
SDLS502 - Square plate “solid” simply supported

Summary:

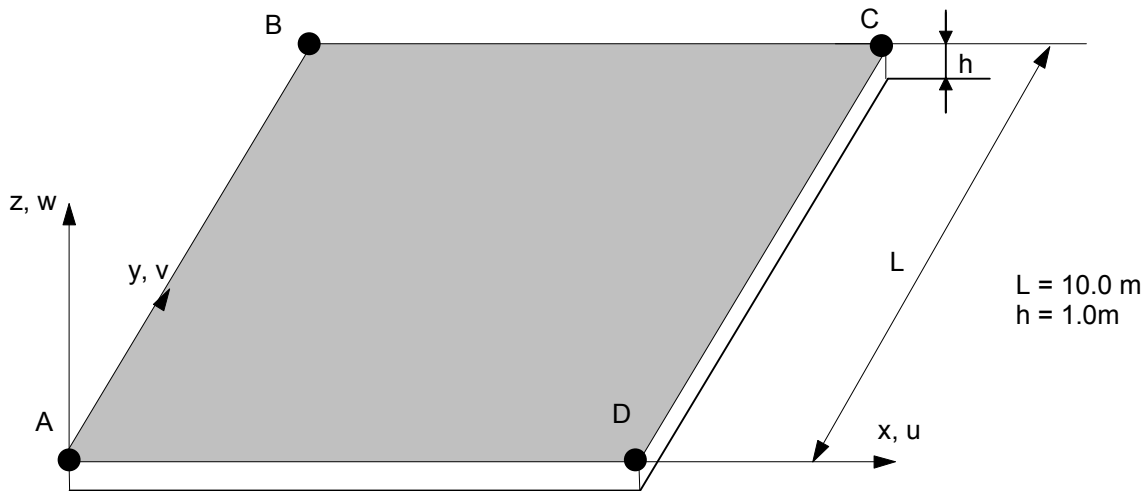
This test represents a calculation in dynamic modal analysis of a thick square plate simply supported. This test makes it possible to validate:

- modelings finite elements DST, DKT, COQUE_3D with meshes QUAD4 and TRIA3, QUAD8 and TRIA6, and 3D with meshes HEXA20,
- the taking into account of rigidity in transverse shearing.

The frequencies and the modes obtained are compared with a reference solution, suggested by NAFEMS, obtained with a calculation finite elements of voluminal type.

1 Problem of reference

1.1 Geometry



1.2 Properties of material

The properties of material constituting the plate are:

$$\begin{aligned} E &= 2.10^{11} && \text{Pa Modulus Young} \\ \nu &= 0.3 && \text{Poisson's ratio} \\ \rho &= 8000. \text{kg/m}^3 && \text{Density} \end{aligned}$$

1.3 Boundary conditions and loadings

Plate simply supported on its contour.

1.4 Initial conditions

Without object

2 Reference solution

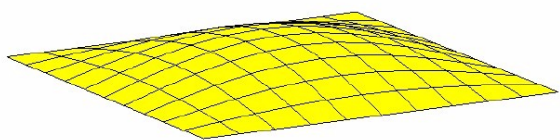
2.1 Method of calculating used for the reference solution

the reference solution suggested by NAFEMS [bib1] was obtained to leave a calculation finite elements 3D with elements bricks with 20 nodes and with a grid 4×4 (plan xy) and 1 element following the thickness.

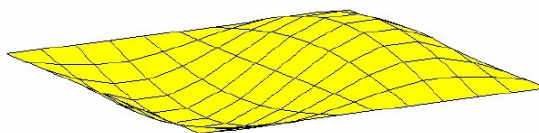
2.2 Results of reference

the first 7 nonworthless frequencies and the associated clean modes, the first three modes are those of body rigid:

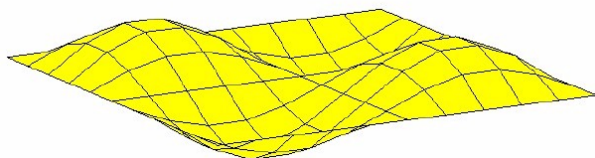
- Frequency (mode 4 except plan) : 44.762 hz
- Frequency (modes 5 & 6 except plan) : 110.52 hz
- Frequency (mode 7 except plan) : 169.08 hz
- Frequency (Mode 8 in the plan) : 193.93 hz
- Frequency (mode 9 & 10 in the plan) : 206.64 hz



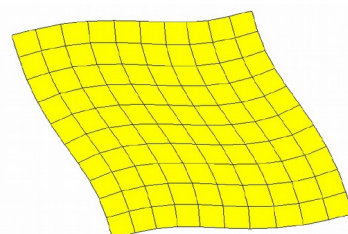
mode 4 except plan



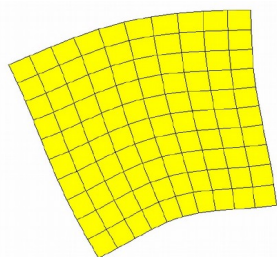
5&6 mode except plan



mode 7 except plan



mode 8 in the plan



9&10 mode in the plan

2.3 Uncertainties on the solution

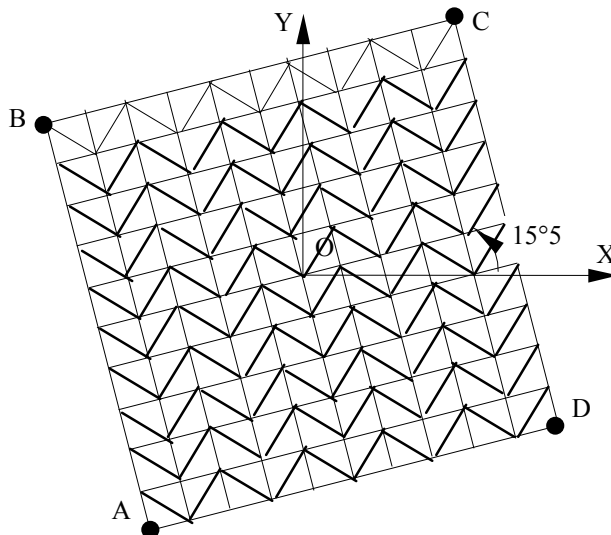
< 2% for a grid identical to that of [2.1], i.e. with few elements.

2.4 Bibliographical references

1. NAFEMS: Standard The NAFEMS Benchmarks, TNSB, rev. 3, October 5th, 1990.

3 Modeling A

3.1 Characteristics of modeling



Modélisation DST (TRIA3)

- La plaque est située dans le plan $Z=2.3$
- Point O : (0. ;0. ;2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w=0$

To validate modeling in a reference mark different from the total reference mark, the plate is turned of $15,5^\circ$. This should not change the Eigen frequencies obtained.

3.2 Characteristics of the grid

Many nodes: 122
Many meshes and types: 200 TRIA3

3.3 Sizes tested and results

Identification	Moments	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	44,989	0,507
Frequency (modes 5 & 6 except plan)		110.52	107,608 107,880	- 2,634 - 2,388
Frequency (mode 7 except plan)		169.08	165,454	- 2,144
Frequency (Mode 8 in the plan)		193.93	196,089	1,114
Frequency (mode 9 & 10 in the plan)		206.64	211,658 212,000	2,428 2,594

3.4 Remarks

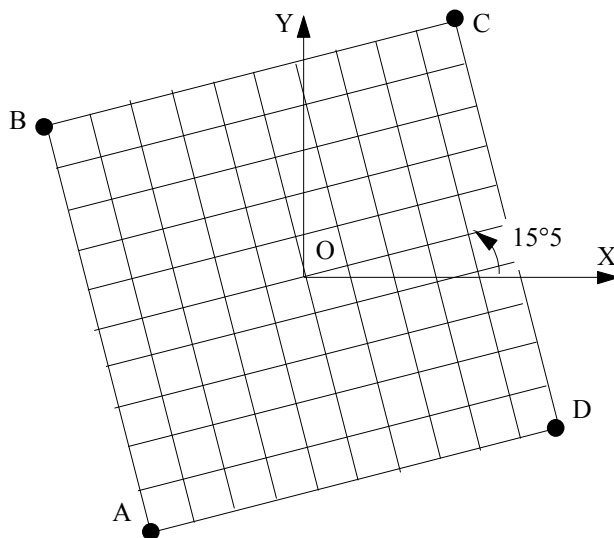
- In Aster, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by *Code_Aster*.
- Appearance of two modes of inflection enters modes 8 and 9 of reference: these modes are modes 6 and 7 of *Code_Aster*.

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	44.98
2	107.61
3	107.88
4	165.45
5	196.09
6	202.80
7	203.54
8	211.66
9	212.00
10	222.53
11	254.74
12	255.62
13	264.73
14	289.85

4 Modeling B

4.1 Characteristics of modeling



Modélisation DST (QUAD4)

- La plaque est située dans le plan $Z= 2.3$
- Point O : (0. ;0. ;2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w=0$

To validate modeling in a reference mark different from the total reference mark, the plate is turned of $15,5^\circ$. This should not change the Eigen frequencies obtained.

4.2 Characteristics of the grid

Many nodes: 122

Many meshes and types: 100 QUAD4

4.3 Sizes tested and results

Identification	Moment s	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	44.64	- 0,273
Frequency (modes 5 & 6 except plan)		110.52	108.04 108.26	- 2,247 - 2,041
Frequency (mode 7 except plan)		169.08	162.86	- 3,681
Frequency (Mode 8 in the plan)		193.93	195.70	0,912
Frequency (mode 9 & 10 in the plan)		206.64	208.89 208.89	1,088

4.4 Remarks

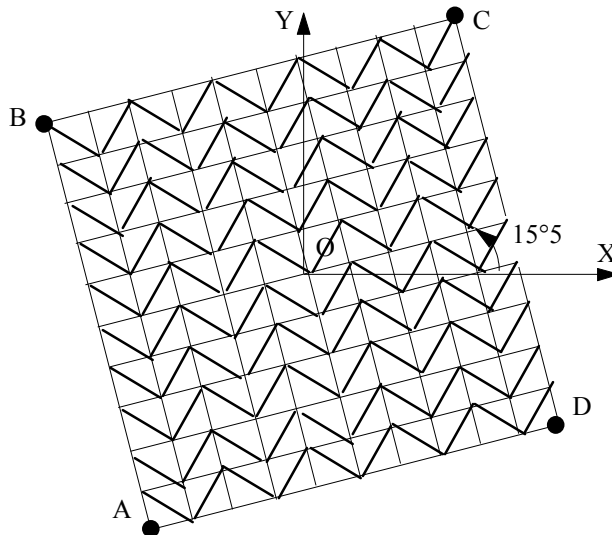
- In Code_Aster, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by Code_Aster.
- Appearance of two modes of inflection enters modes 8 and 9 of reference: these modes are modes 6 and 7 of Code_Aster.

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	44.64
2	108.04
3	108.26
4	162.86
5	195.70
6	203.97
7	206.08
8	208.89
9	208.89
10	220.92
11	248.12
12	250.10
13	252.49
14	289.79

5 Modeling C

5.1 Characteristics of modeling



Modélisation DKT (TRIA6)

- La plaque est située dans le plan $Z=2.3$
- Point O : (0. ;0. ;2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w=0$

To validate modeling in a reference mark different from the total reference mark, the plate is turned of $15,5^\circ$. This should not change the Eigen frequencies obtained.

5.2 Characteristics of the grid

Many nodes: 122
Many meshes and types: 200 TRIA3

5.3 Sizes tested and results

Identification	Moment s	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	47,358	5,799
Frequency (modes 5 & 6 except plan)		110.52	118,029 118,059	6,795 6,822
Frequency (mode 7 except plan)		169.08	187,504	10,897
Frequency (Mode 8 in the plan)		193.93	196,089	1,114
Frequency (mode 9 & 10 in the plan)		206.64	211,658 212,000	2,428 2,594

5.4 Remarks

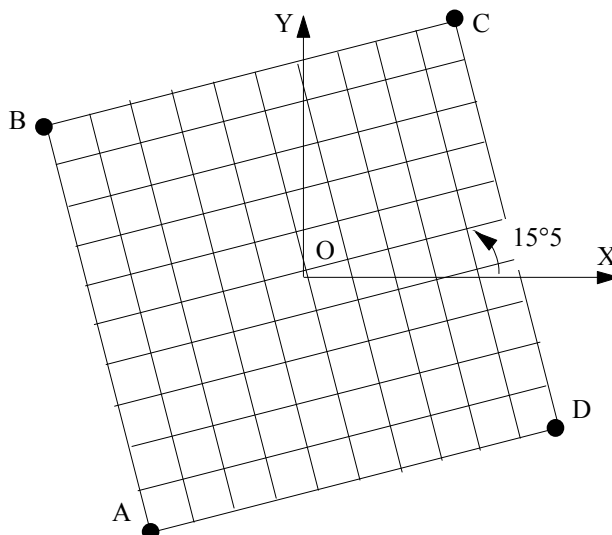
- In Aster, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by *Code_Aster*.
- Appearance of two modes of inflection after mode 11 of reference (mode 8 of *Code_Aster*), same form as those appeared in DST modeling enters modes 8 and 9 of reference (modes 6 and 7 of *Code_Aster*).

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	47,358
2	118.03
3	118.06
4	187.50
5	196.09
6	211.66
7	212.00
8	222.53
9	235.41
10	235.56
11	264.73
12	289.85
13	302.84
14	303.15

6 Modeling D

6.1 Characteristics of modeling



Modélisation DKT (QUAD4)

- La plaque est située dans le plan $Z=2.3$
- Point O : (0. ;0. ;2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w=0$

To validate modeling in a reference mark different from the total reference mark, the plate is turned of $15,5^\circ$. This should not change the Eigen frequencies obtained.

6.2 Characteristics of the grid

Many nodes: 122

Many meshes and types: 100 QUAD4

6.3 Sizes tested and results

Identification	Moment s	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	47,182	5,408
Frequency (modes 5 & 6 except plan)		110.52	117,463	6,283
Frequency (mode 7 except plan)		169.08	184,746	9,266
Frequency (Mode 8 in the plan)		193.93	195,699	0,912
Frequency (mode 9 & 10 in the plan)		206.64	208,887	1,088

6.4 Remarks

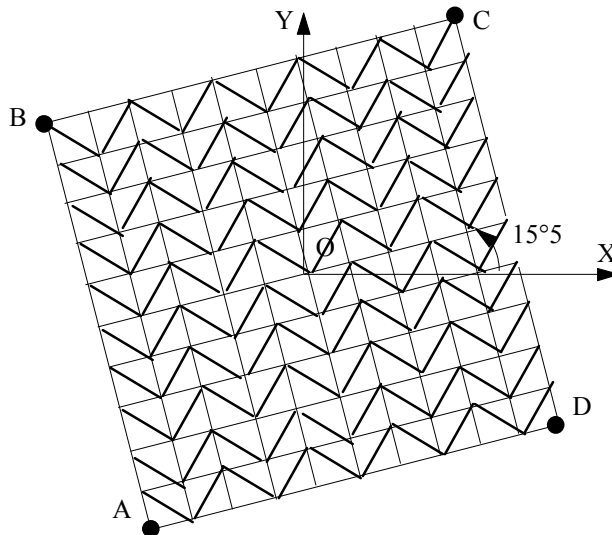
- In Aster, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by *Code_Aster*.
- Appearance of two modes of inflection after mode 11 of reference (mode 8 of *Code_Aster*), same form as those appeared in modeling *DST* between modes 8 and 9 of reference (modes 6 and 7 of *Code_Aster*).

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	47.183
2	117.46
3	117.46
4	184.75
5	195.70
6	208.89
7	208.89
8	220.92
9	234.74
10	234.74
11	252.49
12	289.79
13	297.27
14	297.27

7 Modeling E

7.1 Characteristics of modeling



Modélisation COQUE_3D (TRIA6)

- La plaque est située dans le plan $Z = 2.3$
- Point O : (0. ; 0. ; 2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w = 0$

To validate modeling in a reference mark different from the total reference mark, the plate is turned of $15,5^\circ$. This should not change the Eigen frequencies obtained.

7.2 Characteristics of the grid

Many nodes: 122
Many meshes and types: 200 TRIA6

7.3 Sizes tested and results

Identification	Moment s	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	43,867	2.00
Frequency (modes 5 & 6 except plan)		110.52	106,058 106,066	- 4,037 - 4,029
Frequency (mode 7 except plan)		169.08	160,010	- 5,305
Frequency (Mode 8 in the plan)		193.93	193,600	- 0,170
Frequency (mode 9 & 10 in the plan)		206.64	206,209 206,211	0,208 0,207

7.4 Remarks

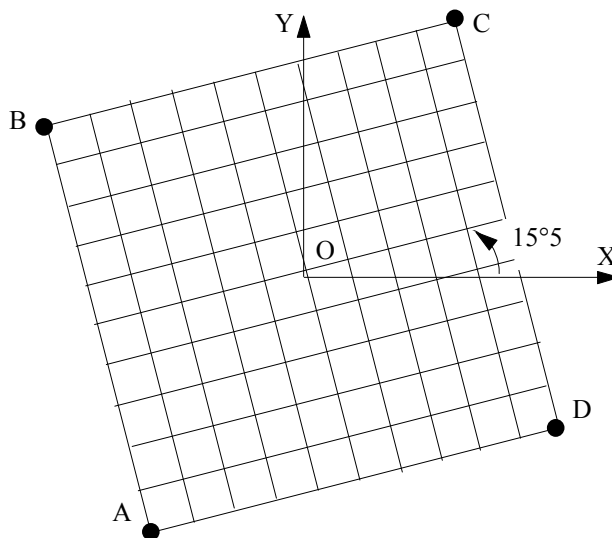
- In Aster, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by *Code_Aster*.
- Appearance of two modes of inflection enters modes 8 and 9 of reference: they are modes 6 and 7 of *Code_Aster*.

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	43.867
2	106.06
3	106.07
4	160.11
5	186.72
6	193.60
7	199.76
8	200.23
9	206.21
10	206.21
11	219.28
12	245.91
13	245.94
14	249.27

8 Modeling F

8.1 Characteristics of modeling



Modélisation COQUE_3D (QUAD8)

- La plaque est située dans le plan $Z=2.3$
- Point O : (0. ;0. ;2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w=0$

To validate modeling in a reference mark different from the total reference mark, the plate is turned of $15,5^\circ$. This should not change the Eigen frequencies obtained.

8.2 Characteristics of the grid

Many nodes: 122
Many meshes and types: 100 QUAD8

8.3 Sizes tested and results

Identification	Moment s	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	43,870	- 1,993
Frequency (modes 5 & 6 except plan)		110.52	106,041	- 4,052
Frequency (mode 7 except plan)		169.08	160,055	- 5,337
Frequency (Mode 8 in the plan)		193.93	193,588	- 0,176
Frequency (mode 9 & 10 in the plan)		206.64	206,192	- 0,216

8.4 Remarks

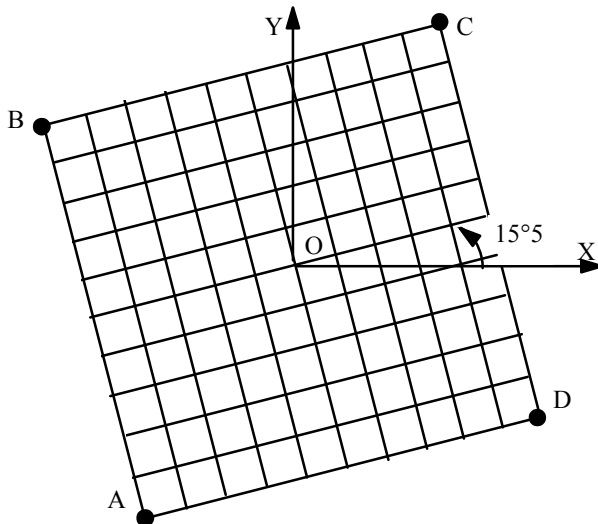
- In *Code_Aster*, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by *Code_Aster*.
- Appearance of two modes of inflection enters modes 8 and 9 of reference: they are modes 6 and 7 of *Code_Aster*.

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	43.87
2	106.04
3	106.04
4	160.06
5	193.59
6	199.64
7	200.13
8	206.19
9	206.19
10	219.26
11	245.68
12	245.68
13	249.20
14	287.99

9 Modeling G

9.1 Characteristics of modeling

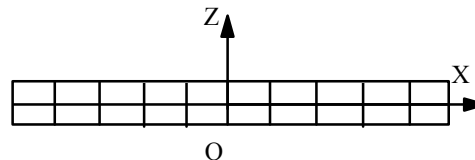


Modélisation 3D (HEXA20)

- Le plan moyen de la plaque est situé dans le plan $Z = 2.3$
- Point O : (0. ; 0. ; 2.3)

Conditions aux limites :

- Cotés AB, BC, CD, DA : $w=0$



9.2 Characteristics of the grid

Many nodes: 1266
Many meshes and types: 200 HEXA20

9.3 Sizes tested and results

Identification	Moment s	Reference	Aster	% difference
Frequency (mode 4 except plan)		44,762	43,862	- 2,009
Frequency (modes 5 & 6 except plan)		110.52	105,953	- 4,132
Frequency (mode 7 except plan)		169.08	159,749	- 5,518
Frequency (Mode 8 in the plan)		193.93	193,590	- 0,175
Frequency (mode 9 & 10 in the plan)		206.64	199,410 199,903	- 3,498 - 3,260

9.4 Remarks

- In *Code_Aster*, the calculated modes are those of rigid body: the fourth mode of reference is the first mode calculated by *Code_Aster*.
- Appearance of two modes of inflection enters modes 8 and 9 of reference: they are modes 6 and 7 of *Code_Aster*.

In the table below we deferred the first 14 found Eigen frequencies.

N° mode	Frequency (Hz)
1	43.86
2	105.95
3	105.95
4	159.75
5	193.59
6	199.41
7	199.90
8	206.16
9	206.16
10	219.27
11	245.07
12	245.07
13	249.13
14	287.75

10 Summary of the results

Taking into account the nature of the digital solution (voluminal finite elements), the got results are satisfactory for:

- modeling A and B (DST) the maximum change is of less 4% for the first 5 modes,
- modeling E and F (COQUE_3D), the maximum change is of approximately 5% for the first 5 modes,
- modeling G (3D), the maximum change is of approximately 5% for the first 5 modes,
- the modes of reference 5 and 6 except plan have symmetry different from those met in modelings E, F and G, but they are equivalent because they are modal recombinations.

Modelings C and D (DKT) are less satisfactory with relative variations reaching 10% on mode 7 except plan, this due to is not taken into account of transverse shearing for this relatively thick plate.

Moreover one observes the appearance of modes of inflection and membrane for all these modelings, including modeling 3D voluminal G. When one refines sufficiently the grids, this tendency is confirmed and the relative variations regress. Calculation 3D in addition showed that in on this side grid 6×6 in the plan (XY), the modes of inflection and membrane were not detected.