SDLL311 - Transitory dynamic response of a beam in traction under imposed displacement

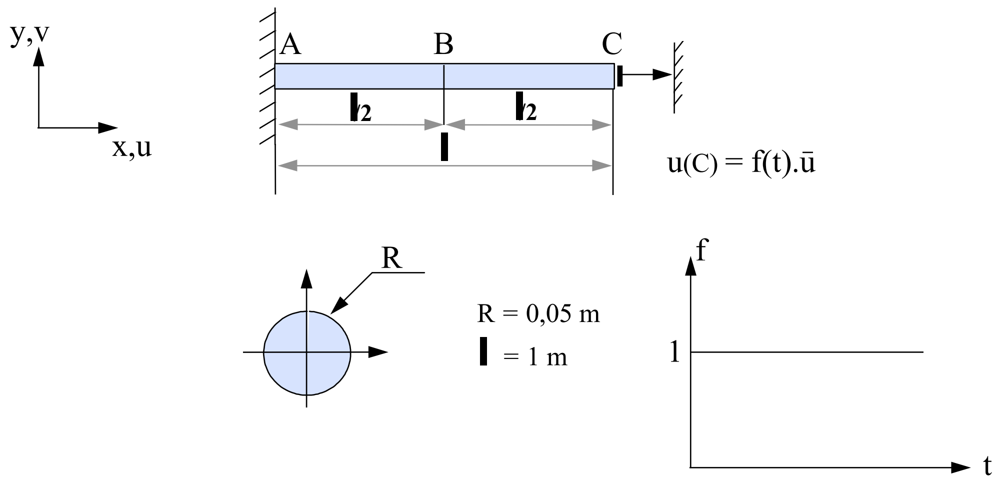
Summary:

This problem-test corresponds to a linear transitory analysis of a bar requested in traction by application of a displacement imposed at an end, the other end being embedded. Function displacement of time is of type "Heaviside" imposed as from the initial moment.

The results got in the middle of the beam for a modeling with four elements are compared with the analytical solution of the problem discretized by four elements by not taking into account the instantaneous peaks speed and acceleration at the initial moment on the level of the end where displacement is imposed.

1 Problem of reference

1.1 Geometry



1.2 Material properties

$E=98696{,}044\ MPa$

$\nu=0$

$\rho=3.10^6\ kg/m^3$

Damping proportional of Rayleigh: $\boldsymbol{C}=\lambda\boldsymbol{K}+\mu\boldsymbol{M}$, $\lambda=5.10^{-4}$, $\mu=5$

1.3 Boundary conditions and loadings

Displacement imposed at the end C : $u(C)=\bar{u}\,f(t)$ with $\bar{u}=10^{-3}m$ and $f(t)$ evolution according to the time of the Heaviside type: $f(t)=1$, $t\geq 0$.

End A embedded.

1.4 Initial conditions

Initial displacement no one in any point.

Worthless initial speed in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

The discretized problem checks:

$$\begin{bmatrix} M_{ll} & M_{ld} \\ M_{ld}^T & M_{dd} \end{bmatrix} \begin{Bmatrix} \ddot{u}_l \\ \ddot{u}_d \end{Bmatrix} + \begin{bmatrix} C_{ll} & C_{ld} \\ C_{ld}^T & C_{dd} \end{bmatrix} \begin{Bmatrix} \dot{u}_l \\ \dot{u}_d \end{Bmatrix} + \begin{bmatrix} K_{ll} & K_{ld} \\ K_{ld}^T & K_{dd} \end{bmatrix} \begin{Bmatrix} u_l \\ u_d \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_d \end{Bmatrix} ,$$

with index l : free degree of freedom
index d : degree of freedom imposed

$F_d(t)$ external loadings applied to the nodes ends and leading to imposed displacements u_d are unknown, one thus eliminates these equations and one obtains:

$$[M_{ll}]\{\ddot{u}_l\} + [C_{ll}]\{\dot{u}_l\} + [K_{ll}]\{u_l\} = -[M_{ld}]\{\ddot{u}_d\} - [C_{ld}]\{\dot{u}_d\} - [K_{ld}]\{u_d\} .$$

The only nonworthless terms of the second member of this system are related to variable kinematics relating to the node end where displacement is imposed. However, with $t=0$, $\ddot{u}_{dC}$ and $\dot{u}_{dC}$ are not defined but with $t=0^-$ and $t=0^+$, $\ddot{u}_{dC}$ and $\dot{u}_{dC}$ are worthless. All the complexity of the problem comes from that.

To obtain a reference solution, we considered $\ddot{u}_{dC}$ and $\dot{u}_{dC}$ uniformly worthless what amounts considering only the elastic internal forces at the end C . This is debatable from a physical point of view but, by adopting the same assumptions at the time of the modeling of the problem, the validation of *Code_Aster* can be concluded.

One calculates the reference solution by dealing with the following problem:

$$[M_{ll}]\{\ddot{u}_l\}+[C_{ll}]\{\dot{u}_l\}+[K_{ll}]\{u_l\}=-[K_{ld}]\{u_d(t)\} \text{ with } \{u_l(0)\}=0 \text{ and } \{\dot{u}_l(0)\}=0 .$$

With this intention, one transports the problem in the modal base of the system which checks:

$$[M_{ll}]\{\ddot{u}_l\}+[K_{ll}]\{u_l\}=0 .$$

Damping being diagonal, the diagonal system is obtained:

$$\lfloor m_g \rfloor \{\ddot{X}\} + \lfloor c_g \rfloor \{\dot{X}\} + \lfloor k_g \rfloor \{X\} = \{g(t)\} \text{ où } \{g(t)\} = \{g\} \text{ pour } t \geq 0 ,$$

$$\text{with } \{X(0)\}=0 \text{ and } \{\dot{X}(0)\}=0 .$$

In modal space, one thus solves three equations (3 free degrees of freedom) differential of the second order then one returns in physical space. One obtains then the displacement of the point medium:

$$u_B(t)=\sum_{i+1}^{3} e^{-\lambda i t}(a_i \cos(\tilde{\omega}_i t)+b_i \sin(\tilde{\omega}_i t)) ,$$

with $\tilde{\omega}_i$: i ème own pseudo-pulsation of the deadened system.

2.2 Results of reference

Displacement, speed and acceleration of the point medium B beam.

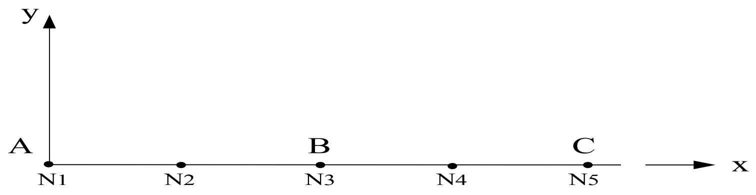
2.3 Uncertainty on the solution

Analytical solution of the problem discretized in four elements length equal by considering speed and acceleration uniformly worthless to the point C where displacement is imposed.

3 Modeling A

3.1 Characteristics of modeling

Modeling in element of beam `3D` : `POU_D_T`



Cutting:
AC = 4 meshs `SEG2` of equal length

Limiting conditions:

- Node $N1(A)$ embedded
  `DDL_IMPO` $DX=DY=DZ=DRX=DRY=DRZ=0$
- Node $N5(C)$ in following imposed displacement x
  `DDL_IMPO` $DY=DZ=DRX=DRY=DRZ=0$ $DX(t)=\bar{u}$

Resolution:
Algorithm of direct integration of Newmark
Pas de time: $\Delta t=10^{-5}s$
Duration of observation: $0{,}03\,s$

3.2 Characteristics of the grid

Number of node S: 5
Number of meshs and type: 4 meshs `SEG2`

3.3 Sizes tested and results

- Displacement at the point medium B

Time (s)	**Displacement Reference** (m)
0.0054	87.376 e-3
0.0055	87.360 e-3
0.0108	26.818 e-3
0.0109	26.800 e-3
0.0163	64.386 e-3
0.0164	64.366 e-3
0.0217	41.083 e-3
0.0218	41.084 e-3
0.0271	55.525 e-3
0.0272	55.530 e-3

4 Modeling B

4.1 Characteristics of modeling

idem that modeling A

4.2 Characteristics of the grid

idem that modeling A

4.3 Sizes tested and results

- Displacement at the point medium B

Time (s)	**Displacement Reference** (m)
0.0054	87.376 e-3
0.0055	87.360 e-3
0.0108	26.818 e-3
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5 Summary of the results

Results given by *Code_Aster* are in perfect agreement with the results of the analytical model, that displacement boils about it beam is imposed by one VECTOR ASSEMBLES or by one LOAD.

Caution: questions of Dirichlet for transitory calculation on physical basis with DYNA_VIBRA are compatible only with the method of integration of NEWMARK.