

SSLP304 - Orthotropic square plate in uniaxial traction out of axes of orthotropism

Summary:

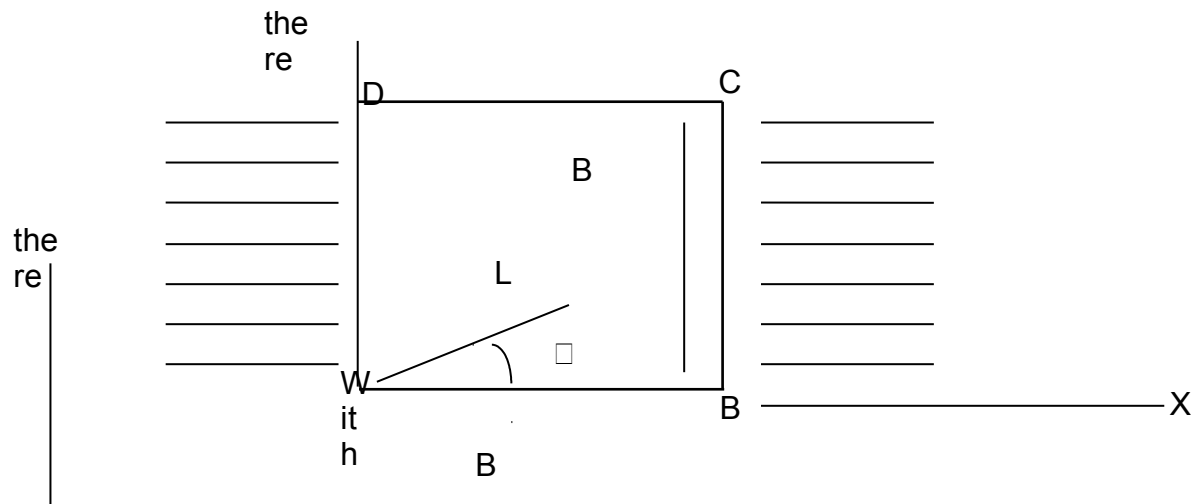
This test represents the static calculation of a square plate, out of orthotropic elastic material, whose axes of orthotropie are tilted of 30 degrees compared to the basic edge, subjected with a uniaxial traction. It makes it possible to validate the good taking into account of orthotropic elastic materials and the change of associated reference mark. 4 modelings are used: C_PLAN with meshes QUAD8 and TRIA6, in a first reference mark, C_PLAN in a second reference mark, COQUE_3D with meshes QUAD9 and TRIA7, in small displacements and COQUE_3D in great displacements. Displacements and the constraints obtained are compared with an analytical reference solution.

The first two modelings of this test result from the validation independent of version 3 of Code_Aster (linear static batch).

1 Problem of reference

1.1 Geometry

A square plate, made up of a tilted orthotropic material of 30 degrees compared to the edge AB .



With $b = 1\text{ m}$, unspecified thickness (plane constraints), angle of orthotropy: $\theta = 30$ degrees.

1.2 Properties of materials

The properties of materials constituting the plate are:

orthotropic rubber band:

$$\begin{aligned} E_L &= 4.E10\text{ Pa} \\ E_T &= 1.E10\text{ Pa} \\ G_{LT} &= 0.45E10\text{ Pa} \\ G_{TN} &= 0.35E10\text{ Pa} \\ NU_{LT} &= 0.075 \end{aligned}$$

The axis L is tilted of 30 degrees compared to AB .

1.3 Boundary conditions and loadings

- At the point A : $DX = 0$, $DY = 0$
- At the point B : $DX = 0$,
- Linear loading distributed: $F_x = 10^4\text{ Pa}$ on BC
- Linear loading distributed: $F_x = -10^4\text{ Pa}$ on DA

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution, obtained with the assumption of uniaxiality of the constraints:

$$\sigma_{xx}(x, y) = F_x \quad \sigma_{xy}(x, y) = \sigma_{yy}(x, y) = \sigma_{zz}(x, y) = 0$$

maybe in the reference mark (A, L, T) :

$$\sigma_{LL}(x, y) = c^2 F_x, \sigma_{TT}(x, y) = s^2 F_x \quad \sigma_{LT}(x, y) = -cs F_x$$

By the law of elastic behavior orthotropic, by using conventions of Code_Aster with regard to NU_{LT} , (cf document of use of the order `DEFI_MATERIAU` [§3.5.2]), one obtains directly (see for example [bib1]):

$$\varepsilon_{xx}(x, y) = \frac{F_x}{E_x}, \varepsilon_{yy}(x, y) = -\frac{\nu_{xy}}{E_x} F_x, 2\varepsilon_{xy}(x, y) = \frac{\eta_x}{E_x} F_x$$

with:

$$\frac{1}{E_x(\theta)} = \frac{c^4}{E_L} + \frac{s^4}{E_T} + c^2 s^2 \left[\frac{1}{G_{LT}} - 2 \frac{\nu_{LT}}{E_T} \right] \quad \frac{\nu_{xy}}{E_x(\theta)} = (c^4 + s^4) \frac{\nu_{LT}}{E_T} - c^2 s^2 \left[\frac{1}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}} \right]$$

$$\frac{\eta_y}{E_x(\theta)} = -2cs \left[\frac{c^2}{E_L} - \frac{s^2}{E_T} \right] + (c^2 - s^2) \left[\frac{\nu_{LT}}{E_T} - \frac{1}{2G_{LT}} \right]$$

avec $c = \cos \theta$

$s = \sin \theta$

As the deformations are uniform in the plate one obtains, by integration, displacements in the reference mark (A, x, y) :

$$u_x(x, y) = \varepsilon_{xx} \cdot x$$

$$u_y(x, y) = \varepsilon_{yy} \cdot y + 2\varepsilon_{xy} \cdot x$$

2.2 Results of reference

Displacements in the reference mark (A, x, y) (in m):

Not	B	C	D
u_x	0.	$5,917 \cdot 10^{-7}$	$5,917 \cdot 10^{-7}$
u_x	$-2,292 \cdot 10^{-7}$	$-5,028 \cdot 10^{-7}$	$-7,319 \cdot 10^{-7}$

Constraints in the reference mark related to the orthotropism:

$$\sigma_{LL}(x, y) = 7500 \text{ Pa}, \quad \sigma_{TT}(x, y) = 2500 \text{ Pa}, \quad \sigma_{LT}(x, y) = 4330.127 \text{ Pa}$$

2.3 Uncertainty on the solution

Analytical solution

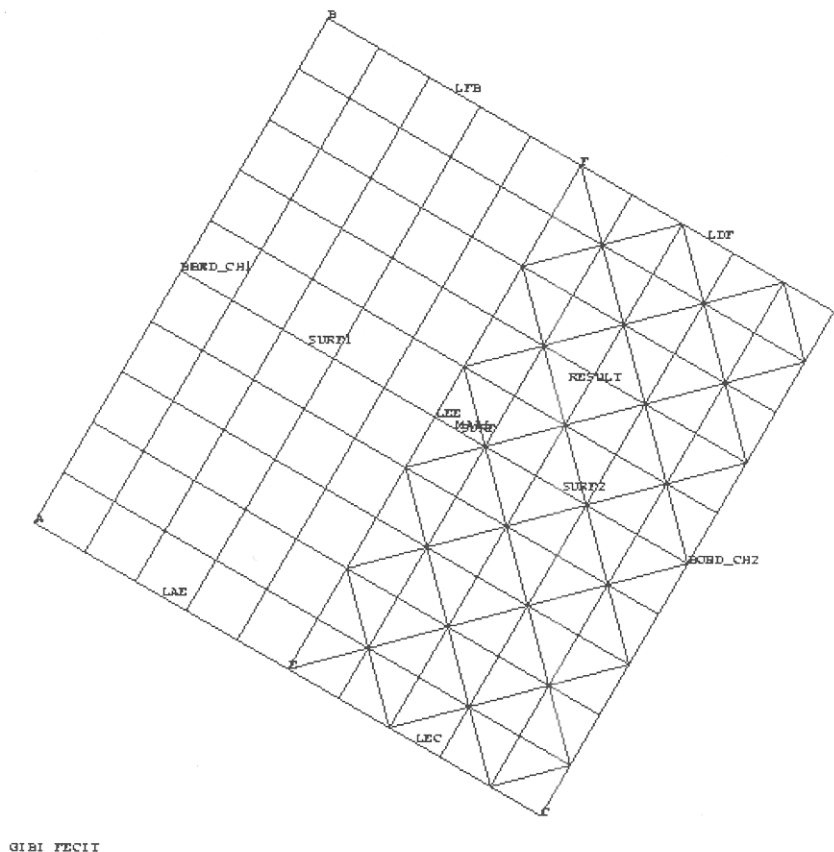
2.4 Bibliographical references

- 1) GAY D: "Composite materials"; 3^{ème} edition, Hermes

3 Modeling A

3.1 Characteristics of modeling

Modeling `C_PLAN`. The plate is turned from -30 degrees around Z , i.e. the axis X total is colinéaire with the axis of orthotropism L . Boundary conditions and loadings, to apply in the reference mark (A, x, y) bound to the plate, are thus projected on the total reference mark (A, X, Y) (use of `LIAISON_DDL` in B).



3.2 Characteristics of the grid

Many nodes: 391

Many meshes and types: 50 QUAD8, 100 TRIA6

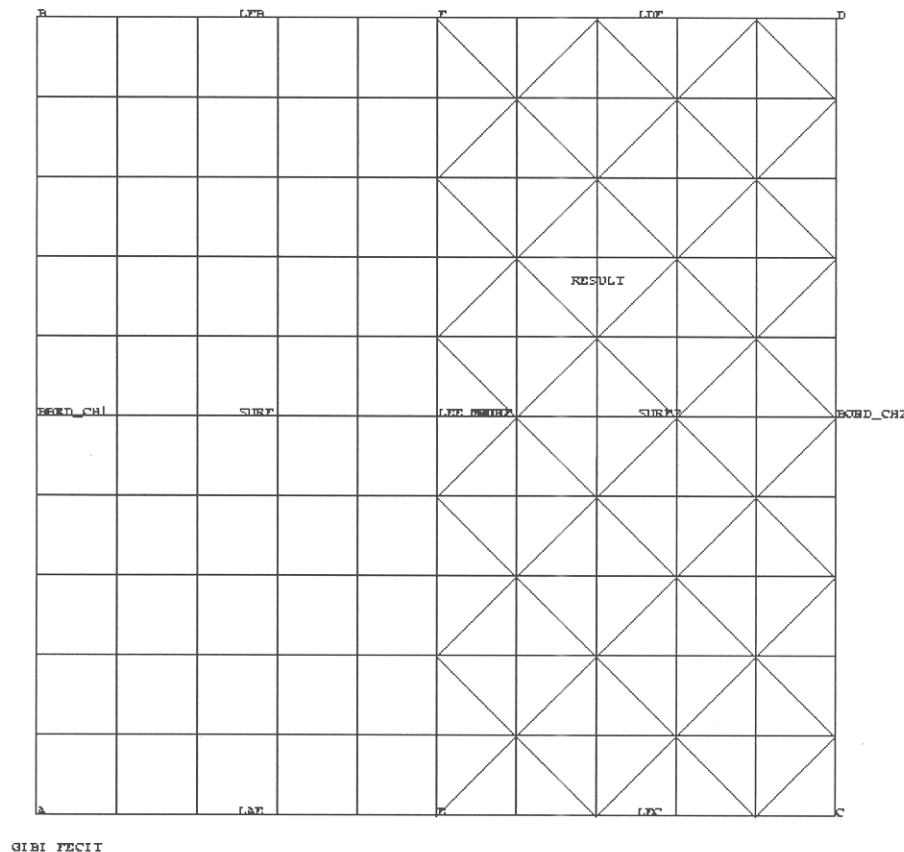
3.3 Values tested

Value	Identification	Reference
$U_x(c) = U_x(D)$	$DX(C)$	$5,917 \cdot 10^{-7}$
$U_y(B)$	$DY(B)$	$-2,292 \cdot 10^{-7}$
$U_y(C)$	$DY(C)$	$-5,028 \cdot 10^{-7}$
$U_y(D)$	$DY(D)$	$-7,319 \cdot 10^{-7}$
σ_{LL}	$SIXX$ (any point)	7500
σ_{TT}	$SIYY$ (any point)	2500
σ_{LL}	$SIXY$ (any point)	4300.127

4 Modeling B

4.1 Characteristics of modeling

Modeling C_PLAN. The plate is parallel to the total axes, i.e. the axis X total is colinéaire with the axis x . It is thus the axis of orthotropism L who is to be directed (using the keyword SOLID MASS of AFFE_CARA_ELEM).



4.2 Characteristics of the grid

Many nodes: 391

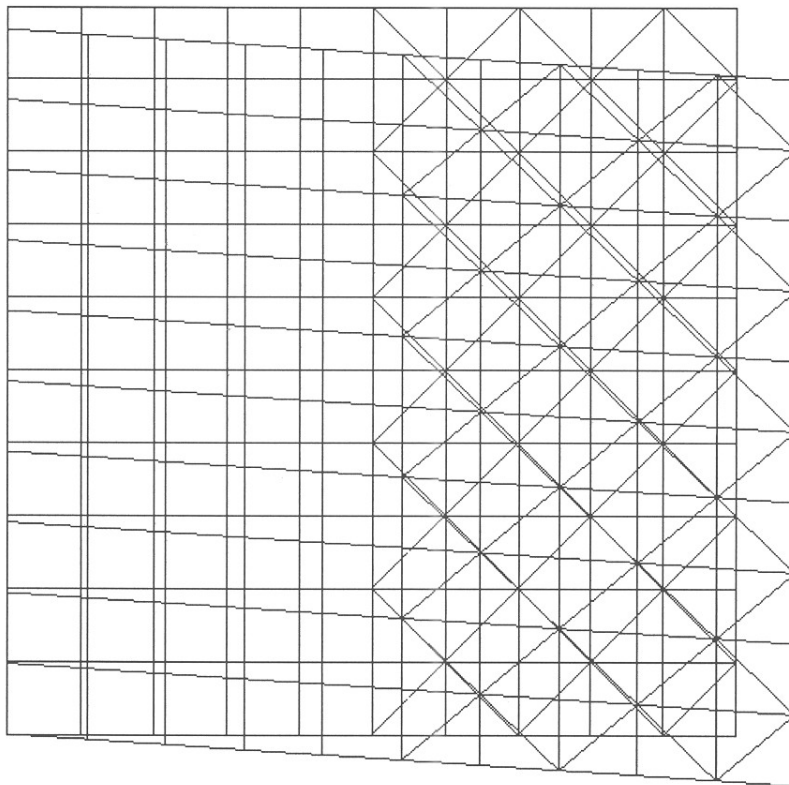
Many meshes and types: 50 QUAD8, 100 TRIA6

4.3 Values tested

Value	Identification	Reference
$U_x(c) = U_x(D)$	$DX(C)$	$5,917 \cdot 10^{-7}$
$U_y(B)$	$DY(B)$	$-2,292 \cdot 10^{-7}$
$U_y(C)$	$DY(C)$	$-5,028 \cdot 10^{-7}$
$U_y(D)$	$DY(D)$	$-7,319 \cdot 10^{-7}$
Σ_{LL}	$SIXX$ (any point)	7500
Σ_{TT}	$SIYY$ (any point)	2500
Σ_{LL}	$SIXY$ (any point)	4300.127

4.4 Remarks

Pace of the deformation: nonsymmetrical because of the orthotropism.



AMPLITUDE
0.
1.37E+05

GIBI FECH

5 Modeling C

5.1 Characteristics of modeling

Modeling COQUE_3D. The plate is parallel to the total axes, i.e. the axis X total is colinéaire with the axis x . It is thus the axis of orthotropism L who is to be directed (using the keyword SOLID MASS of AFFE_CARA_ELEM). The grid is identical to that of modeling B.

5.2 Characteristics of the grid

Many nodes: 541

Many meshes and types: 50 QUAD9, 100 TRIA7

5.3 Values tested

Value	Identification	Reference
$U_x(c) = U_x(D)$	$DX(C)$	$5,917 \cdot 10^{-7}$
$U_y(B)$	$DY(B)$	$-2,292 \cdot 10^{-7}$
$U_y(C)$	$DY(C)$	$-5,028 \cdot 10^{-7}$
$U_y(D)$	$DY(D)$	$-7,319 \cdot 10^{-7}$
Σ_{LL}	$SIXX$ (any point)	7500
Σ_{TT}	$SIYY$ (any point)	2500
Σ_{LL}	$SIXY$ (any point)	4300.127

6 Modeling D

6.1 Characteristics of modeling

Modeling COQUE_3D in great displacements. The plate is parallel to the total axes, it is - with - to say that the axis X total is colinéaire with the axis x . It is thus the axis of orthotropism L who is to be directed (using the keyword SOLID MASS of AFFE_CARA_ELEM). The grid is identical to that of modeling B.

6.2 Characteristics of the grid

Many nodes: 541

Many meshes and types: 50 QUAD9, 100 TRIA7

6.3 Features tested

6.4 Values tested

Value	Identification	Reference
$U_x(c) = U_x(D)$	$DX(C)$	$5,917 \cdot 10^{-7}$
$U_y(B)$	$DY(B)$	$-2,292 \cdot 10^{-7}$
$U_y(C)$	$DY(C)$	$-5,028 \cdot 10^{-7}$
$U_y(D)$	$DY(D)$	$-7,319 \cdot 10^{-7}$
σ_{LL}	$SIXX$ (any point)	7500
σ_{TT}	$SIYY$ (any point)	2500
σ_{LL}	$SIXY$ (any point)	4300.127

7 Summary of the results

The results of four modelings are very close to the analytical solution: to the maximum 0.015% of variation for 4 modelings.

This test thus validates the taking into account of orthotropic elasticity.