

## SSLL111 - Static response of a beam reinforced concrete (section in T) to linear behavior

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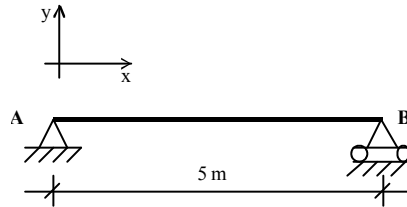
### Summary:

The problem consists in analyzing the answer of a reinforced concrete beam via a modeling multifibre beam. This test corresponds to a static analysis of a beam having a linear behavior. Three successive loading cases are tested: a specific force, the actual weight and a rise in temperature. For the first loading case, two grids of the section, one coarse and the other finer are tested.

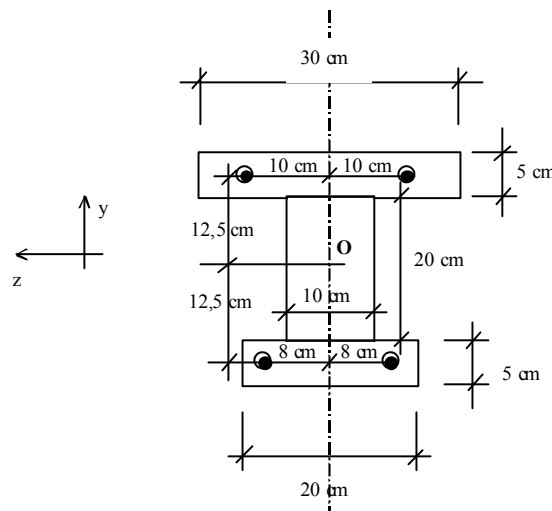
## 1 Problem of reference

### 1.1 Geometry

Beam in inflection three points, defined by:



With a section in double  $T$  :



On this diagram,  $O$  is located at middle height of the section.

The total section of higher steels is  $3 \cdot 10^{-4} m^2$  and that of lower steels is  $4 \cdot 10^{-4} m^2$ .

### 1.2 Material properties

- concrete:  $E = 2 \cdot 10^{10} Pa$  ;  $\nu = 0.2$  ;  $\rho = 2400 kg \cdot m^{-3}$  ;  $\alpha = 10^{-5} K^{-1}$
- steel :  $E = 2,1 \cdot 10^{11} Pa$  ;  $\nu = 0.33$  ;  $\rho = 7800 kg \cdot m^{-3}$  ;  $\alpha = 10^{-5} K^{-1}$

### 1.3 Boundary conditions

Simple support in  $B$  :  $dy = 0$

Support "doubles" in  $A$  :  $dx = dy = dz = 0$  just as  $rx = ry = 0$ .

### 1.4 Loadings

Three loading cases are tested successively:

- Loading 1: effort concentrated in the mediums of the beam,  $F = 10000 N$
- Loading 2: actual weight of the beam,  $g = 9,8 m \cdot s^{-2}$
- Loading 3: homogeneous heating of the beam  $\Delta T = 100 K$

## 2 Reference solution

Calculations of reference are carried out starting from a simple elastic design in RdM.

### 2.1 Elastic center

In pure bending, for an elastic behavior, the neutral axis passes by the elastic center (barycentre of the sections balanced by the modules of materials):

$$C \text{ such as } \int_S E \overrightarrow{CM} dS = \vec{0}$$

One determines initially the position of the centres of gravity of the concrete alone  $G_b$  and of steel alone  $G_a$  compared to the point  $O$ .

$$y_{G_b} = \frac{0.125 \times 0.3 \times 0.05 - 0.125 \times 0.2 \times 0.05}{0.2 \times 0.05 + 0.1 \times 0.2 + 0.3 \times 0.05} = 1.38888 \cdot 10^{-2} \text{ m}$$

$$y_{G_a} = \frac{0.125 \times 3 - 0.125 \times 4}{3 + 4} = -1.78571 \cdot 10^{-2} \text{ m}$$

$$z_{G_a} = z_{G_b} = 0 \text{ m}$$

One can then determine the position compared to  $O$  elastic center  $C$ .

$$\overrightarrow{OC} = \frac{E_a S_a \overrightarrow{OG_a} + E_b S_b \overrightarrow{OG_b}}{E_a S_a + E_b S_b}$$

The concrete section  $S_b$  is  $0,045 \text{ m}^2$  and the steel section  $S_a$  is  $7 \cdot 10^{-4} \text{ m}^2$ . The Young modulus of the concrete is  $2 \cdot 10^{10} \text{ MPa}$  and that of steel  $21 \cdot 10^{10} \text{ MPa}$ . One thus has

$$y_c = \frac{2 \times 0.045 \times 1.38888 - 21 \times 7 \cdot 10^{-4} \times 1.78571}{2 \times 0.045 + 21 \times 7 \cdot 10^{-4}} = 0.94317 \cdot 10^{-2} \text{ m}$$

$$z_c = 0 \text{ m}$$

### 2.2 Quadratic moments

The quadratic moments of the rectangular concrete sections are calculated by the following formula:

$$\frac{bh^3}{12} + b \times h \times d^2$$

Where, B represents the width, H the height and D the distance from the centre of gravity of the section compared to the axis for which one calculates the moment.

One then obtains the quadratic moment of the concrete section compared to the axis  $z$  passing by the elastic center:

$$I_{\text{béton}} = \frac{0.3 \times 0.05^3}{12} + (0.3 \times 0.05) \cdot (0.125 - 0.94317 \cdot 10^{-2})^2 + \frac{0.1 \times 0.2^3}{12} + (0.1 \times 0.2) \cdot (0.94317 \cdot 10^{-2})^2$$

$$+ \frac{0.2 \times 0.05^3}{12} + (0.2 \times 0.05) \cdot (0.125 + 0.94317 \cdot 10^{-2})^2 = 0.4547 \cdot 10^{-3} \text{ m}^4$$

Inertias of steels are calculated by the following formula:

$$\frac{\pi \phi^4}{64} + S \times d^2 \approx S \times d^2$$

Where,  $\phi$  represent the diameter of steel,  $S$  the section of steel and  $D$  the distance from the centre of gravity of the section compared to the axis for which one calculates the moment. The diameter of steels being small, one neglects the first term.

One then obtains the quadratic moment of the steel sections compared to the axis  $z$  passing by the elastic center:

$$3 \cdot 10^{-4} \times (0.125 - 0.94317 \cdot 10^{-2})^2 + 4 \cdot 10^{-4} \times (0.125 + 0.94317 \cdot 10^{-2})^2 = 0.1124 \cdot 10^{-4} \text{ m}^4$$

For the complete section of the beam, the quadratic moment balanced by the Young moduli of materials is:

$$EI = 2 \cdot 10^{10} \times 0.4547 \cdot 10^{-3} + 21 \cdot 10^{10} \times 0.1124 \cdot 10^{-4} = 11.4544 \cdot 10^6 \text{ Pa} \cdot \text{m}^4$$

## 2.3 Loading case 1

In the case of load 1 (loading concentrated in the middle of the beam), the arrow is calculated by the formula of following RDM:

$$f = \frac{F \times l^3}{48 EI}$$

What gives the arrow:

$$f = \frac{10000 \times 5^3}{48 \times 11.4544 \cdot 10^6} = 2.2735 \cdot 10^{-3} \text{ m}$$

One can also calculate the following generalized efforts:

- the shearing action at the beginning of the beam (left left) is worth  $\frac{F}{2} = 5000 \text{ N}$ ,
- the bending moment in the middle of the beam is worth:  $\frac{F \times l}{4} = 1.25 \cdot 10^4 \text{ N.m}$ .

## 2.4 Loading case 2

In the case of load 2 (actual weight of the beam), the arrow is calculated by the formula of following resistance of materials:

$$f = \frac{5 \times p \times l^4}{384 EI}$$

where  $p$  is the linear load due to the weight of materials:

$$p = g(\rho_a S_a + \rho_b S_b) = 9.8 \times (2800 \times 7 \cdot 10^{-4} + 2400 \times 0.045) = 1111.9 \text{ N.m}^{-1}$$

What gives the arrow:

$$f = \frac{5 \times 1111.9 \times 5^4}{384 \times 11.4544 \cdot 10^6} = 7.9 \cdot 10^{-4} \text{ m}$$

## 2.5 Loading case 3

In the case of load 3 (homogeneous rise in temperature), the beam being isostatic and the dilation coefficients of the concrete and steel being identical, the solution is simple:

The generalized constraints and efforts are worthless.

The lengthening of the beam is:  $\Delta l = \alpha \times l \times \Delta T$

What gives with the values of our case:

$$\Delta l = 10^{-5} \times 5 \times 100 = 5 \cdot 10^{-3} \text{ m}$$

## 3 Modeling A

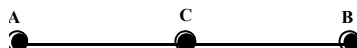
### 3.1 Characteristics of modeling

A modeling is used `POU_D_EM`.

### 3.2 Characteristics of the grid

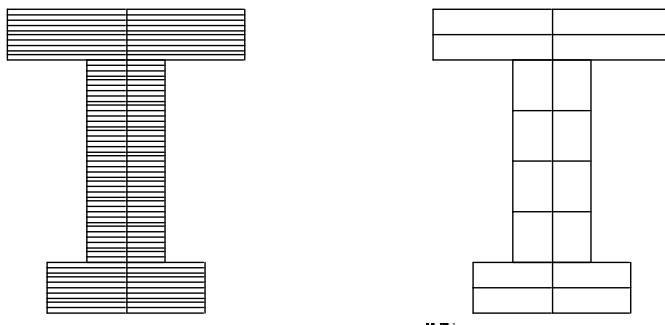
Longitudinal grid of the beam:

We have 3 nodes and two elements (`POU_D_EM`).



The concrete part of the cross section of the beam is with a grid (`DEFI_GEOM_FIBRE/ SECT`) while steels are given directly in the form of 4 specific fibres in `DEFI_GEOM_FIBRE/ FIBRE`.

Two grids of the concrete part are tested in the case of load 1. The fine grid consists of 120 fibres and the coarse grid consists of 16 fibres:



#### Note:

The problem being 2D, only one fibre in the width could seem sufficient (multi-layer), but that would result in having worthless terms in the matrix of rigidity (the own inertia of fibres not being taken into account) and with an error at the time of the solution of the system of equations.

## 3.3 Sizes tested and results

### 3.3.1 Loading case 1

	Reference
Arrow (fine grid)	$2.2735 \cdot 10^{-3}$
Arrow (coarse grid)	$2.2735 \cdot 10^{-3}$
Shearing action (supports $A$ )	5000
Bending moment (Medium)	$1.25 \cdot 10^4$

- 1) Calculations are carried out without taking into account the own inertia of each fibre. The results show that it is not nevertheless very useful to take account of it because the difference between a coarse grid and a fine grid is not obvious.  
The grid of the section does not need to be very fine to have precise results (in elasticity).
- 2) The option `EFGE_NOEU` used to calculate the efforts generalized with the nodes makes an average of the generalized efforts of all the elements connected to the node. In our case, we have 2 superimposed elements of beam (for the concrete, for steel), the calculated efforts are thus divided by 2.  
If one adds the values with efforts by element (`EFGE_ELNO`) element concrete and element steel, the theoretical values well are found.

**Note:**

*If one does a calculation of arrow while taking  $O$  (middle height) like reference axis instead of elastic center ( `COOR_AXE_POUTRE` ), the relative error on the arrow is of 0.2% (because here the elastic center is practically with middle height (see 1.2.1)).*

The distribution in volume of the component is also tested `SIXX` fields `SIEF_ELGA` and `SIGM_ELNO` as well as the volume of the beam concrete.

### 3.3.2 Loading case 2

	Reference
Arrow (fine grid)	$7.900 \cdot 10^{-4}$

### 3.3.3 Loading case 3

	Reference
Lengthening	$5.00 \cdot 10^{-3}$
Efforts	0.00

## 4 Modeling B

Modeling B takes again case 1 of modeling A.

Grid of the section ( Figure 4-1: Section of the beam for SSSL111b ) is however slightly different and the axes were changed: the inflection is done in the plan  $Z$  .

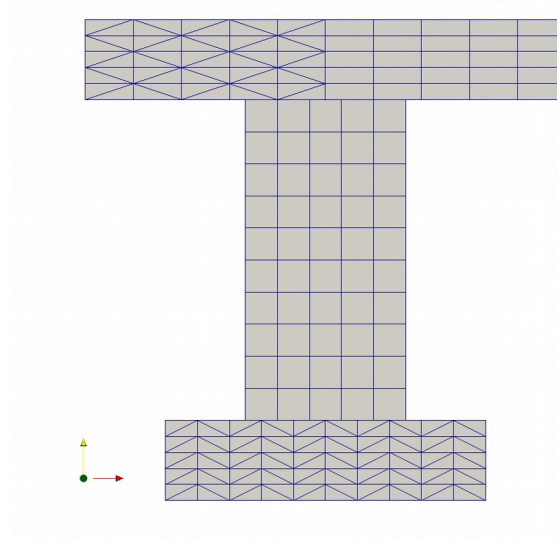


Figure 4-1: Section of the beam for SSSL111b

One wrote for this modeling only tests of not-regression, although one can refer to certain values of modeling A.



## 5 Summary of the results

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The got results are in concord with the results of reference.