SDLL141 - Eigen frequencies of a beam alone, subjected to the gyroscopic effect.

Summary:

This problem consists in seeking the frequencies of vibration of a beam pressed on each one of its ends, on infinitely rigid supports. The beam is full, of section circular and subjected at a number of constant revolutions. It comprises any disc.

Two modeling are studied:

- Modeling a: the beam is along the axis X,
- Modeling b: the beam is along the axis t such as t directing vector of the bisectrix (x, y).
- Modeling C: the beam is along the axis t such as t directing vector of the bisectrix (x, y), and it mass is distributed by discrete elements installed on each node.
- Modeling D: the beam is along the axis X. The section is circular and variable with the two rays $R1$ and $R2$ identical.

This problem thus makes it possible to test the effect of the gyroscopic matrix which was developed for a right beam.

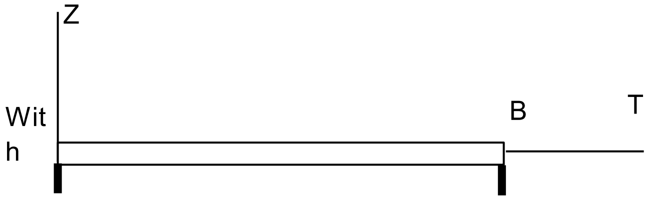
The gyroscopic effect led to the unfolding of the modes. The evolution of the Eigen frequencies according to the number of revolutions makes it possible to build the diagram of Campbell.

The references are based on the theory of Euler-Bernouilli.

The got results are in concord with those given in reference.

1 Problem of reference

1.1 Geometry



Modelings A and D: SDLL141a and SDLL141d

$t=x$

Modelings B and C: SDLL141b and SDLL141c

$\frac{\pi}{4}=(\hat{x},t)$ and $t.z=0$

For modeling C (SDLL141c) the density of material is taken equal to zero. The mass is installed via discrete elements installed on each node.

Length of the beam:

$L=AB=0.9\,m$

Circular section:

Diameter: $D=0.05\,m$

Coordinates of the points (m):

		Modelings With and D	Modelings B and C
	A	B	B
X	0.	0.9	$0.9\cos(\pi/4)$
Y	0.	0.	$0.9\sin(\pi/4)$
Z	0.	0.	0.

Table 1.1-1 : Coordinates of the points A and B

1.2 Material properties

$E=2.10^{11}\,Pa$
$\rho=7\,800\,kg/m^3$ except for modeling C.

1.3 Boundary conditions and loadings

Not A : supported $u=v=w=0$
Not B : supported $u=v=w=0$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is that presented in the work of Rene-Jean GIBERT.
By adopting the following notations:

- beam according to x
- y and z movements of inflection in the plan xOz and xOy
- S : section of the beam
- I : moment of inertia of inflection compared to the axes y and z
- I_x : moment of inertia per unit of length compared to the axis Ox
- ρ , E characteristics of material
- Ω number of revolutions of the beam

The solution singular are controls by the system of equations according to:

$$EI\frac{\partial^4 Y}{\partial x^4}-\rho S\omega^2 Y+i\omega\Omega I_x\frac{\partial^2 Z}{\partial z^2}=0$$
$$EI\frac{\partial^4 Z}{\partial x^4}-\rho S\omega^2 Z-i\omega\Omega I_x\frac{\partial^2 Y}{\partial z^2}=0$$

by observing the boundary conditions following:

$$\begin{cases} Y=Z=0 \\ \dfrac{\partial^2 Y}{\partial z^2}=\dfrac{\partial^2 Z}{\partial z^2}=0 \end{cases} \text{in} \begin{cases} x=0 \\ x=L \end{cases}$$

One obtains two families of clean modes:

- Mode retrogresses:

$$Y_1=-i.Z_1=\sin\frac{n\pi x}{L} \text{ with } \left(\frac{\omega_1}{\omega_0}\right)=\sqrt{\lambda^2+1}-\lambda$$

- Direct mode:

$$Y_2=-i.Z_2=\sin\frac{n\pi x}{L} \text{ with } \left(\frac{\omega_2}{\omega_0}\right)=\sqrt{\lambda^2+1}+\lambda$$

while posing:

own pulsation without rotation: $\omega_0=\left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{\rho S}}$

$$\lambda=\frac{1}{2}\cdot\frac{\Omega I_x}{\sqrt{EI\rho S}} \text{ with } I_x=\frac{\rho S D^2}{8} \text{ and } I=\frac{\pi D^4}{64}$$

2.2 Results of reference

the first 4 clean modes of inflection.

2.3 Uncertainty on the solution

Analytical solution with the assumption of beam of Euler.

2.4 Bibliographical references

Rene-Jean GIBERT, Vibrations of the structures, n°69 of the collection R & D of EDF at EYROLLES, p. 235-237 (1988).

3 Modeling A

3.1 Characteristics of modeling

Modeling : 18 Elements équirépartis of beam POU_D_E

Limiting conditions:

Node at the end A	DDL_IMPO: $(DX=0.0, DY=0.0, DZ=0.0)$
Node at the end B	DDL_IMPO: $(DX=0.0, DY=0.0, DZ=0.0)$
	ANGL_NAUT (45. , 0.0) for modelings B and C
Names of the nodes:	Not A = $N1$
	Not B = $N19$

For modeling D
The elements are of circular section variables with the two rays $R1$ and $R2$ identical.

For modeling C

Beam according to t with $\frac{\pi}{4}=(\hat{x}, t)$ and $t.z=0$

Worthless density
Length of an element

$$e=\frac{L}{18}=0.05\,m$$

Characteristics of the discrete elements in the base (t, v, z)

Nodes $N2$ with $N18$	Nodes $N1$ and $N19$
$m=\rho e \pi \frac{D^2}{4}=0.7657\,kg$	$m'=\rho \frac{e}{2} \pi \frac{D^2}{4}=0.3829\,kg$
$I_{tt}=m.\frac{D^2}{8}=2,393.10^{-4}\ kg.m^2$	$I'_{xx}=m'.\frac{D^2}{8}=1,196.10^{-4}\ kg.m^2$
$I_{vv}=I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2,791.10^{-4}\ kg.m^2$	$I'_{vv}=I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{3}=1,395.10^{-4}\ kg.m^2$

Table 3.1-1 : Calculation of the characteristics of the discrete elements

Two solutions are possible to define the characteristics in the base:
- that is to say to carry out a basic change of the local reference mark of the beam (t, v, z) with the total reference mark (x, y, z). For that, it is necessary to carry out a basic change by a rotation of axis z and of value $-45°$. One obtains:

$$\overline{\overline{I}}=\begin{bmatrix} I_{xx} & I_{xy} & 0 \\ I_{xy} & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \text{ with:}$$

$$I_{xx}=\cos^2(\pi/4) I_{tt}+\sin^2(\pi/4) I_{vv}$$
$$I_{yy}=\sin^2(\pi/4) I_{tt}+\cos^2(\pi/4) I_{vv}$$
$$I_{xy}=\cos(\pi/4)\sin(\pi/4)(I_{tt}-I_{vv})$$

Characteristics of the discrete elements in the base (x, y, z)

Nodes $N2$ with $N18$	Nodes $N1$ and $N19$
$m=\rho.e.\pi.\frac{D^2}{4}=0,7657\ kg$	$m'=\rho.\frac{e}{2}.\pi.\frac{D^2}{4}=0,3829\ kg$
$I_{xx}=I_{yy}=\frac{1}{2}\left(m.\frac{D^2}{8}+\frac{1}{2}m.\frac{D^2}{8}+m.\frac{e^2}{12}\right)$ $=2,592.10^{-4}\ kg.m^2$	$I'_{xx}=I'_{yy}=\frac{1}{2}\left(m'.\frac{D^2}{8}+\frac{1}{2}m'.\frac{D^2}{8}+m'.\frac{e^2}{12}\right)$ $=1,296.10^{-4}\ kg.m^2$
$I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2,792.10^{-4}\ kg.m^2$	$I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{12}=1,396.10^{-4}\ kg.m^2$
$I_{xy}=I_{yx}=\frac{1}{2}\left\{m.\frac{D^2}{8}-\left(\frac{1}{2}m.\frac{D^2}{8}+m.\frac{e^2}{12}\right)\right\}$ $=-1,994.10^{-5}\ kg.m^2$	$I'_{xy}=I'_{yx}=\frac{1}{2}\left\{m'.\frac{D^2}{8}-\left(\frac{1}{2}m'.\frac{D^2}{8}+m'.\frac{e^2}{12}\right)\right\}$ $=-9,971.10^{-6}\ kg.m^2$

Table 3.1-2 : Calculation of the characteristics of the discrete elements

– that is to say to declare the characteristics in the local reference mark of the beam and to use the nautical angles to lay down the direction of the local reference mark. It is this method which was used for modeling C.

3.2 Characteristics of the grid

Grid: Many nodes: 19
Many meshs and types: 18 SEG2

3.3 Sizes tested and results

Rotor with the stop ($\Omega=0$) (frequencies in Hz)

Reference	Modeling A		Modeling B		Modeling C		Modeling D	
frequency	ASTER	% reference	ASTER	% reference	ASTER	% reference	ASTER	% reference
122.7475	122.7475	0.000	122.7461	0.001	122.4789	-0.219	122.7475	0.000
490.9899	490.9949	0.001	490.9889	0.001	486.6954	-0.875	490.9949	0.001
1104.7273	1104.7844	0.005	1104.7710	0.004	1083.2630	-1.943	1104.7844	0.005
1963.9596	1964.2791	0.016	1964.2551	0.015	1897.3908	-3.390	1964.2791	0.016

Table 3.2-1 : Calculation of the frequencies of the rotor to the stop

Calculation of the Eigen frequencies using the algorithm of Sorensen
Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , direct modes (frequencies in Hz)

Reference	Modeling A		Modeling B		Modeling C		Modeling D	
frequency	ASTER	% reference	ASTER	% reference	ASTER	% reference	ASTER	% reference
125.8150	125.8150	0.000	125.8135	0.001	125.5324	-0.225	125.8150	0.000
503.2598	503.2649	0.001	503.2587	0.000	498.7463	-0.897	503.2649	0.001
1132.3346	1132.3918	0.005	1132.3779	0.004	1109.7993	-1.990	1132.3918	0.005
2013.0393	2013.3589	0.016	2013.3342	0.015	1986.6704	-1.310	2013.3589	0.016

Table 3.2-2 : Calculation of the frequencies of the direct modes of the rotor at the speed $10000\,rad/s$

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , retrograde modes (frequencies in Hz)

Reference	Modeling A		Modeling B		Modeling C		Modeling D	
frequency	ASTER	% reference	ASTER	% reference	ASTER	% reference	ASTER	% reference
119.7548	119.7549	0.000	119.7534	0.001	119.4990	-0.214	119.7549	0.000
479.0191	479.0242	0.001	479.0183	0.000	474.9244	-0.855	479.0242	0.001
1077.7931	1077.8502	0.005	1077.8370	0.004	1057.3064	-1.901	1077.8502	0.005
1916.0765	1916.3958	0.017	1916.3723	0.015	1852.4597	-3.320	1916.3958	0.017

Table 3.2-3 : Calculation of the frequencies of the retrograde modes of the rotor at the speed $10000\,rad/s$

4 Summary of the results

Good establishment of the gyroscopic effect for the element of beam. Change of axis of the beam x (direction according to which the elements were established) (modeling A) with a direction $x+y$ (modeling D) does not generate variations of results.

In analytical absence of reference for the validation of the discrete elements subjected to the gyroscopic effect, modeling C makes it possible all the same to check the gyroscopic matrix installation of a presumedly indeformable disc. The movement of each disc is fixed by that of the nodes and thus follows the deformation of neutral fibre only in a discrete way. This explains the variations noted on modeling C, all the more for the high modes characterized by a concavity of the more important modal deformation.