

SHLL100 - Harmonic answer of a bar by dynamic under-structuring

Summary:

The scope of application of this test relates to the dynamics of the structures, and more particularly the harmonic calculation of answer per dynamic under-structuring.

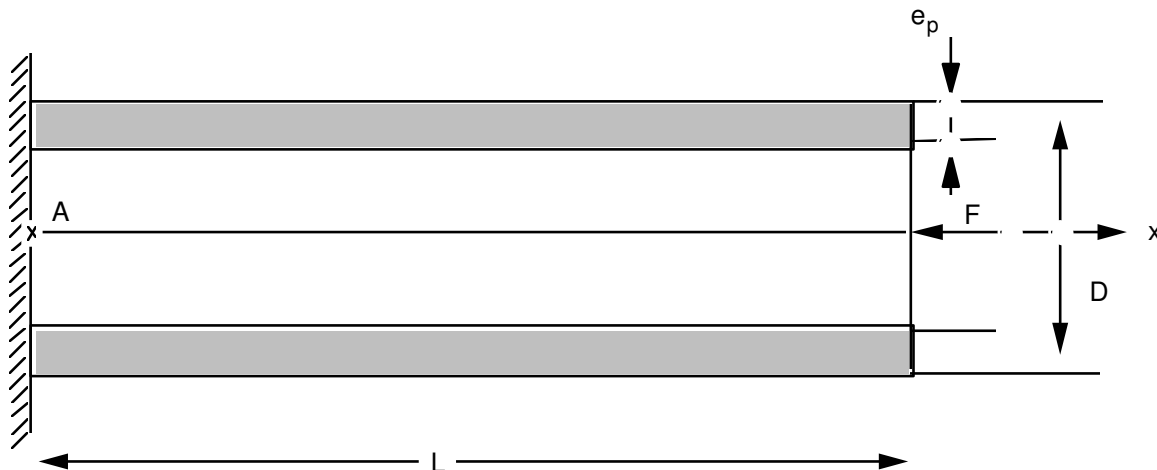
It is a question of calculating the harmonic answer in traction and compression of a fixed beam - free modelled by elements of the type "bars". The modelled structure is deadened (damping of Rayleigh by elements).

The results of reference result from a direct harmonic calculation. This test thus makes it possible to validate the computational tools of harmonic answer per under-structuring established in *Code_Aster* and more particularly:

- the catch in depreciation account by element,
- the calculation of the second member including the harmonic loading,
- restitution of the harmonic answer on a grid skeleton, including the fields of displacement, speed and acceleration.

1 Problem of reference

1.1 Geometry



$$L = 1 \text{ m}$$

$$D = 0,2 \text{ m} - \text{Circular section}$$

1.2 Material properties

$$E = 1.10^{10} \text{ Pa}$$

$$\nu = 0.3$$

$$\rho = 1.10^4 \text{ kg/m}^3$$

Damping of Rayleigh per element: $\alpha_e = 0.1$ $\beta_e = 0.1$

1.3 Boundary conditions and loadings

Embedding in the end A : $u(0) = n(0) = w(0) = 0$.

For any point $M(x)$: $n(0) = w(0) = 0$.

Harmonic loading in time, at the loose lead:

- orientation: according to x ,
- amplitude: 100 N ,
- frequency: 100 Hz .

1.4 Initial conditions

Without object for a harmonic calculation of answer.

2 Reference solution

2.1 Method of calculating used for the reference solution

There exists an analytical solution detailed in the reference [bib2].

Let us use the following notations:

| | |
|-----------------|--|
| E | : Young modulus |
| L | : length of the bar |
| A | : section of the bar |
| N | : normal effort directed according to the axis X |
| α, β | : damping coefficients of Rayleigh |
| Ω | : frequency of excitation |

and let us pose

$$r = \sqrt{\frac{1 + \beta^2 / \Omega^2}{1 + \alpha^2 / \Omega^2}}$$

$$k = p + iq = \Omega \sqrt{\frac{p}{2E}} \left[\sqrt{r - \frac{1 - \alpha\beta}{1 + \alpha^2 \Omega^2}} + i \sqrt{r + \frac{1 - \alpha\beta}{1 + \alpha^2 \Omega^2}} \right]$$

Displacement in a point $M(x)$ unspecified is given by:

$$V(x) = \frac{N}{EA} \frac{A}{(p + iq)(1 + i\Omega\alpha)} \frac{sh\ px \cos qx + i ch\ px \sin qx}{ch\ L \cos qL + i sh\ pL \sin qL}$$

| | Displacement (m) | Speed (m/s) | Acceleration (m/s ²) |
|----------------|--------------------------|-------------------------|----------------------------------|
| Real part | - 7.00 10 ⁻¹¹ | - 3.18 10 ⁻⁶ | 2.76 10 ⁻⁵ |
| Imaginary part | 5.07 10 ⁻⁹ | - 4.40 10 ⁻⁸ | - 2.00 10 ⁻³ |

2.2 Results of reference

Fields of displacement, speed and acceleration of the loose lead of the bar.

2.3 Uncertainty on the solution

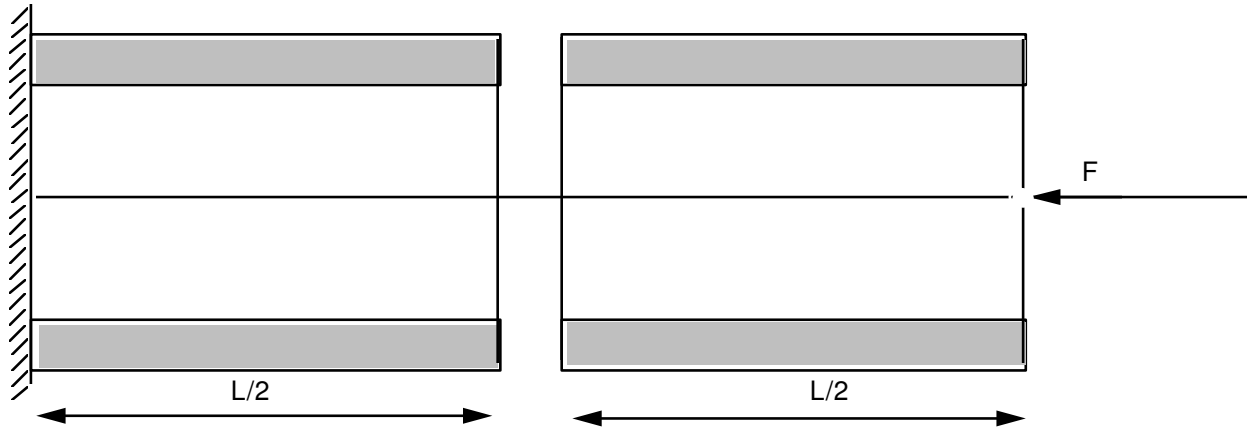
Digital solution.

2.4 Bibliographical references

- 1) T. KERBER "harmonic Under-structuring in *Code_Aster*", EDF Report, HP - 61/93 - 104.
- 2) G. ROBERT, analytical Solutions in dynamics of the structures, Report Samtech n°121, March 1996.
- 3) P. RICHARD, Methods of under-structuring in *Code_Aster*, Report interns EDF - DER, HP-61/92-149.

3 Modeling A

3.1 Characteristics of modeling



The bar is cut out in 2 parts of equal size. Each substructure considered is with a grid in segments to which elements are affected "bars".

The structure is studied using the method of the harmonic under-structuring with interfaces of the HARMONIC type CRAIG-BAMPTON.

The modal base used is made up of 4 clean modes for the substructure of right-hand side, of 5 clean modes for the substructure of left to which the harmonic constrained modes associated are added with the interfaces (calculated with 300 Hz . This value of the pulsation does not have any influence on the result, it is arbitrary [bib3]).

3.2 Characteristics of the grid

Many nodes: 5

Many meshes and types: 5 SEG 2

3.3 Sizes tested and results

| Displacement (m) | | |
|--------------------------|------------------------|-------------------|
| | Reference | Tolerance |
| Real part | $-7.00 \cdot 10^{-11}$ | $2 \cdot 10^{-3}$ |
| Imaginary part | $5.07 \cdot 10^{-9}$ | $2 \cdot 10^{-3}$ |
| Speed (m/s) | | |
| Real part | $-3.18 \cdot 10^{-6}$ | $2 \cdot 10^{-3}$ |
| Imaginary part | $-4.40 \cdot 10^{-8}$ | $2 \cdot 10^{-3}$ |
| Acceleration (m/s^2) | | |
| Real part | $2.76 \cdot 10^{-5}$ | $2 \cdot 10^{-3}$ |
| Imaginary part | $-2.00 \cdot 10^{-3}$ | $2 \cdot 10^{-3}$ |

4 Summary of the results

The precision on the complex coordinates of the fields of displacement speed and acceleration is lower than 0,1% .

This test thus validates the operators of harmonic under-structuring.