

SDLV100 - Vibration of a slim beam of variable rectangular section (embed-free)

Summary:

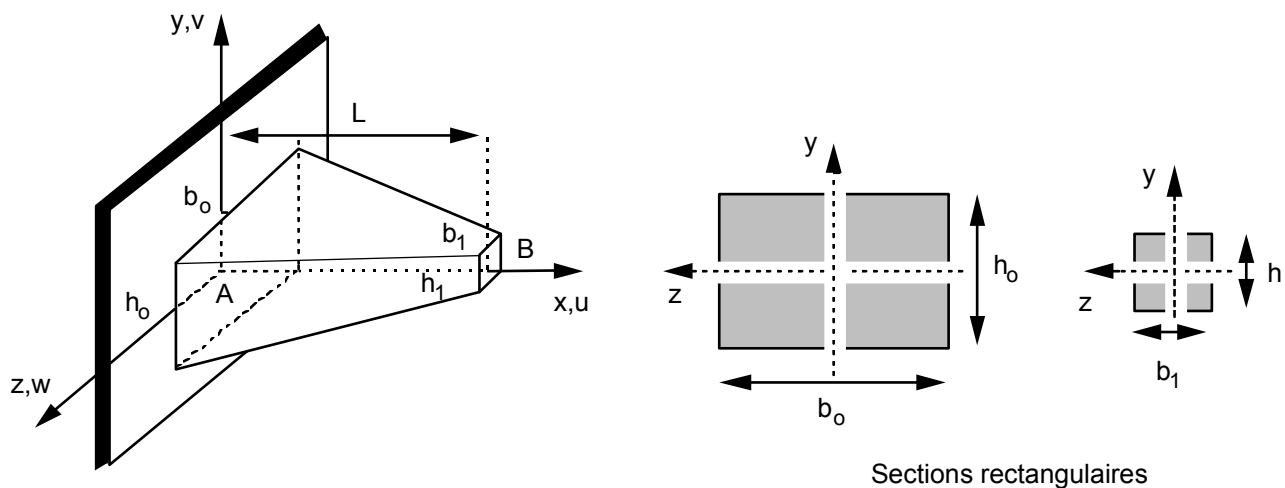
The studied structure is a beam out of free steel embedded with rectangular variable section modelled by voluminal elements. One is interested in his Eigen frequencies in inflection. With the same problem is dealt in modeling beam in the case test SDLL09.

This problem makes it possible to test the voluminal elements `MECA_HEX20` and `MECA_PENTA15` in modal analysis. It also makes it possible to test the option `MASS_MECA_DIAG` of calculation of the matrices of mass diagonalized for voluminal modelings.

The reference solution is a digital solution obtained using the computer code by finite elements the SAMCEF software for similar modelings. The got results are also in concord with the semi-analytical results given in guide VPCS.

1 Problem of reference

1.1 Geometry



Length of the beam:

$$L = 1 \text{ m}$$

Rectangular section:

| | Initial cross-section | Final cross-section |
|----------|---|--|
| height: | $h_o = 0.04 \text{ m}$ | $h_1 = 0.01 \text{ m}$ |
| width: | $b_o = 0.04 \text{ m}$ | $b_1 = 0.01 \text{ m}$ |
| surface: | $A_o = 1.6 \cdot 10^{-3} \text{ m}^2$ | $A_1 = 1 \cdot 10^{-4} \text{ m}^2$ |
| inertia: | $Iz_o = 2.1333 \cdot 10^{-7} \text{ m}^4$ | $Iz_1 = 8.3333 \cdot 10^{-10} \text{ m}^4$ |

Coordinates of the points (in meters)

| | A | B |
|---|----|----|
| x | 0. | 1. |
| y | 0. | 0. |
| z | 0. | 0. |

1.2 Properties of steel

$$E = 2.10^{11} \text{ Pa}$$

$$\rho = 7800 \text{ kg/m}^3$$

1.3 Boundary conditions and loadings

Not A : embedded $u = v = z = 0$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is obtained using the computation software by finite elements the SAMCEF software for identical modelings but with elementary matrices of mass coherent.

One points out the analytical solution given in card SDLL09/89 of guide VPCS. The differential equation in inflection of the beam considered, in theory of Euler-Bernoulli is written (Theory of Euler - Bernoulli):

$$\frac{\partial^2 \left(E I_z \frac{\partial^2 v}{\partial x^2} \right)}{\partial x^2} = -\rho A \frac{\partial^2 v}{\partial t^2}$$

where I_z and A vary with the X-coordinate.

The Eigen frequencies are then of the form:

$$f_i = \frac{1}{2\pi} \lambda_i(\alpha, \beta) \frac{h_1}{L^2} \sqrt{\frac{E}{12\rho}}$$

with $\alpha = \frac{h_0}{h_1} = 4$ and $\beta = \frac{b_0}{b_1} = 4$.

For this value of α and β , first values of the continuation (λ_i) are:

| | λ_1 | λ_2 | λ_3 | λ_4 | λ_5 |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $\beta = 4$ | 23,289 | 73.9 | 165.23 | 299.7 | 478.1 |

2.2 Results of reference

The results of reference selected are the first 5 Eigen frequencies of the modes of inflection.

2.3 Uncertainty on the solution

Analytical solution in theory of beam of Bernoulli, and digital solution the SAMCEF software.

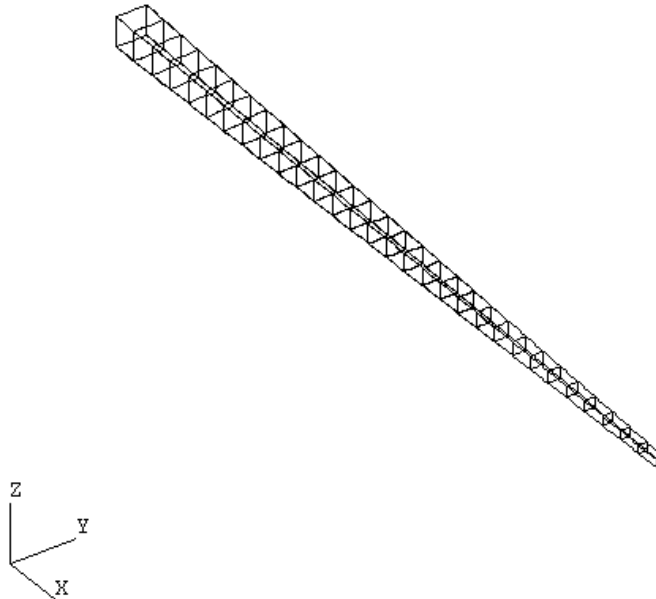
2.4 Bibliographical references

- 1) H.H. MABIE, C.B. ROGERS, Transverse vibrations of double-tapered cantilever beams - Newspaper of the Acoustical Society of America, n° 51, p. 1771-1774 (1972).

3 Modeling A

3.1 Characteristics of modeling

Elements of volume MECA_HEXA20



Discretization:

beam AB: 30 meshes HEXA20
(1 mesh in the section)

Boundary conditions:

- in all the nodes
- at the end *A* (group of G_1 nodes)

DDL_IMPO: (ALL: 'YES' DZ: 0.)
(GROUP_NO: G_1 DX: 0. , DY: 0)

3.2 Characteristics of the grid

Grid:

Many nodes: 368
Number of meshes and type: 30 HEXA20

3.3 Values tested

| Identification | Solution beam analytical | Reference the SAMCEF software | Aster | % difference The Aster- SAMCEF software |
|-----------------|-----------------------------|-------------------------------------|--------|--|
| | frequency | in HZ | in HZ | |
| coherent matrix | | | | |
| inflection 1 | 54.18 | 56.84 | 56.85 | 0.0176% |
| inflection 2 | 171.94 | 180.0 | 180.08 | 0.0444% |

| | | | | |
|--------------|---------|---------|---------|---------|
| inflection 3 | 384.40 | 401.0 | 401.23 | 0.0574% |
| inflection 4 | 697.24 | 723.2 | 724.02 | 0.1134% |
| inflection 5 | 1112.28 | 1145.41 | 1147.51 | 0.1833% |

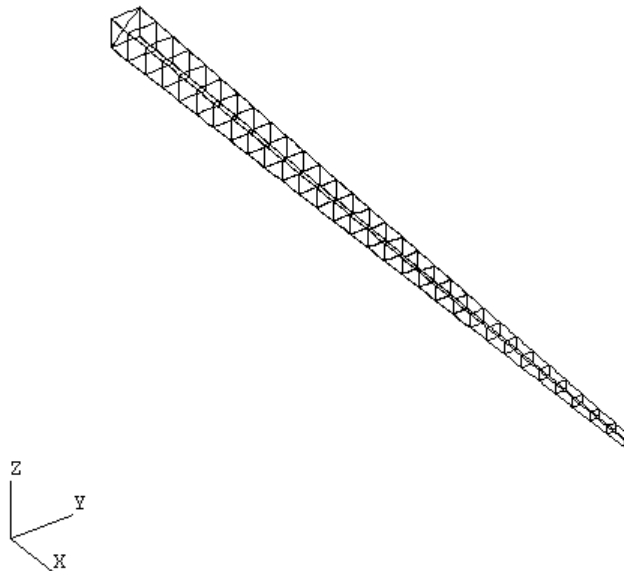
diagonal matrix

| | | | | |
|--------------|---------|---------|---------|-----------|
| inflection 1 | 54.18 | 56.84 | 56.78 | - 0.1033% |
| inflection 2 | 171.94 | 180.00 | 179.57 | - 0.2419% |
| inflection 3 | 384.40 | 401.00 | 399.24 | - 0.4408% |
| inflection 4 | 697.24 | 723.20 | 718.69 | - 0.6273% |
| inflection 5 | 1112.28 | 1145.41 | 1136.01 | - 0.8273% |

4 Modeling B

4.1 Characteristics of modeling

Elements of volume MECA_PENTA15



Discretization: beam *AB* : 60 meshes PENTA15
(2 meshes in the section)

Boundary conditions:

- in all the nodes
 - at the end *A* (group of G_1 nodes)
- DDL_IMPO: (ALL: 'YES' DZ: 0.)
(GROUP_NO: G_1 DX: 0. , DY: 0)

4.2 Characteristics of the grid

Grid: Many nodes: 368
Number of meshes and type: 60 PENTA15

4.3 Values tested

| Identification | Solution beam semi-analytical | Reference the SAMCEF software | Aster | % difference ASTER-SAMCEF |
|-------------------|----------------------------------|-------------------------------------|---------|------------------------------|
| | frequency | in HZ | in HZ | |
| consistent matrix | | | | |
| inflection 1 | 54.18 | 56.84 | 56.82 | - 0,038% |
| inflection 2 | 171.94 | 180.00 | 179.96 | - 0,022% |
| inflection 3 | 384.40 | 401.00 | 400.93 | - 0,018% |
| inflection 4 | 697.24 | 723.20 | 723.41 | 0,029% |
| inflection 5 | 1112.28 | 1145.41 | 1146.41 | 0,088% |

diagonal matrix

| | | | | |
|--------------|---------|---------|--------|---------|
| inflection 1 | 54.18 | 56.84 | 56.76 | -0,149% |
| inflection 2 | 171.94 | 180.00 | 179.51 | -0,272% |
| inflection 3 | 384.40 | 401.00 | 399.25 | -0,437% |
| inflection 4 | 697.24 | 723.20 | 719. | -0,583% |
| inflection 5 | 1112.28 | 1145.41 | 1140. | -0,740% |

5 Summary of the results

Differences between the results of calculations *Code_Aster* and the SAMCEF software with coherent masses are lower than 0.2%.

Differences between the computation results *Code_Aster* with diagonal masses and the SAMCEF software with coherent masses remain lower than 1%.

These results are in conformity so that one could wait, and validate in a reliable way calculations of Eigen frequencies in *Code_Aster* by `CALC_MODES` and the operator `CALC_MATR_ELEM` in coherent masses as in diagonal masses.