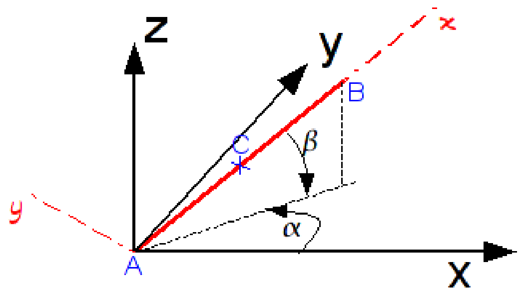
SSLL104 - Predeformations in a beam right-hand side

Summary:
This test validates the taking into account of predeformations in the elastic design of a right beam. The characteristics of calculation are:

- static analysis,
- linear behavior,
- linear model,
- only one modeling testing the elements POU_D_E, POU_D_T, POU_D_TG, POU_D_EM and POU_D_TGM
- the solution is analytical.

1 Problem of reference

1.1 Geometry



A beam AB length $l=100\,mm$ is located on trisecting trihedron (X,Y,Z) : the coordinates of the point B are: $B=\left(\frac{100}{\sqrt{3}},\frac{100}{\sqrt{3}},\frac{100}{\sqrt{3}}\right)$

One defines also a point C medium of A, B.

The local reference mark (A,x,y,z) results from the total reference mark (A,X,Y,Z) by the nautical angles $\begin{cases}\alpha=45°\\ \beta=-35.26°\, solution\, de \cos\beta=\sqrt{\frac{2}{3}}\end{cases}$

1.2 Material properties

The material is elastic linear.

Young modulus $E=1.0\,MPa$ (without influence on the result).

Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

Embedding in A : $DX=DY=DZ=DRX=DRY=DRZ=0$.

Loading: predeformation in the local reference mark (A,x,y,z)

- elongation according to x : $\epsilon_X^0=0.001$
- curve around y : $\chi_y^0=0.002$
- curve around z : $\chi_z^0=0.003$

1.4 Characteristics of the section of beam

All the characteristics (surface, inertias,…) are taken equal to 1. They are without influence on the result.

2 Reference solution

2.1 Method of calculating used for the reference solution

The solution is analytical. It is calculated in the local reference mark.
that is to say:

$U=(u,v,w,\theta_x,\theta_y,\theta_z)$ the displacement of the beam and $E=(\epsilon_x,\chi_y,\chi_z,\gamma_{xy},\gamma_{xz})$ generalized deformation.

That is to say the solution:

$$u=\alpha x \qquad v=\gamma\frac{x^2}{2} \qquad w=-\beta\frac{x^2}{2} \qquad \theta_x=0 \qquad \theta_y=\beta x \qquad \theta_z=\gamma x$$

then:

$$\epsilon_x=u_{,x}=\alpha \qquad \chi_y=\theta_{y,x}=\beta \qquad \chi_z=\theta_{z,x}=\gamma \qquad \gamma_{xy}=v_{,x}-\theta_z=0 \qquad \gamma_{xz}=w_{,x}+\theta_y=0$$

If one chooses $\alpha=\epsilon_X(=0.001)$, $\beta=\chi_y^0(=0.002)$, $\gamma=\chi_z^0(=0.003)$ then $E-E_{init}=0$ and Lbe efforts are worthless: balance is thus checked. In addition, the solution checks the boundary conditions of embedding in A. It is thus the solution of the posed problem.

2.2 Results of reference

The results expressed in the local reference mark are:

In B :

$$Dx=0.10\,mm\,;Dy=15.0\,mm\,;Dz=-10.0\,mm\,;DRx=0.0\,rd\,;DRy=0.2\,rd\,;DRz=0.30\,rd$$

In C :

$$Dx=0.05\,mm\,;Dy=3.75\,mm\,;Dz=-2.50\,mm\,;DRx=0.0\,rd\,;DRy=0.1\,rd\,;DRz=0.15\,rd$$

In the total reference mark, one finds at the points B and C :

$$DX(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}\left(-3\sqrt{6}+2\sqrt{2}\right)[mm] \qquad DX(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}\left(-3\sqrt{6}+2\sqrt{2}\right)[mm]$$

$$DY(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}\left(3\sqrt{6}+2\sqrt{2}\right)[mm] \qquad DY(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}\left(3\sqrt{6}+2\sqrt{2}\right)[mm]$$

$$DZ(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}\left(-4\sqrt{2}\right)[mm] \qquad DZ(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}\left(-4\sqrt{2}\right)[mm]$$

$$DRX(B)=\frac{1}{20}\left(-\sqrt{6}-2\sqrt{2}\right)[rd] \qquad DRX(C)=\frac{1}{40}\left(-\sqrt{6}-2\sqrt{2}\right)[rd]$$

$$DRY(B)=\frac{1}{20}\left(-\sqrt{6}+2\sqrt{2}\right)[rd] \qquad DRY(C)=\frac{1}{40}\left(-\sqrt{6}+2\sqrt{2}\right)[rd]$$

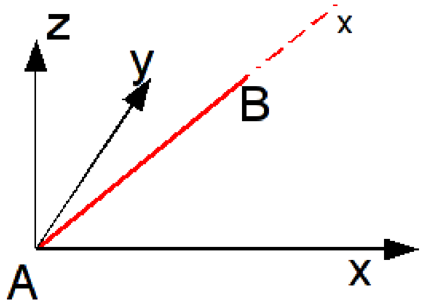
$$DRZ(B)=\frac{1}{20}\left(2\sqrt{6}\right)[rd] \qquad DRZ(C)=\frac{1}{40}\left(2\sqrt{6}\right)[rd]$$

2.3 Uncertainty on the solution

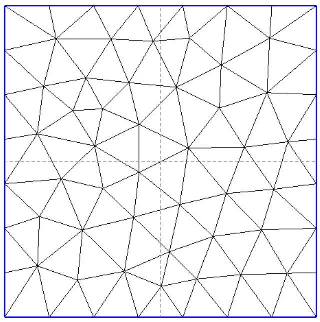
The solution is exact for the theory of the beams of Euler (or of Timoshenko because there is no shearing). Torsion not intervening, the solution is also valid for the elements `POU_D_TG` and `POU_D_TGM`.

3 Modeling A

3.1 Characteristics of modeling



- The segment AB is cut out in 10 of the same elements length (10.). (Only one element would be sufficient).
- 5 identical calculations are successively done on this grid with 4 different modelings:
  - with 10 elements POU_D_E
  - with 10 elements POU_D_T
  - with 10 elements POU_D_TG
  - with 10 elements POU_D_EM
  - with 10 elements POU_D_TGM



Grid of the section:
- 77 nodes
- 124 TRIA3

3.2 Characteristics of the grid

Many nodes: 11
Many meshs and types: 10 SEG2

3.3 Sizes tested and results

Modeling	Identification		Reference	% difference
POU_D_E	B	DX	– 6.4664E+00	< 1.0E-9
		DY	1.4747E+01	< 1.0E-9
		DZ	– 8.1072E+00	< 1.0E-9
		DRX	– 2.6390E-01	< 1.0E-9
		DRY MARTINI	1.8947E-02	< 1.0E-9
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-9
		DY	3.7011E+00	< 1.0E-9
		DZ	– 2.0124E+00	< 1.0E-9
		DRX	– 1.3195E-01	< 1.0E-9
		DRY MARTINI	9.4734E-03	< 1.0E-9
		DRZ	1.2247E-01	< 1.0E-9

Modeling	Identification		Reference	% difference
POU_D_T	B	DX	– 6.4664E+00	< 1.0E-9
		DY	1.4747E+01	< 1.0E-9
		DZ	– 8.1072E+00	< 1.0E-9
		DRX	– 2.6390E-01	< 1.0E-9
		DRY MARTINI	1.8947E-02	< 1.0E-9
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-9
		DY	3.7011E+00	< 1.0E-9
		DZ	– 2.0124E+00	< 1.0E-9
		DRX	– 1.3195E-01	< 1.0E-9
		DRY MARTINI	9.4734E-03	< 1.0E-9
		DRZ	1.2247E-01	< 1.0E-9

Modeling	Identification		Reference	% difference
POU_D_TG	B	DX	– 6.4664E+00	< 1.0E-9
		DY	1.4747E+01	< 1.0E-9
		DZ	– 8.1072E+00	< 1.0E-9
		DRX	– 2.6390E-01	< 1.0E-9
		DRY MARTINI	1.8947E-02	< 1.0E-9
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-9
		DY	3.7011E+00	< 1.0E-9
		DZ	– 2.0124E+00	< 1.0E-9
		DRX	– 1.3195E-01	< 1.0E-9
		DRY MARTINI	9.4734E-03	< 1.0E-9
		DRZ	1.2247E-01	< 1.0E-9

Modeling	Identification	Reference	% difference
POU_D_EM	B DX	– 6.4664E+00	< 1.0E-6
	DY	1.4747E+01	< 1.0E-5
	DZ	– 8.1072E+00	< 1.0E-5
	DRX	– 2.6390E-01	< 1.0E-5
	DRY MARTINI	1.8947E-02	< 1.0E-4
	DRZ	2.4495E-01	< 1.0E-5
	C DX	– 1.6022E+00	< 1.0E-6
	DY	3.7011E+00	< 1.0E-5
	DZ	– 2.0124E+00	< 1.0E-5
	DRX	– 1.3195E-01	< 1.0E-5
	DRY MARTINI	9.4734E-03	< 1.0E-4
	DRZ	1.2247E-01	< 1.0E-5

Modeling	Identification	Reference	% difference
POU_D_TGM	B DX	– 6.4664E+00	< 1.0E-8
	DY	1.4747E+01	< 1.0E-8
	DZ	– 8.1072E+00	< 1.0E-8
	DRX	– 2.6390E-01	< 1.0E-8
	DRY MARTINI	1.8947E-02	< 1.0E-8
	DRZ	2.4495E-01	< 1.0E-9
	C DX	– 1.6022E+00	< 1.0E-10
	DY	3.7011E+00	< 1.0E-8
	DZ	– 2.0124E+00	< 1.0E-8
	DRX	– 1.3195E-01	< 1.0E-8
	DRY MARTINI	9.4734E-03	< 1.0E-7
	DRZ	1.2247E-01	< 1.0E-8

4 Summary of the results

As one could expect it, the results are very precise. They validate the good taking into account of the predeformations in the elements of beam.

The test does not relate to the curved beams (POU_C_T) because one does not have reference solution. Moreover the option CHAR_MECA_EPSI_R is not available any more for this element.