SDLL146 - Validation of the elements “bars” in dynamics

Summary:
The objective of this test is to validate the calculation of the matrix of mass of the elements BAR . It is checked that this matrix is quite threedirectional contrary to the matrix of rigidity (one-way parallel to the orientation of the element). For that one does the following calculations:

- Calculation of the internal forces and the reactions of support of an embedded element subjected to a field of gravity according to 3 directions (parallels with X , Y and Z) .
- Projection of the matrix of mass on a basis made up of three modes (parallel with X , Y and Z).
- Calculation of the kinetic energy for three modes speed (parallel with X , Y and Z).

Note: all the calculations are done with the matrix of complete mass and the matrix of diagonal mass except for the calculation of the kinetic energy which is carried out with the matrix of complete mass.

1 Problem of reference

1.1 Geometry

One considers a mesh SEG2 of with dimensions $1\,m$, directed the axis parallel to X .

1.2 Properties of material

The material is elastic isotropic whose properties are:

$E=37\,000\,MPa$

$\nu=0.2$

$\rho=100\,kg/m^3$

1.3 Boundary conditions and loadings

The node in O is embedded. In B , DY and DZ are blocked. The imposed loading is carried out with CALC_CHAR_SEISME and directed according to X , then Y and Z , it corresponds to a field of gravity according to X , then Y and Z .

2 Reference solution

2.1 Method of calculating

2.1.1 Recalls

Modeling BAR neither shearing action transmits nor bending moment. So if one notes E the Young modulus of the element, A the surface of its section and L its length, the elementary matrix of rigidity K^{elem} of a bar is the following one (with the components in the order $(DX_1\, DY_1\, DZ_1\, DX_2\, DY_2\, DZ_2)$):

$$K^{elem}=\begin{pmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Profile of the matrix of mass M^{elem} is different from the matrix of rigidity because the mass must be taken into account in all the directions of space. Thus, if one notes ρ density of the element, the elementary matrix of mass, with $m=\rho\, A\, L$, is the following one:

$$M^{elem}=\begin{pmatrix} m/3 & 0 & 0 & m/6 & 0 & 0 \\ 0 & m/3 & 0 & 0 & m/6 & 0 \\ 0 & 0 & m/3 & 0 & 0 & m/6 \\ m/6 & 0 & 0 & m/3 & 0 & 0 \\ 0 & m/6 & 0 & 0 & m/3 & 0 \\ 0 & 0 & m/6 & 0 & 0 & m/3 \end{pmatrix}$$

2.1.2 Calculation of the internal forces and the reactions of support

That is to say m mass of the element and U displacement, then if one chooses a field of gravity according to X , external forces in each node are the following ones: $(-m/2, 0\, , 0)$. For an elastic behavior of rigidity K , only displacement not imposed U_{B_X} is equal to $\frac{F^{ext}_{B_X}}{K}=\frac{-m}{2\mathrm{K}}$.

If one notes K^{elem} the elementary matrix of rigidity, one has the relation $F^{int}=K^{elem}U$. One gets the following results for the internal forces: $F^{int}_{O_X}=\frac{m}{2}$, $F^{int}_{O_Y}=0$, $F^{int}_{O_Z}=0$, $F^{int}_{B_X}=\frac{-m}{2}$, $F^{int}_{B_Y}=0$ and $F^{int}_{B_Z}=0$.

While noting R^{ap} the reactions of support, there is the relation $R^{ap}=F^{int}-F^{ext}$. One thus has easily: $R^{ap}_{O_X}=m$ and all other worthless components.

Note: for the directions of loading Y and Z , there is no displacement thus the internal forces are worthless and the reactions of support are equal to the external forces.

2.1.3 Projection of the matrix of mass on a modal basis

The purpose of this calculation is to check that for a unit mode of displacement ϕ according to a given direction, there is the equality:

$$\phi^T M \phi = m$$

2.1.4 Calculation of the kinetic energy

The purpose of this calculation is to check that for a unit mode speed ϕ according to a given direction, there is the equality:

$$\frac{1}{2}\phi^T M \phi = \frac{mv}{2} \text{ with } v=1$$

3 Modeling A

3.1 Characteristics of modeling

A modeling is used BAR.

3.2 Characteristics of the grid

The grid contains 1 element of the type SEG2.

3.3 Sizes tested and results

3.3.1 Reactions of support and internal forces to the two nodes of the mesh.

Matrix supplements gravity according to X :

Size	Place	Component	Type of reference	Value of reference (NR)	Tolerance (%)
REAC_NODA	N001 node	DX	'ANALYTICAL'	-100	1.0E-4
REAC_NODA	N002 node	DX	'ANALYTICAL'	0	1.0E-4
FORC_NODA	N001 node	DX	'ANALYTICAL'	-50	1.0E-4
FORC_NODA	N002 node	DX	'ANALYTICAL'	-50	1.0E-4

Matrix supplements gravity according to Y :

Size	Place	Component	Type of reference	Value of reference (NR)	Tolerance (%)
REAC_NODA	N001 node	DY	'ANALYTICAL'	-50	1.0E-4
REAC_NODA	N002 node	DY	'ANALYTICAL'	-50	1.0E-4
FORC_NODA	N001 node	DY	'ANALYTICAL'	0	1.0E-4
FORC_NODA	N002 node	DY	'ANALYTICAL'	0	1.0E-4

Matrix supplements gravity according to Z :

Size	Place	Component	Type of reference	Value of reference (NR)	Tolerance (%)
REAC_NODA	N001 node	DZ	'ANALYTICAL'	-50	1.0E-4
REAC_NODA	N002 node	DZ	'ANALYTICAL'	-50	1.0E-4
FORC_NODA	N001 node	DZ	'ANALYTICAL'	0	1.0E-4
FORC_NODA	N002 node	DZ	'ANALYTICAL'	0	1.0E-4

Diagonal matrix gravity according to X :

Size	Place	Component	Type of reference	Value of reference (NR)	Tolerance (%)
REAC_NODA	N001 node	DX	'ANALYTICAL'	-100	1.0E-4
REAC_NODA	N002 node	DX	'ANALYTICAL'	0	1.0E-4
FORC_NODA	N001 node	DX	'ANALYTICAL'	-50	1.0E-4
FORC_NODA	N002 node	DX	'ANALYTICAL'	-50	1.0E-4

Diagonal matrix gravity according to Y :

Size	Place	Component	Type of reference	Value of reference (NR)	Tolerance (%)
REAC_NODA	N001 node	DY	'ANALYTICAL'	-50	1.0E-4
REAC_NODA	N002 node	DY	'ANALYTICAL'	-50	1.0E-4
FORC_NODA	N001 node	DY	'ANALYTICAL'	0	1.0E-4
FORC_NODA	N002 node	DY	'ANALYTICAL'	0	1.0E-4

Diagonal matrix gravity according to Z :

Size	Place	Component	Type of reference	Value of reference (NR)	Tolerance (%)
REAC_NODA	N001 node	DZ	'ANALYTICAL'	-50	1.0E-4
REAC_NODA	N002 node	DZ	'ANALYTICAL'	-50	1.0E-4
FORC_NODA	N001 node	DZ	'ANALYTICAL'	0	1.0E-4
FORC_NODA	N002 node	DZ	'ANALYTICAL'	0	1.0E-4

3.3.2 Projection of the matrix of mass on the modal basis

Complete matrix

TABLE	Type of reference	Value of reference (kg)	Tolerance (%)
TABX	'ANALYTICAL'	100.0	1.0E-4
TABY	'ANALYTICAL'	100.0	1.0E-4
TABZ	'ANALYTICAL'	100.0	1.0E-4

Diagonal matrix

TABLE	Type of reference	Value of reference (kg)	Tolerance (%)
TABX	'ANALYTICAL'	100.0	1.0E-4
TABY	'ANALYTICAL'	100.0	1.0E-4
TABZ	'ANALYTICAL'	100.0	1.0E-4

3.3.3 Kinetic energy

Complete matrix

TABLE	NOM_PARA	Type of reference	Value of reference (J)	Tolerance (%)
TABX	TOTAL	'ANALYTICAL'	50.0	1.0E-4
TABY	TOTAL	'ANALYTICAL'	50.0	1.0E-4
TABZ	TOTAL	'ANALYTICAL'	50.0	1.0E-4

4 Summary of the results

The tests carried out in this documentation show that the mass of the element BAR is applied in the three directions of space.