SDLS505 - Buckling of a cylindrical envelope under external pressure

Summary:

This test represents a calculation of stability of a thin cylindrical envelope stopped at its ends subjected to an external pressure and an axial pressure. One calculates the critical loads leading to the elastic buckling of Euler. The geometrical matrix of rigidity used in the resolution of the problem to the eigenvalues is that which is due to the initial constraints.

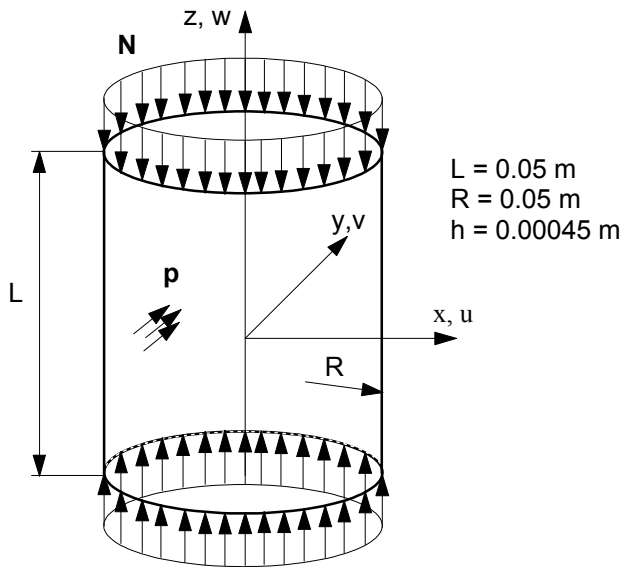
It makes it possible to validate modeling finite elements:

- COQUE_3D with the meshs TRIA7 and QUAD9.
- DKT with the meshs TRIA3 and QUAD4.

The critical load and the clean mode obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



The symmetry of the problem makes it possible to model a half rolls length $L/2$, stopped with the one of its ends, with conditions of symmetry specific to the lower edge.

1.2 Properties of material

The properties of material constituting the plate are:

$E=2.10^{11} Pa$ Young modulus
$\nu=0.3$ Poisson's ratio

1.3 Boundary conditions and loadings

- Loading:
  - pressure uniformly distributed of $p_{cr}=1.523\,10^{6} Pa$ on the cylindrical part. This pressure corresponds to the value of the critical load,
  - effort distributed on the contour of the bottom $N=0.5\times R\cdot p_{cr}=3.8075\,10^{4} N/m$.

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating used for the reference solution

The critical pressure is given in [bib1] or [bib2] by the following expression:

$$P_{cr}=\frac{Eh}{R}\frac{1}{\left(n^2+\frac{b^2}{2}\right)}\left[\frac{1}{\left(\frac{n^2}{b^2}+1\right)^2}+\frac{h^2}{12R^2(1-\nu^2)}(n^2+b^2)^2\right]$$

with $b=\frac{\pi R}{L}$

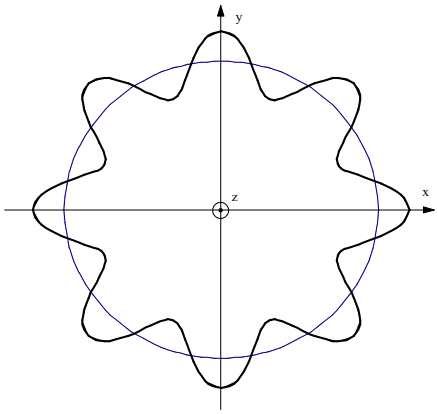
n represent the number of circumferential modes

This formula is valid if $N=0.5\,R\,p_{cr}$.

2.2 Results of reference

Pour les caractéristiques géométriques données, la pression critique est minimum pour un nombre n=8 et vaut p_{cr} = 1.523 10^6Pa.

Le nombre n est obtenu à partir d'un abaque.



2.3 Uncertainties on the solution

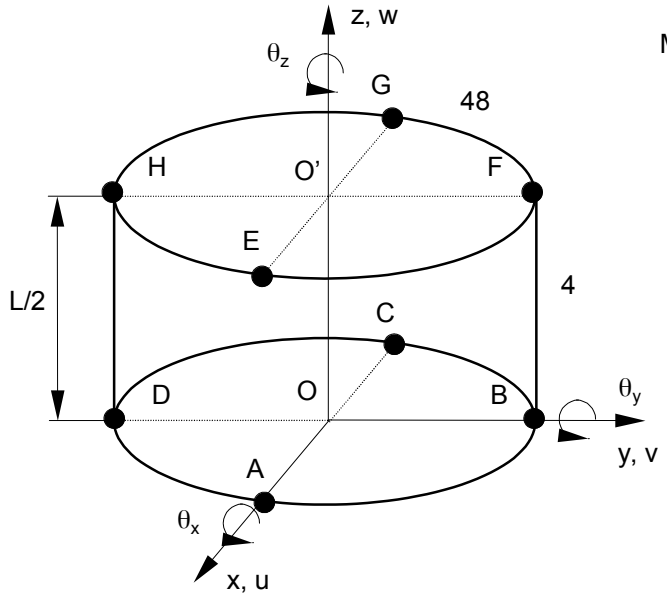
Analytical solution

2.4 Bibliographical references

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, page 500, second edition, DUNOD 1966.
2) BO O. ALMROTH, D.O. BRUSH: Buckling of bars, punts and shells, page 173, Mc Graw-Hill, New York, 1975.

3 Modeling A

3.1 Characteristics of modeling



Modélisation COQUE_3D (TRIA7)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

3.2 Characteristics of the grid

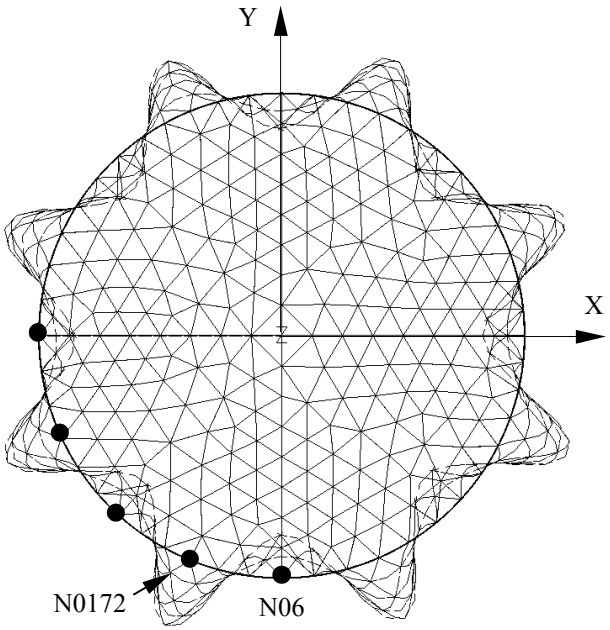
Many nodes: 2464
Many meshs and types: 834 TRIA7

3.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Pressure criticizes ($n=8$)	– 1,523 10^6 Pa	– 1.6862 10^6 Pa	10,715
Displacement x with the node $N06$	– 0.0102.	– 0.0102.	0.
Displacement y with the node $N06$	1.	1.	0.
Displacement x with the node $N0172$	– 0,392	– 0,376	– 4.09
Displacement y with the node $N0172$	– 0,928	– 0.9498	2,352

3.4 Remarks

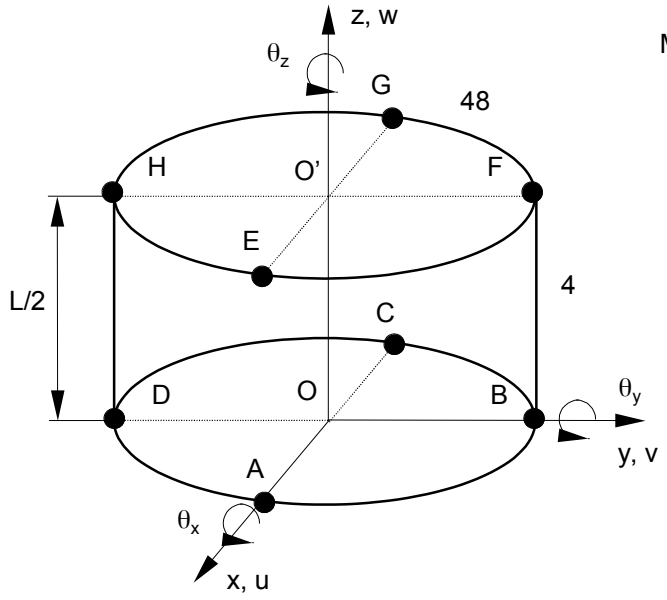
The angular position of the "bumps" is defined except for an angle, which makes difficult the checking of the deformation modal. To check the modal deformation associated with the mode with buckling, we validated the deformation graphically. Then we deferred in the command file (Operator TEST_RESU) displacements DX and DY node $N06$ characterizing this deformation, which explains the null variation obtained on displacements. The displacement of the node $N0172$ can then be deduced immediately from that of the node $N06$ in an analytical way by using the forms of the clean modes given in [bib1] and by considering that it is in opposition of phase with the node $N06$. The value of displacements obtained of the kind is not very precise owing to the fact that the angle between the two nodes is not worth exactly $22°5$.



- Les nœuds N06 et N0172 sont situés dans le plan Z=0. L'angle entre ces deux nœuds est de 22°5 (Demi longueur d'onde) approximativement.

4 Modeling B

4.1 Characteristics of modeling



Modélisation COQUE_3D (QUAD9)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

4.2 Characteristics of the grid

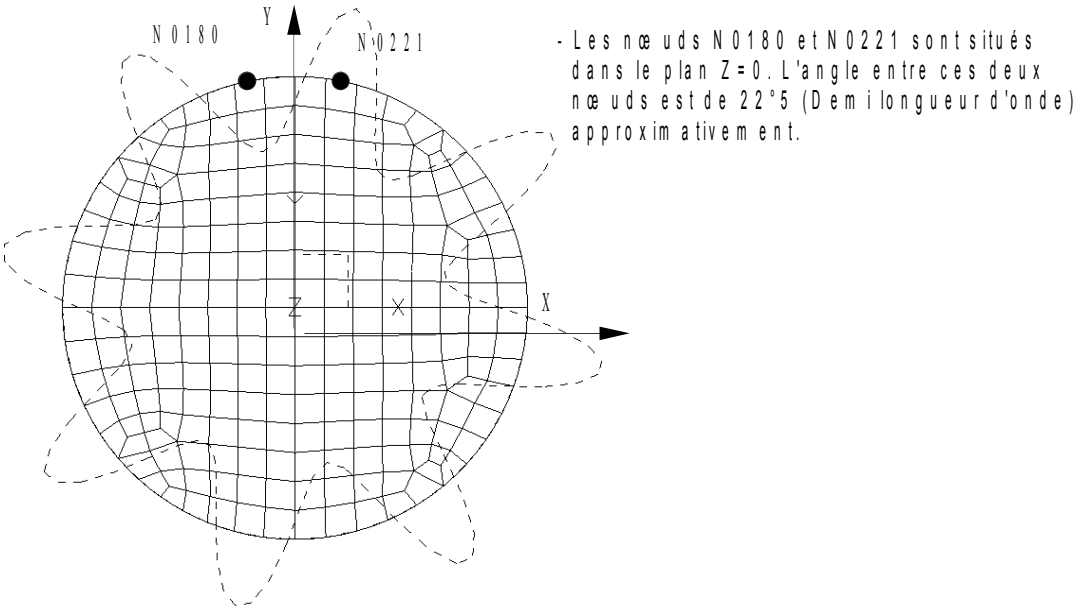
Many nodes: 1802
Many meshs and types: 472 QUAD9

4.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Pressure criticizes ($n=8$)	– 1,523 10^6 Pa	1.5576 10^6 Pa	2,272
Displacement x node $N0180$	0.13596	0.1003	26,258
Displacement y node $N0180$	– 0.7744	– 0.66172	– 14,551
Displacement x node $N0221$	0.17743	0.17743	0.
Displacement y node $N0221$	0.765965	0.765965	0.

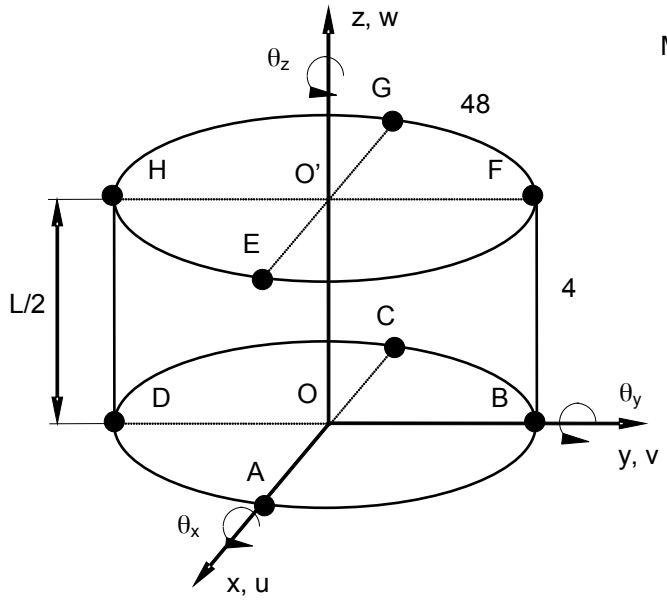
4.4 Remarks

The angular position of the "bumps" is defined except for an angle, which makes difficult the checking of the deformation modal. To check the modal deformation associated with the mode with buckling, we validated the deformation graphically. Then we deferred in the command file (Operator TEST_RESU) displacements DX and DY node $N0221$ characterizing this deformation, which explains the null variation obtained on displacements. The displacement of the node $N0180$ can then be deduced immediately from that of the node $N0221$ in an analytical way by using the forms of the clean modes given in [bib1] and by considering that it is in opposition of phase with the node $N0221$. The value of displacements obtained of the kind is not very precise owing to the fact that the angle between the two nodes is not worth exactly $22°5$.



5 Modeling C

5.1 Characteristics of modeling



Modélisation DKT (TRIA3)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

5.2 Characteristics of the grid

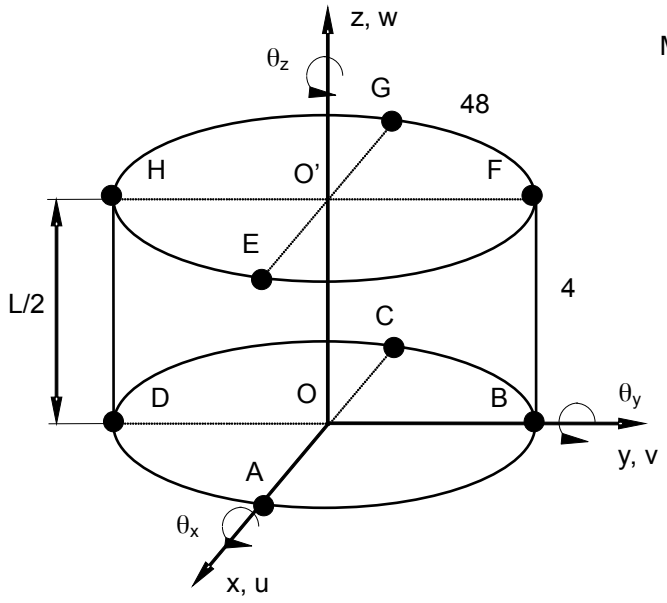
Many nodes: 418
Many meshs and types: 786 TRIA3

5.3 Sizes tested and results

Identification	Type of reference	Reference	% Tolerance
Pressure criticizes ($n=8$)	'ANALYTICAL'	– 1,523 10^6 Pa	12.5

6 Modeling D

6.1 Characteristics of modeling



Modélisation DKT (QUAD4)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

6.2 Characteristics of the grid

Many nodes: 449
Many meshs and types: 424 QUAD4

6.3 Sizes tested and results

Identification	Type of reference	Reference	% Tolerance
Pressure criticizes ($n=8$)	'ANALYTICAL'	– 1,523 10^6 Pa	12.5

7 Summary of the results

Modeling COQUE_3D

The got results are satisfactory. Uncertainties on the critical pressure do not exceed:

- 10.71% with meshs TRIA7,
- 2.3% with meshs QUAD9.

The modal deformation obtained corresponds well to the expected circumferential mode: $n=8$ for two modelings.

It is noted that modeling COQUE_3D with meshs QUAD9 is more precise than modeling COQUE_3D with meshs TRIA7.

Modeling DKT

The got results are satisfactory for elements of plates. Uncertainties on the critical pressure do not exceed 12.5% some is the mesh (QUAD4, TRIA3).

The modal deformation obtained corresponds well to the expected circumferential mode: $n=8$ for two modelings.

This test made it possible to test modelings COQUE_3D and DKT in linear buckling of Euler of a mean structure subjected to an external pressure.