SHLL101 - Right beam. Harmonic analysis

Summary:

This two-dimensional problem consists in calculating the efforts present in a beam subjected to a traction or an inflection during a harmonic analysis. The reference solution is obtained starting from the discretized equations.

This test comprises two modelings.

For the first modeling, four requests are tested:

- force of traction,
- force of traction and material presenting a damping,
- flexural strength,
- material and flexural strength presenting a damping.

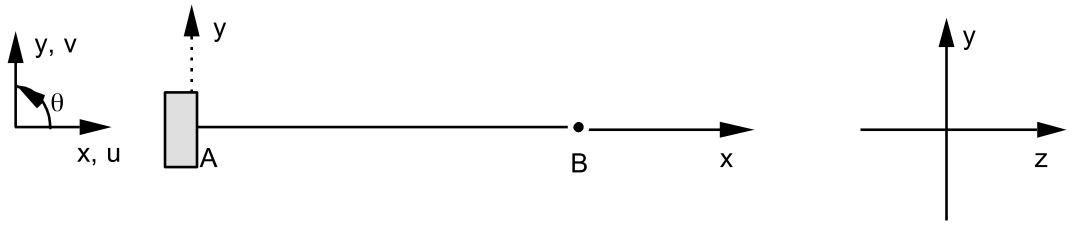
For the second modeling, two requests are tested:

- force of traction,
- force of traction and material presenting a damping.

The second modeling makes it possible to test the complex loadings imposed by the order `AFFE_CHAR_MECA_C`.

1 Problem of reference

1.1 Geometry



The geometrical characteristics of the beam constituting the mechanical model are the following ones:

Length: $L=10\,m$

Cross section	Surface	$IZ=IY$	JX
	$3.439\,10^{-3}\,m^2$	$1.377\,10^{-5}\,m^4$	$2.754\,10^{-5}\,m^4$

The coordinates (in meters) of the points characteristic of the beam are:

	A	B
x	0.	10.
y	0.	0.

1.2 Material properties

The properties of material constituting the beam are:

$E=1.658\,1010\,Pa$

$\nu=0.3$

$\rho=1.3404106\,104^{kg}/m^3$

$\alpha=\mathrm{AMOR\_ALPHA}=0.001$

$\beta=\mathrm{AMOR\_BETA}=0.$

1.3 Boundary conditions and loadings

The boundary condition which characterizes this problem is the embedding of the point A and is written:

$u=v=0.$

$\theta=0.$

For the loading one a:

$Fx=3000.\,N$		$Fy=Fz=0.$	(tractive effort)
$Fx=0.$	$Fy=3000.\,N$	$Fz=0.$	(bending stress)

2 Reference solution

2.1 Method of calculating used for the reference solution

If the beam is modelled by a beam of Euler-Bernoulli and only one finite element, the harmonic problem can be written in the following way:

problem in traction:

$$(1+i\,\alpha\omega)\frac{ES}{L}u(B)-\omega^2\frac{\rho S\,L}{6}u(B)=F_x(B)$$

from where $u(B)=\dfrac{F(B)}{\dfrac{ES}{L}-\omega^2\dfrac{\rho S\,L}{6}+i\,\alpha\,\omega\dfrac{ES}{L}}$

problem in inflection:

$$\left[-\omega^2\begin{bmatrix}\dfrac{13L}{35} & \dfrac{-11L^2}{210}\\ \dfrac{-11L^2}{210} & \dfrac{L^3}{105}\end{bmatrix}+(1+i\alpha\omega)\frac{12EI_y}{L^3}\begin{bmatrix}1 & \dfrac{-L}{2}\\ \dfrac{-L}{2} & \dfrac{L^2}{3}\end{bmatrix}\right]\begin{Bmatrix}v(B)\\ \theta(B)\end{Bmatrix}=\begin{Bmatrix}F_y(B)\\ 0\end{Bmatrix}$$

Note:

If the material does not present damping, one has then: `AMOR_ALPHA` = $\alpha=0$.

Efforts at the point B are calculated in the following way:

problem in traction:

$$N(B)=\left(\frac{ES}{L}-\omega^2\frac{\rho S\,L}{6}\right)u(B)$$

problem in inflection:

$$\begin{Bmatrix}VY(B)\\ MFZ(B)\end{Bmatrix}=\left[-\omega^2\begin{bmatrix}\dfrac{13L}{35} & \dfrac{-11L^2}{210}\\ \dfrac{-11L^2}{210} & \dfrac{L^3}{105}\end{bmatrix}+\frac{12EI_y}{L^3}\begin{bmatrix}1 & \dfrac{-L}{2}\\ \dfrac{-L}{2} & \dfrac{L^2}{3}\end{bmatrix}\right]\begin{Bmatrix}v(B)\\ \theta(B)\end{Bmatrix}$$

The systems analytically are solved 2×2 to obtain the solution.

2.2 Results of reference

The results of reference are the displacements, the speeds, the accelerations and the generalized efforts obtained at the point B during the harmonic analysis.

2.3 Notice for modeling B

For modeling B, one wants to test the problem in traction in the case of the keyword `FORCE_POUTRE` who allows to apply efforts distributed. To obtain the same solution as the beam subjected to nodal force in its end, the relation between the effort distributed constant and the nodal force are:

$$F_x(B)=\frac{f\,L}{2}$$

With the values given to paragraph 1.3, one a: $f=600\,N/m$

2.4 Uncertainty on the solution

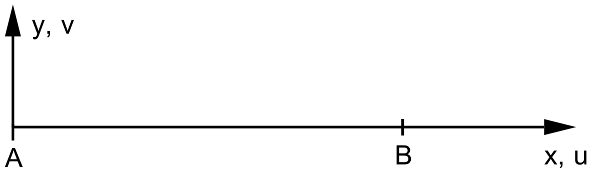
If the assumptions are checked (beam of Euler-Bernoulli), the solution is analytical.

2.5 Bibliographical references

1) Reference material of *Code_Aster* : “Exact” elements of beams (right and curved) - [R3.08.01].

3 Modeling A

3.1 Characteristics of modeling



The beam consists of only one nets.

The modeling used for the beam is that of Euler-Bernoulli (`POU_D_E`).

The end A is embedded:

`DX = DY = DZ = 0.` `DRX = DRY MARTINI = DRZ = 0.`

3.2 Characteristics of the grid

Many nodes: 2

Many meshs and types: 1 mesh of the type `SEG 2`

The points characteristic of the grid are the following:

Not A = A
Not B = B

3.3 Sizes tested (reality-imaginary form)

Problem 1: traction

Not/Size			Reference	*Aster*	% difference
displacement	B	DX	(5,318 10^{-5}, 0.)	(5,318 10^{-5}, 0.)	0.
speed	B	DX	(0. , 3,341 10^{-3})	(0. , 3,341 10^{-3})	0.
acceleration	B	DX	(– 2,099 10^{-1}, 0.)	(– 2,099 10^{-1}, 0.)	0.
generalized effort	B	NR	(3000. , 0.)	(3000. , 0.)	0.

Problem 2: inflection

Not/Size			Reference	*Aster*	% difference
displacement	B	DY	(1,828 10^{-2}, 0.)	(1,828 10^{-2}, 0.)	0.
		DRZ	(1.82 10^{-2}, 0.)	(1.82 10^{-2}, 0.)	0.
speed	B	DY	(0. , 1.1489)	(0. , 1.1489)	0.
		DRZ	(0. , 1.1438)	(0. , 1.1438)	0.
acceleration	B	DY	(– 7,219 10^{-1})	(– 7,219 10^{-1}, 0.)	0.
		DRZ	(– 7,186 10^{-1}, 0.)	(– 7,186 10^{-1}, 0.)	0.
generalized effort	B	VY	(3000. , 0.)	(3000. , 0.)	0.
		MFZ	(0. , 0.)	(– 1,164 10^{-10}, 0.)	0.

Problem 3: traction + damping

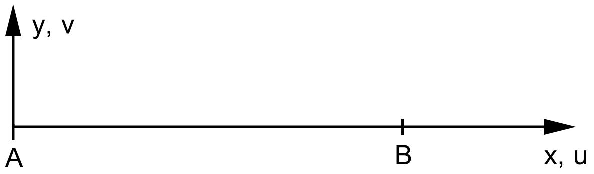
Not/Size			Reference	*Aster*	% diff
displacement	B	DX	(5,296 10^{-5}, – 3,363 10^{-3})	(5,296 10^{-5}, – 3,363 10^{-3})	0.
speed	B	DX	(2,113 10^{-4}, 3,327 10^{-3})	(2,113 10^{-4}, 3,327 10^{-3})	0.
acceleration	B	DX	(– 2,091 10^{-1}, 1,327 10^{-2})	(– 2,091 10^{-1}, 1,327 10^{-2})	0.
generalized effort	B	NR	(2,987 10^{3}, – 1.8975 10^{2})	(2,987 10^{3}, – 1.8975 10^{2})	0.

Problem 4: inflection + damping

Not/Size			Reference	*Aster*	% diff
displacement	B	DY	(1,746 10^{-2}, – 4,469 10^{-3})	(1,746 10^{-2}, – 4,469 10^{-3})	0.
		DRZ	(1,757 10^{-2}, – 3,402 10^{-3})	(1,757 10^{-2}, – 3,402 10^{-3})	0.
speed	B	DY	(2,808 10^{-1}, 1,097)	(2,808 10^{-1}, 1,097)	0.
		DRZ	(2,138 10^{-1}, 1,104)	(2,138 10^{-1}, 1,104)	0.
acceleration	B	DY	(– 6,895 10^{-1}, 1,764 10^{-1})	(– 6,895 10^{-1}, 1,764 10^{-1})	0.
		DRZ	(– 6.94 10^{-1}, 1,343 10^{-1})	(– 6.94 10^{-1}, 1,343 10^{-1})	0.
generalized effort	B	VY	(3,021 10^{3}, 1,212 10^{2})	(3,021 10^{3}, 1,212 10^{2})	0.
		MFZ	(– 1,567 10^{2}, – 8,583 10^{2})	(– 1,567 10^{2}, – 8,583 10^{2})	0.

4 Modeling B

4.1 Characteristics of modeling



The beam consists of only one nets.

The modeling used for the beam is that of Euler-Bernoulli (`POU_D_E`).

The end A is embedded:

`DX = DY = DZ = 0.      DRX = DRY MARTINI = DRZ = 0.`

4.2 Characteristics of the grid

Many nodes: 2

Many meshs and types: 1 mesh of the type `SEG 2`

The points characteristic of the grid are the following:

Not A = A
Not B = B

4.3 Sizes tested (reality-imaginary form)

Problem 1: traction (effort distributed real: worthless imaginary part)

Not/Size			Reference	*Aster*	% difference
displacement	B	DX	(5,318 10^{-5}, 0.)	(5,318 10^{-5}, 0.)	0.
speed	B	DX	(0. , 3,341 10^{-3})	(0. , 3.3414 10^{-3})	0.
acceleration	B	DX	(– 2,099 10^{-1}, 0.)	(– 2.0994 10^{-1}, 0.)	0.
generalized effort	B	NR	(3000. , 0.)	(3000. , 0.)	0.

Problem 2: traction (effort distributed complex: worthless rélle part)

Not/Size			Reference	*Aster*	% difference
displacement	B	DX	(0. , 5,318 10^{-5})	(0. , 5,318 10^{-5})	0.
speed	B	DX	(- 3,341 10^{-3} , 0.)	(- 3.3414 10^{-3} , 0.)	0.
acceleration	B	DX	(0. , – 2,099 10^{-1})	(0. , – 2.0994 10^{-1})	0.
generalized effort	B	NR	(0. , 3000.)	(0. , 3000.)	0.

Problem 3: traction + damping (effort distributed real: worthless imaginary part)

Not/Size			Reference	*Aster*	% diff
displacement	B	DX	(5,296 10^{-5}, – 3,363 10^{-3})	(5.2966 10^{-5}, – 3.3637 10^{-3})	0.
speed	B	DX	(2,113 10^{-4}, 3,327 10^{-3})	(2.1135 10^{-4}, 3.3279 10^{-3})	0.
acceleration	B	DX	(– 2,091 10^{-1}, 1,327 10^{-2})	(– 2,091 10^{-1}, 1.3279 10^{-2})	0.
generalized effort	B	NR	(2.9879 10^{3}, – 1,897 10^{2})	(2,987 10^{3}, – 1.8975 10^{2})	0.

Problem 4: inflection + damping (effort distributed complex: worthless real part)

Not/Size			Reference	*Aster*	% diff
displacement	B	DX	(3,363 10^{-3} , 5,296 10^{-5})	(5,296 10^{-5}, – 3,363 10^{-3})	0.
speed	B	DX	(- 3,327 10^{-3} , 2,113 10^{-4})	(- 3.3279 10^{-3} , 2.1135 10^{-4})	0.
acceleration	B	DX	(- 1,327 10^{-2}, -2,091 10^{-1})	(- 1.3279 10^{-2}, -2,091 10^{-1})	0.
generalized effort	B	NR	(1,897 10^{2} , 2.9879 10^{3})	(1.8975 10^{2} , 2.98794 10^{3})	0.

When the effort distributed is applied as an imaginary part of the loading, the reference solution is obtained from that of real left modeling A while exchanging and imaginary part and by changing the sign of the new real parts.

5 Summary of the results

The analytical results well are found.