SSLL102 - Fixed beam subjected to efforts unit

Summary:
This test allows a simple checking of calculations of right beams and hull $1D$ in linear mechanics of the structures static. The model is linear.

Modelings A, B, C, D, F, G, J and K make it possible to test the various types of elements of right beams in *Code_Aster*. For each modeling, one calculates simultaneously 3 beams of different sections: rectangle, circle, angle.

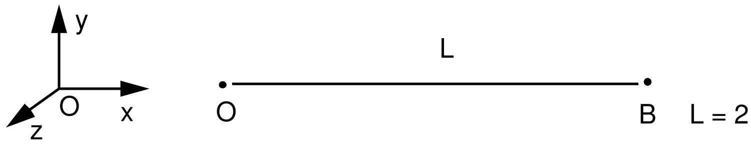
Modeling A makes it possible of more than test the change of reference mark: the beam is directed according to the trisecting one with the total reference mark.
Modeling E tests the loading distributed on voluminal edges of elements.
Modeling F corresponds to a loading distributed varying linearly with modeling POU_D_E.
Modeling G corresponds to a loading distributed varying linearly with modeling POU_D_TG.
Modeling H makes it possible to test the element of hull $1D$ (COQUE_C_PLAN) subjected to unit stresses.
Modeling I makes it possible to test a loading distributed varying linearly with modeling TUYAU_3M.

The values tested are the generalized displacements, efforts and the constraints.

1 Problem of reference

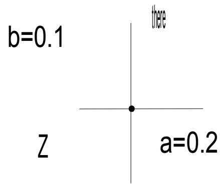
1.1 Geometry

Right beam length L, of direction x. Dimensions are expressed in meters, $[m]$.

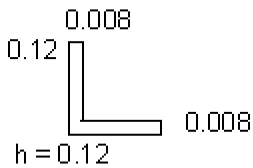


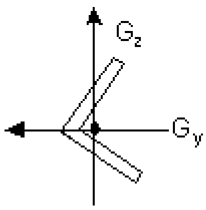
One calculates simultaneously 3 types of different cross sections:

1 rectangular section

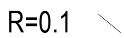


1 corner section with equal wings





1 circular section



1.2 Material properties

Young modulus: $E=2.10^{11} Pa$
Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loadings

Embedding in O
6 unit loadings in B :

$Fx=1$ $Mx=1$
$Fy=1$ $My=1$
$Fz=1$ $Mz=1$

1 loading combined inflection plus traction: $Fx=1$; $My=1$; $Mz=1$;
1 loading combined efforts cutting-edges plus torsion: $Fy=1$ $Fz=1$ $Mx=1$
1 loading distributed linear: $Fy=1000.x$ circular section (modelings F, G, I) (with simple support in A and B in this case)

1.4 Notation of the characteristics of cross sections

The geometrical characteristics of the cross sections are noted:

A	surface of the section
I_y, I_z	geometrical moments of inertia compared to the main axes of inertia of the section
JX	constant of torsion
ay, az	coefficients of shearing in the directions Gy and Gz
$A'_y=\frac{A}{ay}$ and $A'_z=\frac{A}{az}$	equivalent reduced surfaces
e_y, e_z	eccentricity of the center of torsion
JG	constant of warping

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution [bib1] and [bib2]: displacements in B

Simple traction	$u_x=\frac{F_x L}{E S}$		
Pure bending	$u_y=\frac{F_y L^3(4+\phi_y)}{12 E I_z}$	$\theta_z=\frac{L^2 F_y}{2 E I_z}$	$\phi_y=\frac{12 E I_y}{L^2 G A_y'}$
Pure bending	$u_z=\frac{F_z L^3(4+\phi_z)}{12 E I_y}$	$\theta_y=\frac{-L^2 F_z}{2 E I_y}$	$\phi_z=\frac{12 E I_z}{L^2 G A_z'}$
Torsion		$\theta_x=\frac{M_x L}{G J_x}$	
Pure inflection	$u_z=-\frac{M_y L^2}{2 E I_y}$	$\theta_y=\frac{M_y L}{E I_y}$	
Pure inflection	$u_y=\frac{M_z L^2}{2 E I_z}$	$\theta_z=\frac{M_z L}{E I_z}$	

Notice 1:

For the corner section, as the center of shearing is not confused with the centre of gravity $(e_y \neq 0)$ *, it is necessary to add the torque:* $M_x=F_z e_y$ *with the loading* $F_z=1$ *.*

This modifies displacement:

$$u_z=\frac{F_z L^3(4+\phi_z)}{12 E I_y}+\theta_x e_y \quad \theta_x=\frac{M_x L}{G J_x}$$

In the same way, the loading $M_x=1$ *involve a displacement* $u_z=\theta_x e_y$ *.*

Loading distributed linear:

$$u_y(x)=\frac{p x}{360 L E I}(3x^4-10L^2x^2+7L^4) \quad u_y^{max}=\frac{0.00652\, p L^4}{E I}$$

$$en\ x=0.519\, L$$

Notice 2:

With regard to modeling A, the beam is carried by the vector $e_1=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$ *. The other vectors of the local reference mark are:* $e_2=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$ and $e_3=\frac{1}{\sqrt{6}}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$

The components of the vector displacement in the total reference mark are obtained by:

$$u_G=\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}}\end{pmatrix} u_{local}$$

Generalized efforts and constraints in O :

$N(O)=F_x \qquad \sigma_{xx}=\frac{N}{S}$

$M_z(O)=T_y L \qquad T_y=F_y \qquad \sigma_{xx}(y)=\frac{M_z y}{I_z} \qquad \sigma_{xy}=\frac{T_y}{k_y S}$

$M_y(O)=-T_z L \qquad T_z(O)=F_z \qquad \sigma_{xx}(y)=\frac{-M_y z}{I_y} \qquad \sigma_{xz}=\frac{T_z}{k_z S}$

$M_x(0)=M_x(B) \qquad \sigma_{xy}=\sigma_{xz}=\frac{M_x R_T}{J_x}$

$M_y(0)=M_y(B) \qquad \sigma_{xx}(z)=\frac{M_y z}{I_y}$

$M_z(0)=M_z(B) \qquad \sigma_{xx}(y)=\frac{M_y y}{I_z}$

Loading distributed linear:

$$M_z(x)=\frac{-1000}{6}\left(L^2 x-x^3\right) \quad V_y(x)=\frac{1000\,L^2}{6}-\frac{1000\,x^2}{2} \quad \sigma_{xx}^{max}=\frac{M_z^{max} R}{I_z} \; en\; x=\frac{L\sqrt{3}}{3}$$

2.2 Results of reference

Displacement of the point B ,
Efforts generalized at the point O ,
Constraints of the point O .

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

1.J.L. BATOZ, G. DHATT: “Modeling of the structures by finite elements” - Volume 2 ED. HERMES.
2.N.D. PIKLEY: “Formulated for Stress, Stain & Structural Matrices” ED. John Wiley & Sounds.

3 Modeling A

3.1 Characteristics of modeling

2 elements POU_D_E $k_y = k_z = 1$ $\phi = 0$ by type of section

$S1$: Rectangular section modelled by SECTION: 'GENERAL'

$$A = 0.02 \quad Iy = 0.1666\text{E}-4 \quad Iz = 0.6666\text{E}-4 \quad J_x = 0.45776\text{E}-4$$
$$Ry = 0.1 \quad Rz = 0.05 \quad R_T = 0.0892632$$
$$\textit{Point de calcul des contraintes}$$

$S2$: Corner section

$$A = 1.856\text{E}-3 \quad I_y = 4.167339\text{E}-4 \quad I_z = 1.045547\text{E}-4$$
$$J_x = 03.9595\text{E}-8 \quad e_y = 41.012\text{E}-3 \quad e_z = 0.0$$

$S3$: Rectangular section modelled by SECTION: RECTANGLE

$Hy = 0.2$ $Hz = 0.1$

$S4$: Section CIRCLE $R = 0.1$

$$I_Y = I_Z = \frac{\pi R^4}{4} = \frac{\pi}{4} 10^{-4}$$

3.2 Characteristics of the grid

4×2 elements POU_D_E. The beam is directed according to the vector $(1,1,1)$.

3.3 Sizes tested and results

Loading case	Beam	Identification	Reference
$F_x = 1$	$S1 = S3$	$u_x(B)$	2.887E-10
		$\theta_{xx}(0)$	50.
	$S2$	$u_x(B)$	3.11E-9
	$S4$	$u_x(B)$	1.838E-10
		σ_{xx}	31.83
$F_y = 1$	$S1 = S3$	$u_y(B)$	+1.414E-7
		$\theta_z(B)$	1.225E-7
		$\sigma_{xx}(0)$	3000
	$S2$	$u_y(B)$	9.017E-8
	$S4$	$\sigma_{xx}(0)$	2546.479
$F_z = 1$	$S1 = S3$	$u_z(B)$	6.532E-7
		$\theta_y(B)$	-4.243E-7
		$\sigma_{xx}(0)$	6000
		$\sigma_{xz}(0)$	50
	$S2$	$u_z(B)$	9.279E-7
		$\theta_y(B)$	1.553E-5
		$\theta_x(B)$	1.555E-5
	$S4$	$u_z(B)$	1.386E-7
		$\theta_y(B)$	9th-8

		$\sigma_{xx}(0)$	2546.479
		$\sigma_{xz}(0)$	31,831
$M_x=1$	$S1=S3$	$\theta_x(B)$	3.279E-7
		$\sigma_{xy}=\sigma_{xz}(0)$	1950.0
	$S2$	$\theta_x(B)$	3.791E-4
		$u_z(B)$	2.199E-5
	$S4$	$\theta_x(B)$	9.556E-8
		$\sigma_{xy}=\sigma_{xz}(0)$	636.62
$M_y=1$	$S1=S3$	$u_z(B)$	-4.899E-7
		$\theta_y(B)$	4.243E-7
		$\sigma_{xx}(0)$	3000
	$S2$	$u_z(B)$	-1.959E-8
		$\theta_y(B)$	1.697E-8
	$S4$	$u_z(B)$	-1.04E-7
		$\theta_y(B)$	9.0E-8
		$\sigma_{xx}(0)$	1273.2395
$M_z=1$	$S1=S3$	$u_y(B)$	1.061E-7
		$\theta_z(B)$	1.225E-7
		$\sigma_{xx}(0)$	1500.0
	$S2$	$u_y(B)$	6.763E-8
		$\theta_z(B)$	7.809E-8
	$S4$	$u_y(B)$	9.0E-7
		$\sigma_z(B)$	1.04E-7
		$\sigma_{xx}(0)$	1273.2395
$M_y=1$ $M_z=1$ $F_x=1$	$S1=S3$	$\sigma_{xx} max(0)$	4550.0
		$\sigma_{xx}\left(\frac{a}{2,}\frac{b}{2}\right)$	1550.0
	$S4$	$\sigma_{xx} max(0)$	1832.4636
$F_y=1$ $F_z=1$ $M_x=1$	$S1, S3$	$\sigma_{xy}(0)$	2000.0
		$\sigma_{xz}(0)$	2000.0
		$\sigma_{xx} max(0)$	9000.0
	$S1, S3$	$\sigma_{xx}\left(\frac{a}{2,}\frac{b}{2}\right)$	-9000.0
	$S4$	$\sigma_{xx} max(0)$	3601.27
		$\sigma_{xy}(0)$	668,451

4 Modeling B

4.1 Characteristics of modeling

2 elements POU_D_T.
The coefficients of shearing are:
$S1$: Rectangular section

$$AY = AZ = 1.2 = \frac{1}{k_y}$$

$S2$: Corner section

$$AY = AZ = \frac{1}{0.358}$$

$S4$: Section CIRCLE

$$AY = AZ = \frac{10}{9}$$

4.2 Characteristics of the grid

4×2 elements POU_D_T

4.3 Sizes tested and results

One gives only the values which differ from modeling A (because of the taking into account of transverse shearing).

Loading	Section	Identification	Reference
$F_y = 1$	$S1 = S3$	$u_y(B)$	2.0156E-7
		$\sigma_{xy}(0)$	60.
	$S2$	$u_y(B)$	1.666552E-7
	$S4$	$u\,y(B)$	1.707308E-7
		$\sigma_{xy}(0)$	37.13615
$F_z = 1$	$S1, S3$	$u_z(B)$	8.0156E-7
		$\sigma_{xz}(0)$	60.
	$S2$	$u_z(B)$	1.17559754E-6
	$S4$	$u_z(B)$	1.707308E-7
		$\sigma_{xz}(0)$	37.13615
$F_y = 1$ $F_z = 1$ $M_x = 1$	$S4$	$\sigma_{xz}(0)$ $\sigma_{xy}(0)$	673.75592 673.75592

5 Modeling C

5.1 Characteristics of modeling

2 elements POU_D_TG.
Warping is not constrained.
The coefficients of shearing are identical to those of modeling B.

5.2 Characteristics of the grid

4×2 elements POU_D_TG

5.3 Sizes tested and results

Loading	Section	Identification	Reference
$F_y=1$	$S1=S3$	$u_y(B)$	2.0156E-7
		$\theta_{xy}(0)$	60.
	$S2$	$u_y(B)$	1.666552E-7
	$S4$	$u_y(B)$	1.70684E-7
		$\theta_{xy}(0)$	35.367765
$F_z=1$	$S1, S3$	$u_z(B)$	8.0156E-7
		$\theta_{xz}(0)$	60.
	$S2$	$u_z(B)$	1.17559754E-6
	$S4$	$u_z(B)$	1.70684E-7
		$\theta_{xz}(0)$	35.367765

5.4 Notice

Warping is not constrained. The results are thus identical to those of modeling B.

6 Modeling D

6.1 Characteristics of modeling

Elements `POU_D_TG`, constrained torsion

$$JG=\begin{cases}5.5556\mathrm{E}-8 \; pour\; S_1\\ 4.439822\mathrm{E}-11 \; pour\; S_2\end{cases}$$

in 0 $GRX=0$

6.2 Characteristics of the grid

•10 elements,
•refinement towards embedding.

6.3 Sizes tested and results

Same results as for modeling C, except those which relate to the effects of warping.

Loading	Section	Identification	Reference
$F_z=1$	$S2$	$\theta_x=DRX$	2.62034E-5
		$u_z=DZ$	1.14578E-6
		GRX	1.34652E-5
$M_x=1$	$S1$	$u_z=DZ$	5.52E-7
		GRX	2.84E-7
	$S2$	u_z	2.6203E-5
		θ_x	6.3892E-4
		GRX	3.28324E-4

6.4 Remarks

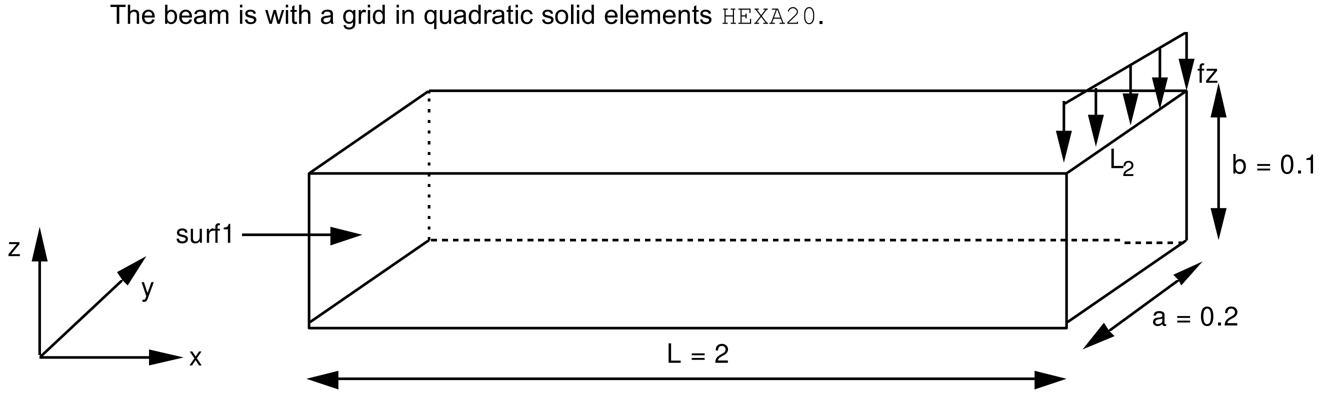
For θ_x the solution is (cf [bib1]):

$$\theta_x=\frac{M_x L}{G J_x}+\frac{M_x\left(1-e^{2\alpha L}-2e^{\alpha L}\right)}{\alpha^3 E J G\left(1+e^{2\alpha L}\right)} \quad \alpha^2=\frac{G J}{E J G}$$

7 Modeling E

7.1 Characteristics of modeling

The beam is with a grid in quadratic solid elements HEXA20.



The beam is embedded on the level of the section $surf1$. It is subjected to a unit shearing action which is modelled by a linear density of load fz applying to the 4 meshs SEG3 constituting the higher edge $L2$.

7.2 Characteristics of the grid

The beam is with a grid with 640 quadratic solid elements HEXA20.
The model comprises 3665 nodes.

7.3 Sizes tested and results

One tests the value of the arrow according to z node medium of the section where one applies the loading (node $N62$).

Identification	Reference	Aster	% difference
dz node $N62$	-8.0E-7	-7.9523E-7	-0,596

7.4 Remarks

The value of reference corresponds to the value given by the Resistance Of Materials.

8 Modeling F

8.1 Characteristics of modeling

The model is composed of 10 elements right beam of Euler. The section is circular full, of ray $0.1\mathrm{m}$.

8.2 Characteristics of the grid

It consists of 10 elements POU_D_E. The length of the beam is $L=6m$

8.3 Sizes tested and results

8.3.1 Interior efforts

	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ\ (2\sqrt{3})$	-1.3856E+04

8.3.2 Constraint

	Analytical results
$SIXX\ (2\sqrt{3})$	1.7642E+07

9 Modeling G

9.1 Characteristics of modeling

The model is composed of 10 elements right beam of Timoshenko with warping. The section is circular full, of ray 0.1m .

9.2 Characteristics of the grid

It consists of 10 elements `POU_D_TG`. The length of the beam is $L=6m$

9.3 Sizes tested and results

9.3.1 Interior efforts

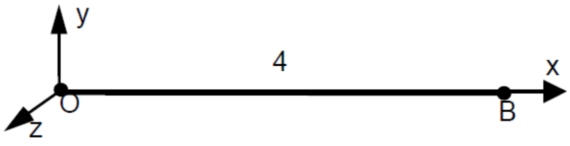
	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ\ (2\sqrt{3})$	-1.3856E+04

9.3.2 Displacement (marks with arrows near to the medium of the beam)

	Results *Aster* (not regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4

10 Modeling H

10.1 Characteristics of modeling



Modélisation COQUE_C_PLAN

- Section rectangulaire
- Conditions limites : Point O $u = v = \theta_z = 0$
- Chargement unitaire : Point B F_x , F_y et M_z

10.2 Characteristics of the grid

Many nodes: 9
Many meshs and types: 4 SEG3

10.3 Sizes tested and results

Loading case	Beam	Identification	Reference
$F_x=1$	*S1*	$u_x(B)$	5.E-10
		$\sigma_{xx}(0)$	5.0
$F_y=1$	*S1*	$u_y(B)$	2.E-7
		$\theta_z(B)$	1.5E-7
		$\sigma_{xx}(0)$	300.0
$M_z=1$	*S1*	$u_y(B)$	1.5E-7
		$\theta_z(B)$	1.5E-7
		$\sigma_{xx}(0)$	150.0

Loading case	Beam	Identification	Value *Aster*	% difference
$OMEGA=100$	*S1*	$u_x(B)$	0.0104	NON_REGRESSION
		$\sigma_{xx}(0)$	1.576E+8	NON_REGRESSION

10.4 Remarks

The width for modeling COQUE_C_PLAN is imposed on 1 in *Code_Aster*. Consequently, we multiplied by 0.1 the Young modulus to take account of the real width of the beam. This width of 1 modifies the inertia of the beam and consequently the value of the constraint σ_{xx} who is 10 times lower than the value of reference. Moreover, for displacements, the results differ from modeling A because of change of reference mark.

For the loading in rotation, one compares a calculation where the axis of rotation is confused with the origin with same calculation where the grid and the axis of rotation are relocated (to test the keyword CENTER).

The computed values quite equal and are tested in NON_REGRESSION.

11 Modeling I

11.1 Characteristics of modeling

The model is composed of 21 elements TUYAU_3M being pressed on meshs SEG4.
The effort distributed is imposed along the axis y. the inflection thus takes place around z.

11.2 Characteristics of the grid

It consists of 21 meshs SEG3. The length of the pipe is $L=6m$

11.3 Sizes tested and results

11.3.1 Displacements

	Analytical results
$D_y maxi$	9.38888E-03

11.3.2 Interior efforts

	Analytical results
$V_y(x=0)$	6.0000E+03
$V_y(x=L=6)$	-1.2000E+04
$MFZ 2\sqrt{3}$	-1.3856E+04

11.3.3 Constraints

It are calculated at the point of X-coordinate $x=\frac{L\sqrt{3}}{3}$ who corresponds to the maximum moment:

$$M_z(x)=\frac{-1000}{9\sqrt{(3)}}L^3=-13856.41\,N.m$$

For the angle 0 on the circumference of the pipe (the origin of the angles being the axis z), the constraints are worthless, and for angle 90, they are maximum:

$$\sigma_{xx}^{max}=\frac{M_z^{max}(R-e/2)}{I_z}=-4.87363\text{E}+07\,Pa$$

	Reference	Tolerance
$\sigma_{xx}(\alpha=0)$	0	0,10%
$\sigma_{xx}(\alpha=90)$	-4.87363E+07	1,00%
MFZ	-1.3856E+04	1.%

12 Modeling J

12.1 Characteristics of modeling

The model is composed of 2 elements POU_D_EM.
The loading is similar to that of modeling A (torque only)

12.2 Characteristics of the grid

It consists of 2 meshs SEG2. The length of the beam is $L=2m$

The beam is directed according to the vector $(1,1,1)$.

The section is rectangular, identical to that of modeling A.

12.3 Sizes tested and results

12.3.1 Displacement (rotation due to the torque)

	Results *Aster* (not regression)	**Tolerance (%)**
$DX=DY=DZ$	3.2792525E-07	1.E-6

13 Modeling K

13.1 Characteristics of modeling

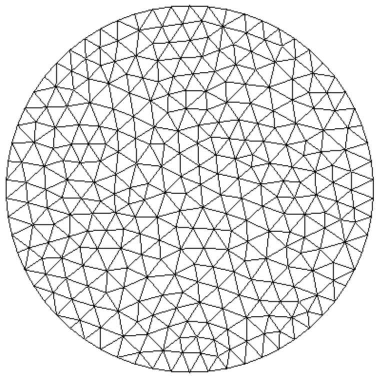
The model is composed of 10 elements POU_D_EM.
One applies a force distributed of $6000\mathrm{N}/m$ on all the beam.

13.2 Characteristics of the grid

It consists of 10 meshs SEG2. The length of the beam is $L=6m$

The grid of the section consists of:

- 373 nodes
- 62 SEG2
- 682 TRIA3



13.3 Sizes tested and results

13.3.1 Interior efforts

Interior efforts	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
MFZ	-1.3856E+04

13.3.2 Displacement (marks with arrows near to the medium of the beam)

	Results ***Aster*** (not regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4

14 Summary of the results

This test makes it possible to check the good performance of the elements simultaneously POU_D_E, POU_D_T and POU_D_TG on 3 types of different sections. The perfect coincidence of the results with the analytical solutions (RDM) is normal, and must always be observed, since the solution is contained in the functions of form of the elements.

Moreover, modeling E makes it possible to test the loading distributed on voluminal edges of elements. The variation with the analytical solution (RDM) is lower than 0.6% .

Modelings F, G, I and K make it possible to test the loading distributed (linear variation) for the elements of beam POU_D_E, POU_D_TG, POU_D_EM and elements of PIPE. The variation with the analytical solution (Resistance of Materials) is lower than 0.6% .

For modeling COQUE_C_PLAN the results are satisfactory (displacements and constraints) for the unit loadings of standard extension and inflection (imposed moment). For the loading of inflection (load imposed at an end) the error on displacement is weak and lower than 0.5% . It is more important on the constraint: 3.6% .