SSLL402 - Dynamometric ring

Summary:
This test makes it possible to check in linear elasticity the calculation of the interior efforts and the constraints on a curved beam.

A modeling makes it possible to test the curved elements of Timoshenko (POU_C_T).

The reference solution is analytical and the got results are of very good quality.

1 Problem of reference

1.1 Geometry

Circular ring

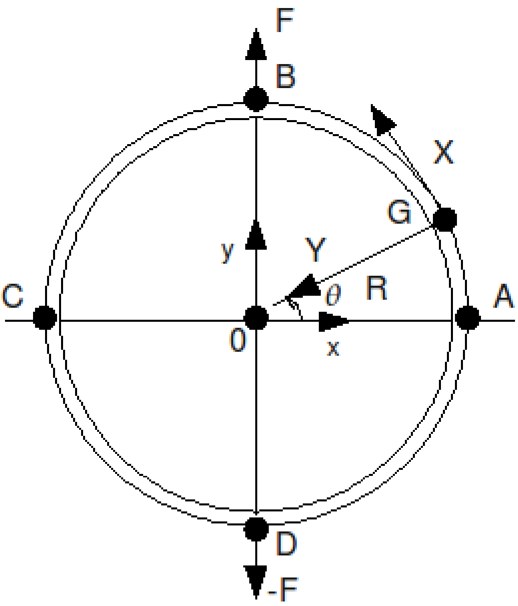

Figure 1-a : Circular ring.

$R=2\,m$

The section (full) is a circle of radius $0{,}01\,m$.

1.2 Properties of materials

Young modulus: $E=2.10^{11}\,Pa$

Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loading

Boundary condition:

$DX=DY=DZ=DRX=0$ on the point A

$DY=DZ=0$ on the point C

Loading:

on B , $F=1\,N$,

on D , $F=-1\,N$.

2 Reference solutions

2.1 Method of calculating used for the reference solutions: analytical

On the section (A,B) $\left(0<\theta<\frac{\pi}{2}\right)$, we have:

$$N=\frac{F}{2}\cos\theta\ ,\ Vy=-\frac{F}{2}\sin\theta\ ,\ M_z=-\frac{FR}{2}(1-\cos\theta)+\Gamma\ .$$

On the section $(B,C)\left(\frac{\pi}{2}<\theta<\pi\right)$, we have:

$$N=-\frac{F}{2}\cos\theta\ ,\ Vy=\frac{F}{2}\sin\theta\ ,\ M_z=-\frac{FR}{2}(1+\cos\theta)+\Gamma\ .$$

By the use of the law of behavior connecting M with the rotation of the normal and given that the latter is worthless in A and in B, we have:

$$\int_0^{\frac{\pi}{2}} M\,d\theta=0\ ,$$

from where: $\Gamma\ =\ M_A\ =\ M_C\ =\frac{\pi-2}{2\pi}F\ R$

2.2 Results of reference

Interior efforts for $\theta=0°$ and $90°$.

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

1. Report n° 2314/A of the Institute Aerotechnics “Proposal and realization for new cases tests missing with the validation of the beams *Aster* ”

3 Modeling A

3.1 Characteristics of modeling

The model is composed of 4 elements curved beam of Timoshenko.

3.2 Characteristics of the grid

It consists of 4 elements POU_C_T.

3.3 Grandeus tested and results

3.3.1 Interior effort with $\theta=0°$

	Type of Reference	Reference	Tolerance
N	ANALYTICAL	5.000E-01	1,00E-003 %
Vy	ANALYTICAL	0.0000	1,00E-005 *
MFz	ANALYTICAL	3.6338E-01	1,00E-003 %

* absolute Deviation

3.3.2 Interior effort with $\theta=90°$

	Type of Reference	Reference	Tolerance
N	ANALYTICAL	0.0000	1,00E-005 *
Vy	ANALYTICAL	– 5.0000E-01	1,00E-003 %
MFz	ANALYTICAL	– 6.3662E-01	1,00E-003 %

*Ecart absolute

3.3.3 Constraint with $\theta=0°$

	Type of Reference	Reference	Tolerance
$SIXXMAX$	ANALYTICAL	4.6426E+05	1,00E-003 %

* absolute Deviation

3.3.4 Constraint with $\theta=90°$

	Type of Reference	Reference	Tolerance
$SIXXMIN$	ANALYTICAL	– 8.1056E+05	1,00E-003 %

3.4 Remarks

Symmetry compared to the axis (A, C) imply the nullity of the shearing action T in A and in C.

Following balance Oy half-ring (A, B, C) impose in A and in C a normal effort equal to $\frac{F}{2}$.

Symmetry compared to the axis (B, D) imply that the moments in A and in C are equal in absolute value and contrary direction.

4 Summary of the results

The results agree with the analytical solution and make it possible to validate the calculation of the internal efforts (EFGE_ELNO) and of the constraints (SIPM_ELNO) by the curved elements of beams (POU_C_T).