SDLX400 – Girder hinged on elastic support

Summary:

This test consists in calculating the Eigen frequencies of a beam whose one of the ends is articulated and the other is either free, or pressed on a spring.
The reference solution of this problem is analytical.
The originality of this test is to model this beam by means of voluminal elements and beams.

This test comprises three modelings:

- Modeling a: POU_D_E ;
- Modeling b: 3D and POU_D_E ;
- Modeling C: POU_D_TG.

The connection 3D-beam is carried out by LIAISON_ELEM in the operator AFFE_CHAR_MECA.

1 Problem of reference

1.1 Geometry

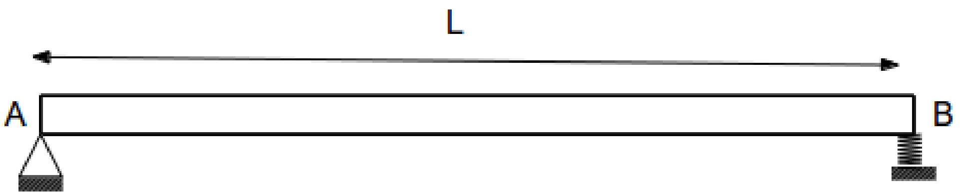

Figure 1.1 Geometry of the problem and system of loading

Beam with square section on side: $c=0.014\ \mathrm{m}$
Length of the beam: $L=0.783\ \mathrm{m}$

1.2 Properties of material

Young modulus	$E=6.07\times10^{10}\,\mathrm{Pa}$
Poisson's ratio	$\nu=0$
Density	$\rho=2400.0\,\mathrm{kg.m^{-3}}$
Stiffness of the spring	$k=18\,000\ \mathrm{N.m^{-1}}$

1.3 Boundary conditions and loadings

Imposed displacement:

At the point A	$DX=0$, $DY=0$
At the point B	Case 1: loose lead Case 2: elastic support

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Case 1: loose lead

The equation of the beam is written:

$$\frac{d^4 v}{dx^4} = \frac{-\rho S}{EI} \frac{d^2 v}{dt^2} \tag{1}$$

A solution of the form is sought:

$$v(x) = \sin(\omega t)\left[A\cos(kx) + B\sin(kx) + C\cosh(kx) + D\sinh(kx)\right] \tag{2}$$

with $\omega = \sqrt{\frac{EI}{\rho S}} k^2$

The boundary conditions are the following ones:

In $x=0$, $v=0$ and $\frac{d^2 v}{dx^2}=0$

In $x=L$, $\frac{d^2 v}{dx^2}=0$ and $\frac{d^3 v}{dx^3}=0$

While replacing v by the expression (2), one obtains:

- In $x=0$, $A=C=0$;
- In $x=L$, $\sin(kL)\cosh(kL)=\cos(kL)\sinh(kL)$ (3)

The first six roots of (3) are given in the following table:

kL
0
3.9266
7.06858
10.2102
13.3518
16.4934

One from of deduced the values from ω_i , $i=1,\ldots,6$.

2.1.2 Case 2: end in elastic support

The equation with the eigenvalues is the following one:

$$\sin(kL)\left(k^3L^3\frac{EI}{L^3}\cosh(kL)-K\sinh(kL)\right)-\sinh(kL)\left(k^3L^3\frac{EI}{L^3}\cos(kL)+K\sin(kL)\right)=0 \quad (4)$$

The own pulsations are given by:

$$\omega=\frac{1}{L^2}\sqrt{\frac{EI}{\rho S}}k^2L^2 \quad (5)$$

Six first roots of (4) are given in the following table:

kL
2.7880886
4.5619625
7.1895724
10.249031
13.368916
16.502406

One from of then deduced the values from ω_i , $i=1,\ldots,6$.

2.2 Results of reference

2.2.1 Case 1: loose lead

The structure has a rigid mode at worthless frequency. The following table presents the nonworthless frequencies.

Mode	Frequency (Hz)
1	85.5
2	277
3	577.9
4	988.2
5	1507.9

2.2.2 Case 2: end in elastic support

Mode	Frequency (Hz)
1	43.1
2	115.4
3	286.5
4	582.3

5	990.7
6	1509.6

2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling A

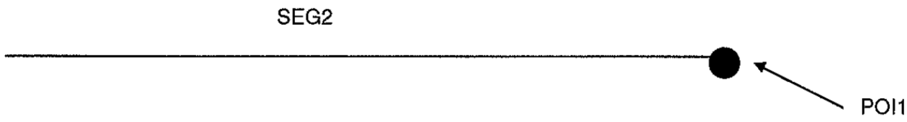

Figure 3.1 Grid of the problem

Modeling POU_D_E and discrete element of type DIS_TR.

3.2 Characteristics of the grid

Many nodes: 11
Many meshs and types: 10 SEG2 and 1 POI1

3.3 Sizes tested and results

3.3.1 Case 1: loose lead

METHODE=' SORENSEN'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	85.5	'ANALYTICAL'	0.1
Mode 2	277	'ANALYTICAL'	0.1
Mode 3	577.9	'ANALYTICAL'	0.1
Mode 4	988.2	'ANALYTICAL'	0.3
Mode 5	1507.9	'ANALYTICAL'	0.5

METHODE=' JACOBI'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	85.5	'ANALYTICAL'	0.1
Mode 2	277	'ANALYTICAL'	0.1
Mode 3	577.9	'ANALYTICAL'	0.1
Mode 4	988.2	'ANALYTICAL'	0.3
Mode 5	1507.9	'ANALYTICAL'	0.5

3.3.2 Case 2: end in simple support

METHODE=' SORENSEN'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	43.1	'ANALYTICAL'	0.1
Mode 2	115.4	'ANALYTICAL'	0.1
Mode 3	286.5	'ANALYTICAL'	0.1

Mode 4	582.3	'ANALYTICAL'	0.1
Mode 5	990.7	'ANALYTICAL'	0.3
Mode 6	1509.6	'ANALYTICAL'	0.5

METHODE=' JACOBI'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	43.1	'ANALYTICAL'	0.1
Mode 2	115.4	'ANALYTICAL'	0.1
Mode 3	286.5	'ANALYTICAL'	0.1
Mode 4	582.3	'ANALYTICAL'	0.1
Mode 5	990.7	'ANALYTICAL'	0.3
Mode 6	1509.6	'ANALYTICAL'	0.5

4 Modeling B

4.1 Characteristics of modeling B

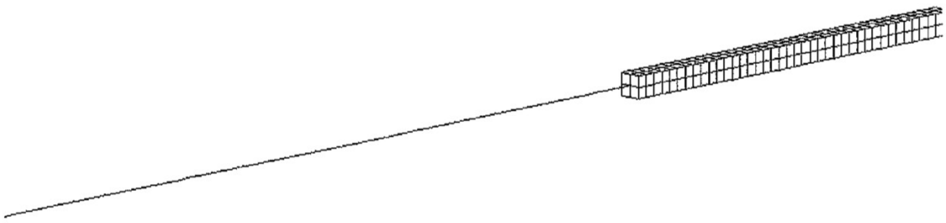

Figure 4.1 Grid of the problem

Modeling POU_D_E for a third of the structure and the rest in modeling 3D, discrete element of type DIS_TR.

4.2 Characteristics of the grid

Many nodes: 1303
Many meshs and types: 160 HEXA20, 8 QUAD8, 80 SEG2 and 1 POI1

4.3 Sizes tested and results

4.3.1 Case 1: loose lead

METHODE=' SORENSEN'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	85.5	'ANALYTICAL'	0.85
Mode 2	277	'ANALYTICAL'	0.7
Mode 3	577.9	'ANALYTICAL'	0.8
Mode 4	988.2	'ANALYTICAL'	1.2
Mode 5	1507.9	'ANALYTICAL'	3.6

METHODE=' JACOBI'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	85.5	'ANALYTICAL'	0.85
Mode 2	277	'ANALYTICAL'	0.7
Mode 3	577.9	'ANALYTICAL'	0.8
Mode 4	988.2	'ANALYTICAL'	1.2
Mode 5	1507.9	'ANALYTICAL'	3.7

4.3.2 Case 2: end in simple support

METHODE=' SORENSEN'

Identification	Value of reference	Type of reference	Tolerance (%)

Mode 1	43.1	'ANALYTICAL'	0.4
Mode 2	115.4	'ANALYTICAL'	0.6
Mode 3	286.5	'ANALYTICAL'	0.6
Mode 4	582.3	'ANALYTICAL'	0.8
Mode 5	990.7	'ANALYTICAL'	1.2
Mode 6	1509.6	'ANALYTICAL'	3.7

METHODE=' JACOBI'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	43.1	'ANALYTICAL'	0.4
Mode 2	115.4	'ANALYTICAL'	0.6
Mode 3	286.5	'ANALYTICAL'	0.6
Mode 4	582.3	'ANALYTICAL'	0.8
Mode 5	990.7	'ANALYTICAL'	1.2
Mode 6	1509.6	'ANALYTICAL'	3.7

5 Modeling C

5.1 Characteristics of modeling C

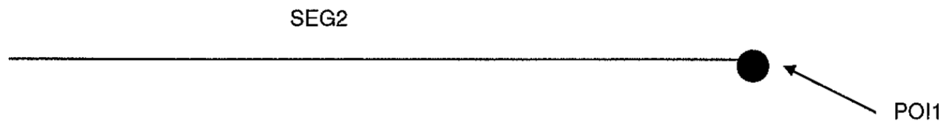

Figure 5.1 Grid of the problem

Modeling POU_D_TG and discrete element of type DIS_TR.

5.2 Characteristics of the grid

Many nodes: 11
Many meshs and types: 10 SEG2 and 1 POI1

5.3 Sizes tested and results

5.3.1 Case 1: loose lead

METHODE=' SORENSEN'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	85.5	'ANALYTICAL'	0.11
Mode 2	277	'ANALYTICAL'	0.3
Mode 3	577.9	'ANALYTICAL'	0.4
Mode 4	988.2	'ANALYTICAL'	0.6
Mode 5	1507.9	'ANALYTICAL'	0.7

METHODE=' JACOBI'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	85.5	'ANALYTICAL'	0.11
Mode 2	277	'ANALYTICAL'	0.3
Mode 3	577.9	'ANALYTICAL'	0.4
Mode 4	988.2	'ANALYTICAL'	0.6
Mode 5	1507.9	'ANALYTICAL'	0.7

5.3.2 Case 2: end in simple support

METHODE=' SORENSEN'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	43.1	'ANALYTICAL'	0.1
Mode 2	115.4	'ANALYTICAL'	0.1

Mode 3	286.5	'ANALYTICAL'	0.2
Mode 4	582.3	'ANALYTICAL'	0.4
Mode 5	990.7	'ANALYTICAL'	0.6
Mode 6	1509.6	'ANALYTICAL'	0.7

METHODE=' JACOBI'

Identification	Value of reference	Type of reference	Tolerance (%)
Mode 1	43.1	'ANALYTICAL'	0.1
Mode 2	115.4	'ANALYTICAL'	0.1
Mode 3	286.5	'ANALYTICAL'	0.2
Mode 4	582.3	'ANALYTICAL'	0.4
Mode 5	990.7	'ANALYTICAL'	0.6
Mode 6	1509.6	'ANALYTICAL'	0.7

6 Summary of the results

Results got with the elements of beam POU_D_E and POU_D_TG are in very good agreement with the analytical solution and respectively lower than 0.46 % and 0.61 %. Results got by mixing two types of elements (beam and voluminal) then by connecting the degrees of freedom by means of the option LIAISON_ELEM are less precise (but the variation remains lower than 3.61 %).
It should be noted that the calculated Eigen frequencies are identical whatever the method used in the modal solvor (SORENSEN or JACOBI).