SSLL404 - Buckling of an arch

Summary
The scope of application of this test is the analysis of stability of the structures. The studied structure is an arch bent by moments applied at the two ends; it is modelled by elements of right beams. The goal is to calculate the breaking values of the moments.

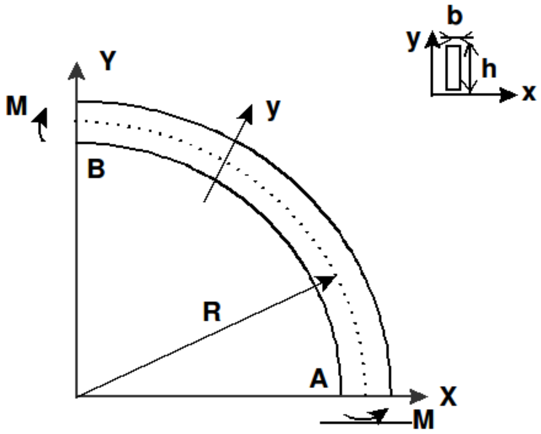
The interest of this test lies in the following aspects:

- calculation of a geometrical matrix of rigidity for the elements POU_D_E.
- test of the modal methods of CALC_MODES in stability
- presence of close eigenvalues

The calculated clean loads are compared with values obtained analytically for a model of beam of Euler-Bernoulli.
In this test, the option is also validated OPTION_INV=' RAYLEIGH' (under SOLVEUR_MODAL) order CALC_MODES.

1 Problem of reference

1.1 Geometry



Radius of curvature	$R=0.3\,m$
Height of the profile	$h=0.015\,m$
Width of the profile	$b=0.002\,m$
Section	$S=bh$
the 1st inertia of inflection	$I_X=bh^3/12$
the 2nd inertia of inflection	$I_Y=hb^3/12$
Inertia of torsion	$J=hb^3/3$

1.2 Properties of materials

Young modulus	$E=7.E\,10\,N/m^2$
Poisson's ratio	$\nu=0.3$
Modulus of rigidity	$G=E/2(1+\nu)$

1.3 Boundary conditions and loading

The beam bi--is supported. One prevents the torsion of the section at the ends A and B. To respect the assumptions of the ideal model taken as reference, it is important that the moment is constant and that the normal effort is null along the beam. This is why free displacement is left u according to X at the point B. The boundary conditions are:

- At the point A : $u=v=w=0$; $\Phi_Y=0$
- At the point B : $v=w=0$; $\Phi_X=0$

The initial state of stress which makes it possible to carry out the analysis of stability is obtained by imposing one bending moment around the axis Z, at the points A and B : $M=1\,Nm$

1.4 Initial conditions

Without object in static analysis of stability.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is obtained analytically for a beam of Euler-Bernoulli. The theoretical aspects are developed in the reference [bib1].
By using the notations of the paragraph [§1], the breaking values are given by the expression:

$$M_{CR}=-\frac{EI_x+GJ}{2\mathrm{R}}\pm\sqrt{\left(\frac{EI_x-GJ}{2\mathrm{R}}\right)^2+4\mathrm{n}^2\frac{EI_x GJ}{R^2}}\quad n=1,2,3,\ldots$$

The plus sign corresponds to positive moments such as they are indicated on the figure of [§1.1].

2.2 Results of reference

The first 5 critical loads are classified by order of increasing module.

Mode	Moment criticizes (Nm)
1	2.86074
2	8.63207
3	– 8.78382
4	14.4147
5	– 14.5551

With *Code_Aster* , the opposites of these critical loads are found (what is logical compared to the formulation of the problem to solve).

2.3 Uncertainty on the solution

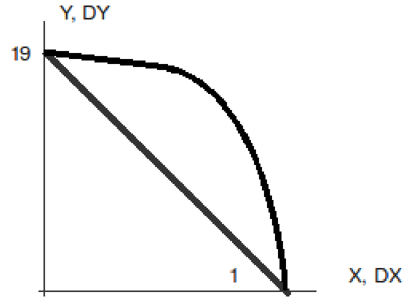
Analytical solution

2.4 Bibliographical references

[1] TIMOSHENKO Stephen P., MANAGES James Mr., Theory of Elastic Stability, McGraw-Hill, International Edition, 1963, pp. 313-318.

3 Modeling A

3.1 Characteristics of modeling



The arch is with a grid by means of elements of right beam of type `POU_D_E`.
Boundary conditions:

- At the point A such as $X=R$, $Y=0$: $DX=DY=DZ=0$ and $RY=0$
- At the point B such as $X=0$, $Y=R$: $DY=DZ=0$ and $RX=0$

For the static analysis, unit moments around Z are defined with nodes 1 and 19.

3.2 Characteristics of the grid

Many nodes: 19
Many meshs: 18 `POU_D_E`

3.3 Sizes tested and results

Critical load

3.3.1 `CALC_MODES` with `SOLVEUR_MODAL=_F (METHOD = 'SORENSEN')`

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3,823
2	– 8.63207	– 8.30613	3,776
3	8.78382	8.39554	4,420
4	– 14.4147	– 13.93216	3,348
5	14.5551	14.01104	3,738

3.3.2 `CALC_MODES` with `OPTION = 'NEAR'`

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3,823
2	– 8.63207	– 8.30613	3,776
3	8.78382	8.39554	4,420
4	– 14.4147	– 13.93216	3,348
5	14.5551	14.01104	3,738

3.3.3 CALC_MODES with OPTION = 'SEPARATE'

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3,823
2	– 8.63207	– 8.30613	3,776
3	8.78382	8.39554	4,420
4	– 14.4147	– 13.93216	3,348
5	14.5551	14.01104	3,738

3.3.4 CALC_MODES with OPTION = 'ADJUSTS'

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3,823
2	– 8.63207	– 8.30613	3,776
3	8.78382	8.39554	4,420
4	– 14.4147	– 13.93216	3,348
5	14.5551	14.01104	3,738

4 Summary of the results

Methods of Sorensen and the iterations opposite (`OPTION=' PROCHE'` or `'SEPARE'` or `'ADJUSTS'`) give identical and satisfactory results since the maximum change with the analytical solution is lower than 4.5% . It is pointed out that the analytical solution takes into account the curve of the structure. Elements `MEPOUCT` could not be used in this test because the calculation of the geometrical matrix of rigidity is not available for this kind of element.