
SDLS114 - Calculation of the stress intensity factors of a plate fissured by modal recombination

Summary

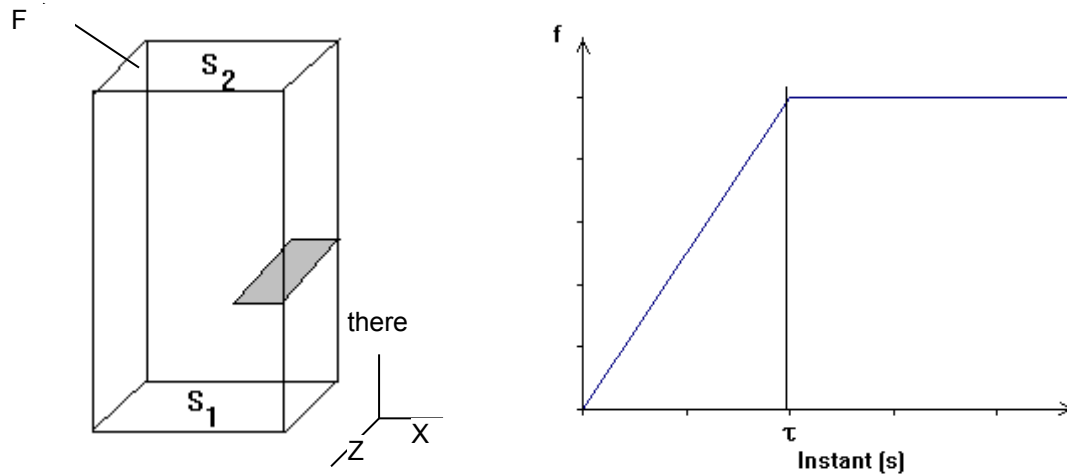
This test aims at validating the calculation of the factors of intensity of a plate fissured by modal recombination. The modal factors of intensity, therefore associated with each clean mode of vibration of the structure, are calculated with the operator `CALC_G` (option `CALC_K_G`). Modelings illustrate the possibility of combining the modal factors of intensity directly in the command file by instructions python via the operator `POST_K_TRANS`.

This test contains two modelings 2D (modeling A and C) and two modelings 3D (B and D). The reference solution is resulting from a direct temporal resolution of the transitory problem. Two modelings A and B are for the cracks with a grid. Two modelings C and D are for the cracks not-with a grid (X-FEM).

1 Problem of reference

1.1 Geometry

A plate height is considered $h=0,1\text{ m}$, of width $l=0,05\text{ m}$ and thickness $e=0.005\text{ m}$. A crack is positioned in the middle of the height of the beam, with a depth of $0,1l$.



1.2 Material properties

One considers the classical properties of a steel:

Young modulus:	$E = 2.10^5 \text{ MPa}$
Poisson's ratio:	$\nu = 0.3$
Density	$\rho = 7800 \text{ kg/m}^3$

1.3 Boundary conditions and loadings

The plate is:

- 1) embedded on surface S_1 ;
- 2) subjected to a force $F(t)$ on surface S_2 .

Evolution of the standard of $F(t)$ is traced on the figure above. One takes $\tau = 0,001\text{ s}$. Direction of the force $F(t)$ is the following one:

- 1) $F(t) = f(t) \cdot e_x$ for modeling A;
- 2) $F(t) = (a e_x + b e_y + c e_z) f(t)$ for modeling B, with $b = 2a$ and $c = 0.4 a$.

For modeling A, one blocks displacements in the direction z (problem plan).

2 Reference solution

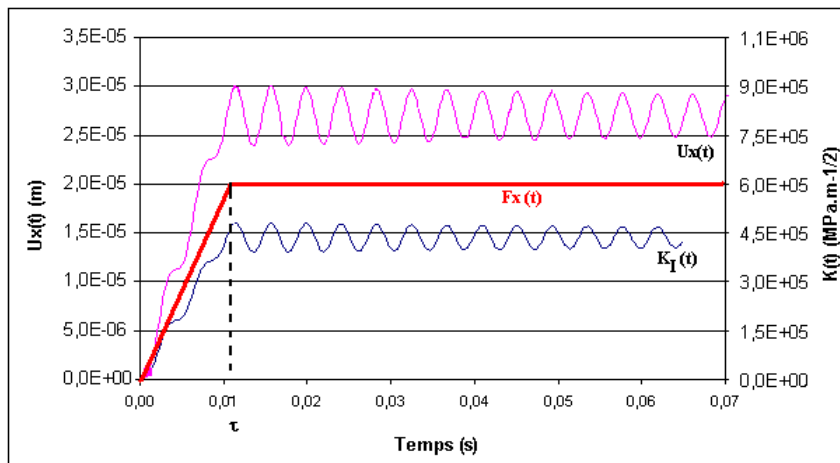
2.1 Method of calculating used

The reference solution is that obtained by a direct temporal resolution of the transitory problem. The operator `DYNA_VIBRA` is used to identify the fields of displacement, with a diagram of integration in times of Newmark.

The evolution of the stress intensity factors according to time is then calculated by interpolation of the jumps of displacements (operator `POST_K1_K2_K3`).

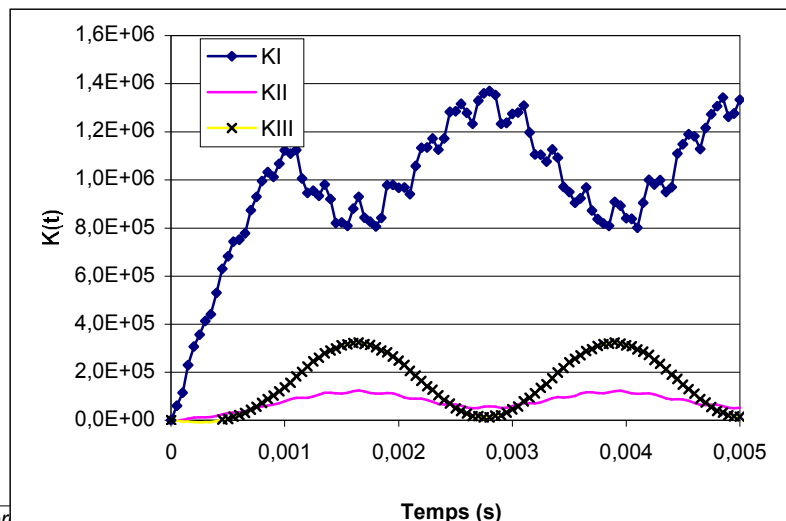
2.2 Results of reference – Modeling A

For modeling A, the plate is solicited by a force in the plan (O, x, y) and displacements in the direction z are blocked. The result of reference, calculated by direct temporal resolution on a grid 2D, is traced on the following figure. Horizontal displacement from the top of the plate and the factor of intensity of the constraints oscillate with a frequency corresponding to the first clean mode of the structure.



2.3 Results of reference Modeling B

The evolution of the three factors of intensity of the constraints is traced on the following figure for the node located in the middle of the bottom of crack. The oscillations of the factors of intensity of the



constraints show dominating contribution of the first mode of inflection of the plate in the direction x and of the first mode of inflection in the direction z .

2.4 Results of reference – Modeling C

Modeling C represents a crack 2D not-with a grid (X-FEM). The other characteristics are identical to modeling A. For modeling C, the reference solution is the solution calculated by Code_Aster starting from a direct temporal resolution of the transitory problem. The orders used to generate this solution are provided in the file orders as comments.

2.5 Results of reference – Modeling D

Modeling D represents a crack 3D not-with a grid (X-FEM). The other characteristics are identical to modeling B. For modeling D, the reference solution is the solution calculated by Code_Aster starting from a direct temporal resolution of the transitory problem. The orders used to generate this solution are provided in the file orders as comments.

2.6 Uncertainty on the solution

The explicit direct resolution of the transitory problem can be regarded as exact. Uncertainty on the identification of the factors of intensity of the constraints by interpolation of the jumps of displacements is about 5%.

2.7 Bibliographical references

1. E. CRYSTAL, S. DI DOMIZIO: Method theta in breaking process: development of the bilinear form G in 3D and application to the case of the dynamic low frequency, Notes EDF HT-65/05/024/A, 2005

3 Modeling A

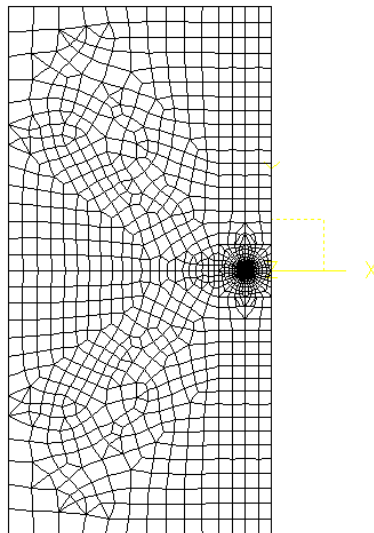
3.1 Characteristics of modeling

It is about a modeling 2D – plane constraints. The calculation of the evolution of the factors of intensity of the constraints according to time is carried out in several stages:

- 1) calculation of the first 15 clean modes of the structure;
- 2) calculation of the modal factors of intensity of the constraints associated with these modes by two methods;
- 3) resolution of the transitory dynamic problem by projection on modal basis;
- 4) recombination of K modal.

3.2 Characteristics of the grid

The grid is composed of quadratic elements. It comprises 2000 nodes and 700 meshes and is refined around the bottom of crack.



3.3 Sizes tested and results

Modal values: case test of not-regression

Number of the mode	K_I (POST_K1_K2_K3)	K_I (CALC_K_G)	% difference
1	-1,921.E+10	-1,898.E+10	1.2
2	-1,166.E+11	-1,152.E+11	1.3
3	8,039.E+10	7,948.E+10	1.1
4	-1,188.E+11	-1,174.E+11	1.2
5	1,723.E+11	1,705.E+11	1.1

Temporal values $K_I(t)$: comparison with the explicit resolution

Moment	Reference	Aster	% difference
0.0005	24055.6	24337.0	1.2
0.001	44676.8	45159.3	1.1
0.002	90592.3	91679.4	1.2
0.003	134065.3	135633.9	1.2
0.004	181113.3	183286.7	1.2

3.4 Notice

The difference between the modal values calculated by interpolation of the jumps in displacement or the method theta is weak and coherent with that observed on the static problems.

The value of $K_I(t)$ is calculated from K_I modal (method CALC_K_G) and of the coefficients of the resolution about modal base directly in the case test by command lines in python:

$$K_I(t) = \sum_{i=1}^M \alpha_i(t) \cdot K_I^i$$

where coefficients $\alpha_i(t)$ are the coefficients of modal participation, extracted the result of the operator DYNA_VIBRA, and them K_I^i are the modal factors of intensity of the constraints.

The precision obtained is satisfactory taking into account the number of elements retained in the modal base. The precision increases quickly with the number of modes [bib1].

4 Modeling B

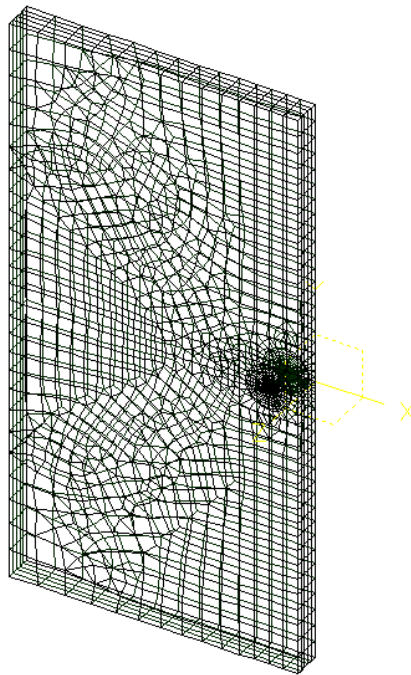
4.1 Characteristics of modeling

It is about a modeling 3D. The calculation of the evolution of the factors of intensity of the constraints according to time is carried out in several stages:

- 1) calculation of the first 50 clean modes of the structure;
- 2) calculation of the modal factors of intensity of the constraints associated with these modes by two methods;
- 3) resolution of the transitory dynamic problem by projection on modal basis;
- 4) recombination of K modal.

4.2 Characteristics of the grid

The grid is composed of linear elements. It comprises 8200 nodes and 8900 meshes and is refined around the bottom of crack.



4.3 Sizes tested and results

The values indicated are those found with the node which is in the middle of the bottom of crack.

Modal values: case test of not-regression

Number of the mode	$K_I (\text{POST_K1_K2_K3})$	$K_I (\text{CALC_K_G})$	% difference
1	5,631E+09	4,790E+09	14.9
2	8,599E+09	7,291E+09	15.2
3	6,940E+10	5,897E+10	15.0
4	-2,702E+11	-2,897E+11	-7.2
5	-9,637E+10	-8,165E+10	15.3

Temporal values $K_I(t)$: comparison with the explicit resolution

Moment (s)	Reference (Pa. \sqrt{m})	Aster (Pa. \sqrt{m})	% difference
0,0005	696752.4	721825.9	3.6
0,001	1153703.3	1239061.8	7.4
0,002	997675.6	1110569.6	11.3
0,003	1305429.9	1364524.8	4.5
0,004	870347.2	1004735.2	15.4

4.4 Notice

The difference between the modal values calculated by interpolation of the jumps in displacement or the method theta is high: that is explained by the linear grid very little refined in the thickness of the plate.

The value of $K_I(t)$ is calculated from K_I modal (method CALC_K_G) and of the coefficients of the resolution about modal base directly in the case test by command lines in python:

$$K_I(s, t) = \sum_{i=1}^M \alpha_i \cdot K_I^i(s)$$

where coefficients $\alpha_i(t)$ are the coefficients of modal participation, extracted the result of the operator DYNA_VIBRA, and them $K_I^i(s)$ are the modal factors of intensity of the constraints.

The precision obtained is satisfactory taking into account the number of elements retained in the modal base (50) and grid cuts it. The precision increases quickly with the number of modes [bib1].

5 Modeling C

5.1 Characteristics of modeling

The crack is modelled by method X-FEM. A modeling is used C_PLAN.

5.2 Characteristics of the grid

The grid contains 96000 elements of the type QUAD4.

5.3 Sizes tested and results

One tests the dynamic stress intensity factors $K_I(t)$ and $K_{II}(t)$ with $t=2.E-3 sec$.

Identification	Type of reference	Value of reference	Tolerance
$K_I(t)$	'AUTRE_ASTER'	2.41542E+06	0.2%
$K_{II}(t)$	'AUTRE_ASTER'	3.57437E+05	0.6%

6 Modeling D

6.1 Characteristics of modeling

The crack is modelled by method X-FEM. A modeling is used 3D.

6.2 Characteristics of the grid

The grid contains 1875 elements of the type HEXA8.

6.3 Sizes tested and results

One tests the dynamic stress intensity factors $K_I(t)$, $K_{II}(t)$ and $K_{III}(t)$ with $t=4.E-3$ sec for the bottom of crack in the middle of thickness.

Identification	Type of reference	Value of reference	Tolerance
$K_I(t)$	'AUTRE_ASTER'	3.5521523 E+06	2.0%
$K_{II}(t)$	'AUTRE_ASTER'	4.2648023 E+05	10.0%
$K_{III}(t)$	'AUTRE_ASTER'	-8.5924717E+05	4%

7 Summary of the results

This test makes it possible to validate the calculation of the modal factors of intensity by the operator `CALC_G` (option `CALC_K_G`) and illustrates their use for the resolution of a problem of breaking process in dynamic low frequency by modal recombination.

The relationship between the computing times of the resolution clarifies and of the resolution about modal base are understood between 10 and 50 according to the type of grid, and the precision of the method of modal recombination is fully satisfactory.

The results of modelings C and D make it possible to extend the application of method X-FEM to calculate the modal factors of intensity of the constraints. The next stage will be for the resolution of a problem of breaking process in dynamic low frequency by modal recombination