SSLL403 - Buckling of a beam under the effect of its actual weight

Summary:
This test makes it possible to validate in linear elasticity the loading due to the forces of gravity for a modeling of type right beam of Euler (POU_D_E). It also allows the implementation and the validation of the calculation of the geometrical matrix of rigidity.

The reference solution is analytical and the results considered to be satisfactory.

1 Problem of reference

1.1 Geometry

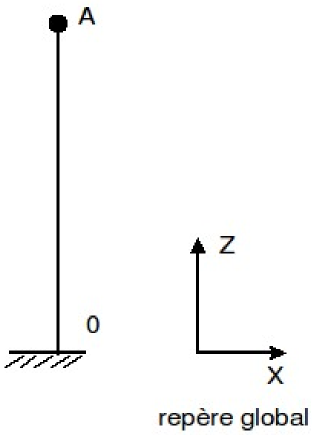

Figure 1.1-a : Vertical beam.

Rectangular section: $H_y = 0.01\,m$, $H_z = 0.01\,m$
Length: $L = 1\,m$

1.2 Properties of materials

Young modulus: $E = 2.10^{11}\,Pa$
Poisson's ratio: $\nu = 0{,}3$
Density: $\rho = 7800\,kg/m^3$

1.3 Boundary conditions and loading

Boundary condition:
Embedded end (0) : $DX = DY = DZ = DRX = DRY = DRZ = 0$.

Loading:
Force of gravity: p weight per unit of length with $g = (0, 0, -9.81)$ (given in total reference mark).

2 Reference solutions

2.1 Method of calculating used for the reference solutions

In local reference mark, x along the axis OA beam, bending moment, with the X-coordinate x , has as an expression:

$$M_{F_y}(x)=p\int_x^L\left[v(\xi)-v(x)\right]d\xi \,.$$

The arrow $v(x)$ satisfied thus the equation:

$$E\ I_z=\frac{d^2 v}{dx^2}=p\int_x^L\left[v(\xi)-v(x)\right]d\xi=-p\left[\int_x^L v(\xi)d\xi+(L-x)v(x)\right]$$

By deriving the two members, one obtains the differential equation:

$$\frac{d^3 v}{dx^3}+\frac{p}{E\ I_z}(L-x)\frac{dv}{dx}=0$$

The function $v'(x)=\frac{dv}{dx}$ satisfied the linear and homogeneous differential equation with the second order:

$$\frac{d^{2v'}}{dx^2}+\frac{p}{E\ I_z}(L-x)v'=0\ ,$$

who can be solved using the functions of Bessel. One finds the value of the linear weight then criticizes equalizes with:

$$p_c=7{,}837\frac{E\ I_z}{L^3}\ .$$

The analytical solution gives numerically:

$$p_c=7{,}837\ \ 2\ \ 10^{11}\cdot\frac{10^{-8}}{12}=1{,}3061667\ \ 10^3\ .$$

2.2 Results of reference

The value criticizes multiplier λ : $\lambda_c=\frac{P_c}{\rho S g}$

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

[1] Report n° 2314/A of the Institute Aerotechnics "Proposal and realization for new cases tests missing with the validation of the beams *Aster* "

3 Modeling A

3.1 Characteristics of modeling

The model is composed of 10 elements right beam of Euler.

3.2 Characteristics of the grid

It consists of 10 elements POU_D_E.

3.3 Sizes tested and results

Eigenvalue of the system ($K+\lambda K_G$) $X=0$:

	Reference	*Aster*	Variation %
λ	– 170,701	– 170.0005	– 0,408

3.4 Notice

Since $p_c=\lambda\rho S g$, ($\rho S g$ represent linear prestressing), we have like critical loading: $p_c=1300{,}84\,N.m^{-1}$.

4 Summary of the results

The results are very close to the analytical solution (variation: $0{,}4\%$ for 10 elements). This variation is function of the smoothness of discretization being given the assumptions used for geometrical rigidity (cf [R3.08.01]). This thus validates this kind of loading for the buckling of Euler.