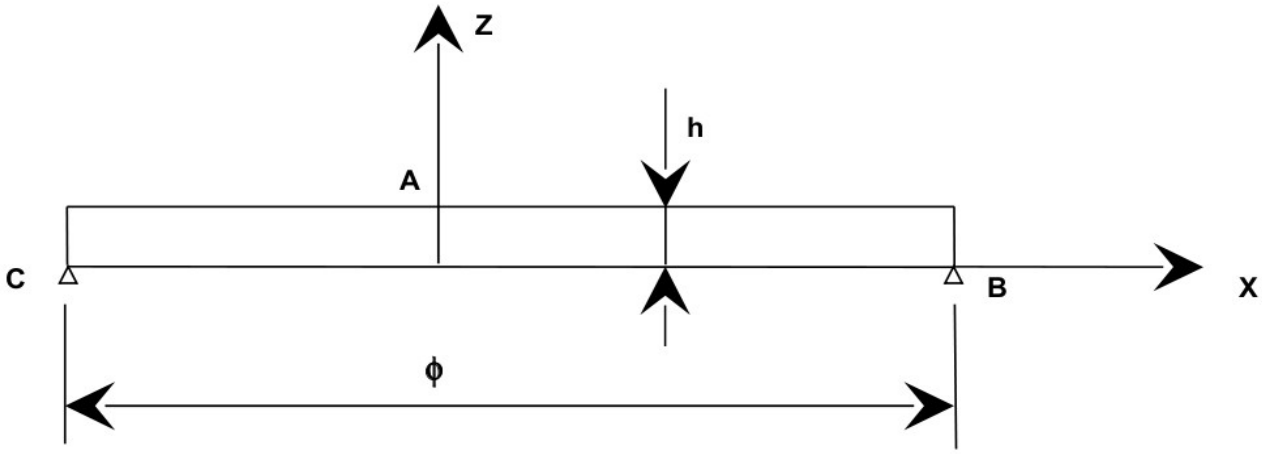
The purpose of the test is to validate the calculation of the potential energy in linear elasticity.

Only one axisymmetric modeling is presented.

The reference solution is analytical.

1 Problem of reference

1.1 Geometry



Diameter: $\phi=0.5\,m$

Thickness: $h=0.005\,m$

1.2 Material properties

Young modulus: $E=2.1\times10^{11}\,Pa$

Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loadings

- Support on the edge ($w=0$)
- Loading concentrated at the point A : $P=-350\,N$

1.4 Initial conditions

Without object for the static analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

- The value of axial displacement in the center of the disc (not A) is given by:

$$W_a = -\frac{P\,\phi^2}{64\pi D} \times \frac{3+\nu}{1+\nu}$$

where $D = \frac{E\,h^3}{12(1-\nu^2)}$

- The value of the potential energy (with balance) is given by:

$$E_p = -\frac{1}{2} P\,W_a$$

- The absolute value of the potential energy by radian is:

$$e_p = \frac{1}{2}\frac{P\,W_a}{2\pi}$$

2.2 Results of reference

- Displacement at the point A : $W_a = -0.4596 \times 10^{-3} m$
- Potential energy by radian: $e_p = 0.012799\, Nm/rd$

2.3 Uncertainty on the solution

Analytical solution.

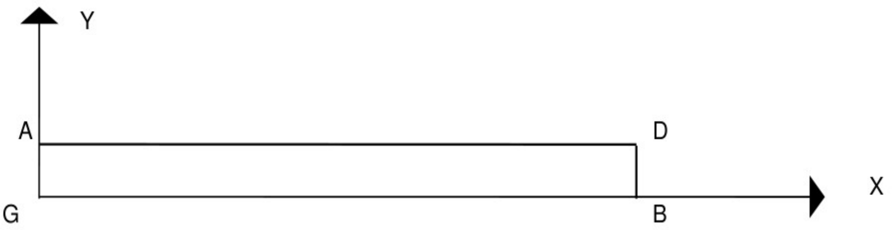
2.4 Bibliographical references

1) R.J. ROARK and W.C. YOUNG Formulated for stress and strain, 5 ème edition, New York, Mc Graw-Hill, 1975

3 Modeling A

3.1 Characteristics of modeling

It is an axisymmetric modeling.



Limiting conditions:

in B `DDL_IMPO: (GROUP_NO: B DY: 0.)`
on AG `DDL_IMPO: (GROUP_NO: lAG DX: 0.)`

Loading:

in A `FORCE_NODALE: (GROUP_NO: With FY: -55,704)`

Name of the nodes:
$A=N1$ $B=N755$ $D=N858$ $G=N201$

Cutting: 100 elements according to the ray
2 elements according to the thickness

3.2 Characteristics of the grid

Many nodes: 905

Many meshs and types: 100 QUAD 8.200 SORTED 6.208 SEG 3

3.3 Values tested

Localization	Type of value	Reference	*Aster*	% difference
Not A	$W_A(m)$	$-0.4596 \quad 10^{-3}$	$-0.4617 \quad 10^{-3}$	0.46
	$e_p(Nm/rd)$	$-1.2799 \quad 10^{-2}$	$-1.2859 \quad 10^{-2}$	0.47

3.4 Remarks

- The value of the load required is brought back to a sector of 1 radian. Consequently, the value of the potential energy given on the file result corresponds to the deformation of this sector (with the sign near).
- The option `ENERPOT` calculate in fact a deformation energy:

$E_d = \frac{1}{2} U^T K U$ who is identical to the potential energy with the sign near:

$$E_p = \frac{1}{2} U^T K U - U^T F = -\frac{1}{2} U^T F = -\frac{1}{2} U^T K U \text{ (because } KU = F\text{)}$$

4 Summary of the results

These good performances on the displacement and the deformation energy (similar variation of 0,5% with the analytical reference solution) show that the calculation of this energy is correct. To approach still best the value of reference, the grid would have to be discretized more.