SDLL135 - Dynamic response of a embed-free beam-pipe

Summary
The scope of application is linear dynamics, and more particularly the modal analysis then the linear transitory analysis.

Various modelings:
1) finite elements of beam of Euler POU_D_E,
2) finite elements of beam of Timoshenko POU_D_T,
3) finite elements of pipes (TUYAU_3M and TUYAU_6M).
4) finite elements of bars
5) finite elements of hull (DKT)

This case test is the study of a homogeneous isotropic linear beam-pipe elastic, embedded at its base, subjected to a level of force and couple at the head.

The objective is to test the Eigen frequencies, displacements and the reactions of supports in the mode transient (propagation of a wave of acceleration), in small transformations with a diagram of temporal integration implicit. This beam-pipe is short and the effect of the transverse energy of shearing is notable. To validate the taking into account of the term due to the inertial forces, one tests the nodal reaction at the charged point, this one having to be worthless.

The comparison is carried out compared to an analytical solution whose broad outlines are presented; intercomparison between modelings. Comparaison is also carried out for solutions obtained with other computation softwares: *Circus* (by transfer transfer functions) and *EuroPlexus* (in explicit fast dynamics). For the bars the comparison is carried out on the results got by the orders DYNA_VIBRA and DYNA_NON_LINE in order to test the option M_GAMMA.

1 Problem of reference

1.1 Assumptions and geometry

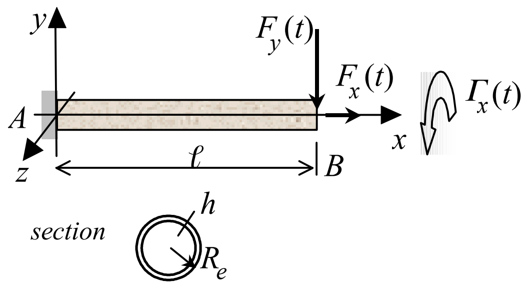

Figure 1.1-a

One considers an isotropic homogeneous beam-pipe elastic, charged at his end and embedded with the other. This problem has an analytical solution in theory of the beams, confer for example [bib1], [bib2].

Dimensions are the following ones (example drawn from [bib3]):

1) length $l=1{,}00\,m$,
2) ray external of the pipe: $R_e=0.16\,m$,
3) thickness: $h=0.01\,m$

1.2 Properties of materials

One chooses the data of the example drawn from [bib3]:

Young modulus: $E=200000\ MPa$

Poisson's ratio: $\nu=0{,}29$

Density: $\rho=7830\ kg/m^3$

One does not introduce viscous damping into the problem.

1.3 Boundary conditions and loading

Boundary conditions

The end A has its displacements of beam blocked in x, y and z as well as rotations; on the other hand, the section can become deformed freely: the modes of ovalization-swelling of the pipe are free.

Initial conditions

The beam-pipe is initially at rest in a virgin state.

Loading

One exerts a level of thrust load $F_x=1{,}0\ N$, of transverse force $F_y=1{,}0\ N$, of axial couple $\Gamma_x=1{,}0\ Nm$ on the end B at the moment $t=0\ s$.

The duration of analysis is sufficient to reach the first reflection of wave on embedding.

Gravity is neglected.

2 Reference solution

2.1 Method of calculating used for the reference solution

One notes $C_l=\sqrt{E/\rho}$ the celerity of the waves of traction and compression.

The section of the beam-pipe ESt: $S=\pi h\left(2R_e-h\right)$.

The inertia of inflection of the beam-pipe is: $I=\pi h\left(R_e\left(R_e^2+h^2\right)-3hR_e^2/2-h^3/4\right)$.

The inertia of torsion of the beam-pipe is: $J_x=2\pi h\left(R_e\left(R_e^2+h^2\right)-3hR_e^2/2-h^3/4\right)$.

The twinge is given by: $\eta=l\sqrt{S/I}$, l being the length of the beam.
For the dimensions given to the § 1, one a:

$$S=0.0097389\,m^2\ ;\ I=0.0001171\,m^4\ ;\ \eta=9.1192\,.$$

The vibratory behavior of the structure consists of modes of “beam”, and modes of “hulls” (ovalization…). This beam-pipe is short: one can expect an effect of the energy of transverse shearing. One builds the transitory dynamic solution under excitation forced by modal recombining.

2.1.1 Elastodynamic solution of the beam in traction and compression

◊ the dynamic elastic balance of the free vibrations in traction and compression is written:

$$ESu_{,xx}-\rho Su_{,tt}=0\text{ , because the normal effort is }N=ESu_{,x}$$

Frequencies of j èmes clean modes of beam are given by [bib2] in the embed-free situation in axial extension:

$$f_j^{axial}=\frac{C_l}{4l}\left(2j-1\right),\quad\text{for } j=1,2\ldots,\quad\text{modes being: } u_j(x)=\sin\frac{\pi(2j-1)x}{2\ell}$$

With the mechanical characteristics of this problem, one finds the frequencies:

$$f_1=1263{,}497\,Hz\ ,\ f_2=3790{,}490Hz\ ,\ f_3=6317{,}484Hz\ ,\ f_4=8844{,}477Hz\ldots$$

This gives the order of magnitude of the periods of harmonic pulsation of the answer to the loading.

◊ now Let us treat the transitory solution in loading imposed at the end.
The transitory axial displacement of the beam, under the action of a force F_x, is given by [bib1]:

$$u(x,t)=\frac{4C_l}{\pi ES}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{2j-1}\sin\left(\frac{(2j-1)\pi x}{2l}\right)\int_0^t F_x(\tau)\sin\left(\frac{(2j-1)\pi C_l(t-\tau)}{2l}\right)d\tau$$

For a force level F_x applied to $t=0$ in B, the wave is propagated with celerity C_l and comes to reflect itself in A at the moment l/C_l ; with the mechanical characteristics of this problem, one finds that this moment is: $0{,}000197864\,s$.
One a:

$$u(x,t)=\frac{8l\,F_x}{\pi^2 ES}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{(2j-1)^2}\sin\frac{(2j-1)\pi x}{2l}\left(1-\cos\frac{(2j-1)\pi C_l t}{2l}\right)$$

This series converges quickly because of term $(2j-1)^2$. In $x=\ell$, one obtains:

$$u(l,t)=\frac{F_x l}{ES}-\frac{8F_x l}{\pi^2 ES}\sum_{j=1,2,3\ldots}^{\infty}\frac{1}{(2j-1)^2}\cos\left(\frac{(2j-1)\pi C_l t}{2l}\right)$$

Because: $\sum_{j=1,2,3\ldots}^{\infty}\frac{1}{(2j-1)^2}=\frac{\pi^2}{8}$, from where $u(l,t)\in\left[0,\frac{2F_x l}{ES}\right]$

Before the first reflection of wave on embedding, one obtains simply: $u(l,t)=\frac{F_x t}{S\sqrt{E\rho}}$.

Calculation of the efforts

The normal effort is simply: $N(x,t)=ESu_{,x}(x,t)$. From where:

$$N(x,t)=\frac{4F_x}{\pi}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{2j-1}\cos\frac{(2j-1)\pi x}{2l}\left(1-\cos\frac{(2j-1)\pi C_l t}{2l}\right)$$

From where reaction in A :

$$R_x^A(t)=-N(0,t)=\frac{4F_x}{\pi}\sum_{j=1,2,3\ldots}^{\infty}\frac{(-1)^{j-1}}{(2j-1)}\cos\left(\frac{(2j-1)\pi C_l t}{2l}\right)-F_x$$

Reaction in A is worthless on the time interval $[0\,s;\ 0{,}000197864\,s]$ before the arrival of the wave, then is worth $-2F_x$.

2.1.2 Elastodynamic solution of the beam in inflection

One limits oneself to the movements in the average plan xOy.

◊ Let us take initially the modeling of the beams of Navier-Bernoulli-Euler. The inertia of rotation is not treated, cf [bib4].

The local equation of dynamic elastic balance is written [bib1,2,3]: $v_{,xxxx}+\frac{\rho S}{EI_z}v_{,tt}=0$.

The equation of "dispersion" connects the pulsation ω and the number of wave k : $\omega=k^2\sqrt{\frac{EI_z}{\rho S}}$.

For the boundary conditions "embed-free", there is the expression of the modes:

$$v(x,t)=e^{i\omega t}\left(c_1(\cos kx-\cosh kx)+c_2(\sin kx-\sinh kx)\right)$$

with $\cos kl\cosh kl=-1$ and $c_2/c_1=(\sin k\ell-\sinh k\ell)/(\cos k\ell+\cosh k\ell)$

The frequencies of the modes are obtained by:

$$f_n=\frac{(kl)_n^2}{2\pi l^2}\sqrt{\frac{EI_z}{\rho S}}=\frac{(kl)_n^2 C_l}{2\pi l^2}\sqrt{\frac{I_z}{S}}$$

with for the first modes:

n° of mode	1	2	3	4	5	$n\geq 6$
$(kl)_n$	1.875104069	4.694091133	7.854757438	10.99554073	14.13716839	$(2n-1)\pi/2$
c_2/c_1	-0.734095514	-1.018467319	-0.999224497	-1.000033553	-0.999998550	≈ -1

With the mechanical characteristics of this problem, one finds the frequencies:

$f_1=310.133\,Hz$, $f_2=1943.568\,Hz$, $f_3=5442.048\,Hz$, $f_4=10664.242\,Hz$, $f_5=17628.755\,Hz$

◊ Let us take then the modeling of the beams of Timoshenko with transverse shearing and inertia of rotation, the latter being then treated, cf [bib4], of *Code_Aster*.

The local equation of dynamic elastic balance couples the arrow and rotation and is written [bib3]:

$$\begin{cases} GS_r(v_{xx}-\theta_{z,x})-\rho S v_{,tt}=0 \\ EI_z\theta_{z,xx}+GS_r(v_{,x}-\theta_z)-\rho I_z\theta_{z,tt}=0 \end{cases}$$

The equation of "dispersion" connects the pulsation ω and the number of wave k :

$$k^4+\left(\frac{\rho}{E}+\frac{\rho S}{GS_r}\right)\omega^2k^2-\frac{\rho S\omega^2}{EI_z}+\frac{\rho^2 S}{GES_r}\omega^4=0$$

Two roots are always imaginary, the two others are complex: real below a cut-off frequency: $\frac{1}{2\pi}\sqrt{\frac{GS_r}{\rho I_z}}$, and imaginary pure beyond this cut-off frequency: the modes are then only sinusoidal (not hyperbolic terms).

With the mechanical characteristics of this problem, one finds [bib3] the frequencies (modes in sinusoids and exponential):

$f_1=269{,}932\,Hz$, $f_2=1077{,}199\,Hz$, $f_3=2270{,}705\,Hz$, $f_4=3249{,}207\,Hz$, $f_5=4649{,}212\,Hz$

and the first frequency of the mode in sinusoids alone: $f_5^{bis}=4002{,}830\,Hz$.

It should be noted that in [bib3] the value of the coefficient of section reduced to the shearing action is selected equal to $0{,}530659727$, while in *Code_Aster*, the selected formula [bib4] gives: $0{,}510805163$. One can expect a light consequence from it on the calculated Eigen frequencies.

◊ now Let us treat the transitory solution in loading imposed at the end $x=\ell$. The solution is written in the form of a sum on the clean modes obtained above (with embedding in A):

$$v(x,t)=\sum_{j=1}^{\infty}\eta_j(t).w_j(x) \quad ; \qquad \theta_z(x,t)=\sum_{j=1}^{\infty}\eta_j(t).\beta_j(x) \text{ (case Timoshenko)}$$

One breaks up the loading at the end $x=\ell$ on the modes. As follows:

$$F_j(t)=w_j(l)\ .F(t)$$

The solution checks:

$$\ddot{\eta}_j(t)+\omega_j^2\eta_j(t)=F_j(t)$$

From where:

$$\eta_j(t)=\frac{1}{\omega_j}\int_0^t F_j(\tau).\sin\omega_j(t-\tau)\,d\tau+\eta_j(0)\cos\omega_j t+\frac{1}{\omega_j}\dot{\eta}_j(0)\sin\omega_j t$$

$\eta_j(0)$ and $\dot{\eta}_j(0)$ are obtained by the initial conditions (here in the case Euler-Bernoulli):

$$\eta_j(0)=\int_0^l {}^t w_j(x).\rho S v(x,0)\,dx \quad \text{and} \quad \dot{\eta}_j(0)=\int_0^l {}^t w_j(x).\rho S\dot{v}(x,0)\,dx$$

2.1.3 Elastodynamic solution of the beam-pipe in torsion

◊ the dynamic elastic balance of the free vibrations in torsion (free model torsion) is written:

$M_{x,x}-\rho J\theta_{x,tt}=0$, because the torque is $M_x=GJ\theta_{x,x}$

Frequencies of j èmes clean modes of beam are given by [bib2] in the embed-free situation in torsion:

$f_j^{tors}=\frac{C_l}{4l}\frac{2j-1}{\sqrt{2(1+\nu)}}$, for $j=1,2\ldots$, modes being: $\theta_{x\,j}(x)=\sin\frac{\pi(2j-1)x}{2\ell}$

With the mechanical characteristics of this problem, one finds the frequencies:

$$f_1 = 786{,}619 Hz\,,\; f_2 = 2359{,}856 Hz\,,\; f_3 = 3933{,}094 Hz\,,\; f_4 = 5506{,}331 Hz \ldots$$

◊ The transitory dynamic solution, under loading imposed at the end, present a complete similarity with traction; the transitory axial rotation of the beam is given by, cf [bib1]:

$$\theta_x(x,t) = \frac{4C_l}{\pi GJ} \sum_{j=1,2,3\ldots}^{\infty} \frac{(-1)^{j-1}}{2j-1} \sin\left(\frac{(2j-1)\pi x}{2l}\right) \int_0^t \Gamma_x(\tau) \sin\left(\frac{(2k-1)\pi C_l (t-\tau)}{2l}\right) d\tau$$

For a couple level Γ_x, applied to $t=0$ in B, the wave is propagated with celerity C_l and comes to reflect itself in A at the moment $l\sqrt{2(1+\nu)}/C_l$; with the mechanical characteristics of this problem, one finds that this moment is: $0{,}000317816\,s$.

One a:

$$\theta_x(x,t) = \frac{8\Gamma_x l}{\pi^2 GJ} \sum_{j=1,2,3\ldots}^{\infty} \frac{(-1)^{j-1}}{(2j-1)^2} \sin\left(\frac{(2j-1)\pi x}{2l}\right)\left(1-\cos\left(\frac{(2j-1)\pi C_l t}{2l}\right)\right)$$

This series converges quickly because of term $(2j-1)^2$. In $x=\ell$, one obtains:

$$\theta_x(l,t) = \frac{\Gamma_x l}{GJ} - \frac{8\Gamma_x l}{\pi^2 GJ} \sum_{j=1,2,3\ldots}^{\infty} \frac{1}{(2j-1)^2} \cos\left(\frac{(2j-1)\pi C_l t}{2l}\right)$$

Because $\sum_{j=1,2,3\ldots}^{\infty} \frac{1}{(2j-1)^2} = \frac{\pi^2}{8}$, from where $\theta_x(l,t) \in \left[0, \frac{2\Gamma_x l}{GJ}\right]$.

Before the first reflection of wave on embedding, one obtains simply: $\theta_x(l,t) = \frac{\Gamma_x t}{J\sqrt{G\rho}}$.

Calculation of the efforts

The torque is simply: $M_x(x,t) = GJ\,\theta_{x,x}(x,t)$. From where:

$$M_x(x,t) = \frac{4\Gamma_x}{\pi} \sum_{j=1,2,3\ldots}^{\infty} \frac{(-1)^{j-1}}{(2j-1)} \cos\left(\frac{(2j-1)\pi x}{2l}\right)\left(1-\cos\left(\frac{(2j-1)\pi C_l t}{2l}\right)\right)$$

From where the couple of embedding in A:

$$\Gamma_x^A(t) = -M_x(0,t) = \frac{4\Gamma_x}{\pi} \sum_{j=1,2,3\ldots}^{\infty} \frac{(-1)^{j-1}}{(2j-1)} \cos\left(\frac{(2j-1)\pi C_l t}{2l}\right) - \Gamma_x$$

Moment of embedding in A is null on the time interval $[0\,s;\; 0{,}000317816\,s]$ before the arrival of the wave, then is worth $-2\Gamma_x$.

2.1.4 Elastodynamic solution of the pipe in model cylindrical hull

The boundary conditions of the studied problem fix only the rigid modes of the structure, on the ddls of beam. The kinematics of the cylindrical hull around that of beam is free. The parameters of configuration of the model of pipe [bib5] are thus free.

For a cylindrical hull finite length (length l, external ray R_e, thickness h average radius $R_m = R_e - h/2$), by admitting that the following membrane deformations are worthless: $\epsilon_{\theta\theta} = \epsilon_{x\theta} \approx 0$ (the inflection being prevalent; but one imposes nothing on $\in_{xx}$), the clean modes are following form [bib2] with $j=2,3,4\ldots$, $n=1,2,3\ldots$:

$$\begin{cases} u_x(x,\theta,t) = -\frac{R_m}{j^2} U_{n,x} \cos(j\theta).\sin(\omega t+\phi) \\ u_\theta(x,\theta,t) = -\frac{1}{j} U_n \sin(j\theta).\sin(\omega t+\phi) \\ u_r(x,\theta,t) = U_n \cos(j\theta).\sin(\omega t+\phi) \end{cases}$$

where n indicate the number of the axial mode and j the number of the circumferential mode.
And the frequencies are [bib2]:

$$f_{jn} = \frac{C_l}{\sqrt{1-\nu^2}} \frac{\lambda_{jn}}{2\pi R_m}$$

where λ_{jn} is obtained by:

$$\lambda_{jn}^2 = \frac{j^4 \left(a_{11} a_{22} a_{33} + 2 a_{12} a_{23} a_{13} - a_{11} a_{23}^2 - a_{22} a_{31}^2 - a_{33} a_{12}^2\right)}{\left(j^4 + j^2 + \lambda_n^2 \alpha_{n2} R_m^2 / l^2\right).\left(a_{11} a_{22} - a_{12}^2\right)}$$

where $\lambda_n = (k\, l)_n$ is associated with the mode of beam of Euler, for the boundary conditions selected (cf [§ 2.1.2]), where $\alpha_{n2} = \frac{1}{l} \int_0^l U_{n,x}^{\ 2} dx$ is calculated starting from the axial mode of standard beam, the boundary conditions of the cylinder, and where constants $a_{11}, a_{22}, a_{33}, a_{12}, a_{23}, a_{13}$ are given by (while noting $\chi = h^2 / (12 R_m^{\ 2})$):

$$a_{11} = \lambda_n^2 R_m^{\ 2} / \ell^2 + \frac{1}{2} j^2 \alpha_{n2} (1-\nu)(1+\chi)$$

$$a_{22} = j^2 + \frac{1}{2} \alpha_{n2} \lambda_n^2 R_m^{\ 2} (1-\nu)(1+3\chi) / \ell^2$$

$$a_{33} = 1 + \chi \left[\lambda_n^4 R_m^{\ 4} / \ell^4 + \left(j^2 - 1\right)^2 + 2\nu\, j^2 \lambda_n^2 R_m^{\ 2} \alpha_{n1} / \ell^2 + 2 j^2 \lambda_n^2 R_m^{\ 2} \alpha_{n2} (1-\nu) / \ell^2 \right]$$

$$a_{12} = -j \lambda_n R_m \left(\nu \alpha_{n1} + \frac{1}{2} \alpha_{n2} (1-\nu)\right) / \ell$$

$$a_{23} = j \left(1 + \chi \lambda_n^2 R_m^{\ 2} \left(\nu \alpha_{n1} + \frac{3}{2} \alpha_{n2} (1-\nu)\right) / \ell^2\right)$$

$$a_{31} = \lambda_n R_m \left(-\nu \alpha_{n1} + \chi \left(\frac{1}{2} j^2 \alpha_{n2} (1-\nu) - \lambda_n^2 R_m^{\ 2} / \ell^2\right)\right) / \ell$$

where $\alpha_{n1} = -\frac{1}{l} \int_0^l U_{n,xx} . U_n \ dx$.

The values are given by the table below for the embed-free case (where displacements and rotations are completely blocked in $x=0$):

n° of beam mode	1	2	3	4	5
$(k\, l)_n$	1.875104069	4.694091133	7.854757438	10.99554073	14.13716839
α_{n1}	-0.2241	0.6033	0.7440	0.8182	0.8585
α_{n2}	1.3219	1.4712	1.2529	1.1820	1.1415

In modeling suggested here, embedding is only assured on the ddls beam, in the manner of the connection COQ_POU, to see [bib6], what ensures that only the averages and the moments of order 1 of displacements at the end $x=0$ are worthless. The kinematics of the elements pipes uncouples the terms generating the deformation from "beam" of those supplementing them to generate a kinematics of hulls, using the first circumferential modes of Fourier, to see [bib5], within the meaning of the scalar product of displacements on the structure, i.e. also within the meaning of the inertial forces (matrix of mass).

This is why, one gives below the two families of clean modes completely inextensionnels $\epsilon_{\theta\theta}=\epsilon_{x\theta}=\epsilon_{xx}=0$, for a "free-free" cylinder, length l, with $j=2,3,4...$:

$$\text{modes de Rayleigh}\quad \begin{cases} u_x(x,\theta,t)=0 \\ u_\theta(x,\theta,t)=-\frac{1}{j}\sin(j\theta).\sin(\omega t+\phi) \\ u_r(x,\theta,t)=\cos(j\theta).\sin(\omega t+\phi) \end{cases}$$

and

$$\text{modes de Love}\quad \begin{cases} u_x(x,\theta,t)=\frac{R_m}{j^2}\cos(j\theta).\sin(\omega t+\phi) \\ u_\theta(x,\theta,t)=-\frac{(x-l/2)}{j}\sin(j\theta).\sin(\omega t+\phi) \\ u_r(x,\theta,t)=(x-l/2)\cos(j\theta).\sin(\omega t+\phi) \end{cases}$$

These modes check the condition well in $x=0$ of embedding on average of "beam".
Frequencies of j èmes clean modes are respectively for the first, then for the seconds:

$$f_j^{Rayleigh}=\frac{C_l\sqrt{\chi}}{2\pi R_m\sqrt{1-\nu^2}}\frac{j(j^2-1)}{\sqrt{j^2+1}}$$

and

$$f_j^{Love}=\frac{C_\ell\sqrt{\chi}}{2\pi R_m\sqrt{1-\nu^2}}j(j^2-1)\sqrt{\frac{j^2\ell^2+24(1-\nu)R_m^2}{j^2\ell^2+12R_m^2}}$$

with $j=2,3,4...$. With the mechanical characteristics of this problem, one finds the frequencies:

Rayleigh: $f_2=78,22613\,Hz$, $f_3=221,2569\,Hz$, $f_4=424,2407\,Hz$, $f_5=686,0885\,Hz$ …

Coil: $f_2=179,7433\,Hz$, $f_3=708,6504\,Hz$, $f_4=1762,019\,Hz$, $f_5=3514,927\,Hz$ …

2.2 Results of reference

- Eigen frequencies, analytical reference;
- Displacements, rotations in B : components DX and DRX analytical reference per series;
- Reactions in A : components DX and DRX analytical reference per series.

2.3 Uncertainty on the solution

Analytical solution.

Comparison of some frequencies in model beam obtained with the software Circus [bib7].
Comparison of some displacements and transitory rotations in model beam of Euler obtained by S.Potapov with the software *EuroPlexus* (in fast dynamics clarifies), for which one chose a coarser grid (100 finite elements of beam, diagram clarifies centered differences and not in optimal time about $\Delta t=10^{-10}s$). One does not compare the efforts obtained by *EuroPlexus*, because they are sullied with oscillations coming from the explicit diagram of integration to centered differences.

2.4 References bibliographical

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3 Modeling A

3.1 Characteristics of modeling

One adopts a modeling by elements of beam of Euler POU_D_E.

Grid of the model calculated with *Code_Aster* consists of 1001 nodes and 1000 meshs SEG2. Is needed a very fine grid, because the inertial forces give solutions which are not in the base of the functions of the shape of the elements of beam, to see [bib4].

One chooses a tight temporal discretization from $t=0\,s$, in order to collect the initial shock as well as possible. One chooses to solve on the interval $[0\,s\,;0{,}00032\,s]$.
The diagram of temporal integration selected is:
diagram of Newmark, in average acceleration (values by default) and $\Delta t=10^{-7}s$.

3.2 Sizes tested and results

3.2.1 Oscillatory modes

One puts for information some values of Eigen frequencies obtained with the software Circus [bib7].

Eigen frequencies of beam of Euler (in Hz)

Type of mode	reference	nature	*Code_Aster*	*Circus*
Inflection 1	310.133	analytical	310,132	310.13
Inflection 2	1943.568	analytical	1943.566	1943.56
Inflection 3	5442.048	analytical	5442.046	
Inflection 4	10664.242	analytical	10664.242	
Inflection 5	17628.755	analytical	17628.756	
Traction 1	1263.497	analytical	1263.497	1263.48
Traction 2	3790.490	analytical	3790.494	
Traction 3	6317.484	analytical	6317.500	
Traction 4	8844.477	analytical	8844.522	
Torsion 1	786.619	analytical	786,619	786.62
Torsion 2	2359.856	analytical	2359.858	
Torsion 3	3933.094	analytical	3933.104	
Torsion 4	5506.331	analytical	5506.359	

3.2.2 Transitory answer

Displacements of the end - not B (in m), rotations of the end - not B

Moment (S)	Component	reference	nature	*Code_Aster*	*EuroPlexus*
0.00010	DX	$2.5947\ 10^{-10}$	analytical	$2.5934\ 10^{-10}$	$2{,}568\ 10^{-10}$
0.00015	DX	$3.8921\ 10^{-10}$	analytical	$3.8908\ 10^{-10}$	$3{,}865\ 10^{-10}$
0.00020	DX	$5.1895\ 10^{-10}$	analytical	$5.1882\ 10^{-10}$	$5{,}163\ 10^{-10}$
0.00010	DRX	$1.7329\ 10^{-8}$	analytical	$1.7321\ 10^{-8}$	$1{,}715\ 10^{-8}$
0.00020	DRX	$3.4659\ 10^{-8}$	analytical	$3.4651\ 10^{-8}$	$3{,}448\ 10^{-8}$

Reactions to embedding - not A (in N), moments with embedding - not A (in $N.m$)

Moment (S)	Component	reference	nature
0.00010	DX	0,00	analytical
0.00015	DX	0,00	analytical

0.00020	DX	-2,00	analytical
0.00010	DRX	0,00	analytical
0.00020	DRX	0,00	analytical
0.00032	DRX	-2,00	analytical

4 Modeling B

4.1 Characteristics of modeling

One adopts a modeling by elements of beam of Timoshenko POU_D_T.

Grid of the model calculated with *Code_Aster* consists of 1001 nodes and 1000 meshs SEG3. Is needed a very fine grid, because the inertial forces give solutions which are not in the base of the functions of the shape of the elements of beam, to see [bib4]. It is the same grid as in modeling A.

One chooses a tight temporal discretization from $t=0\,s$, in order to collect the initial shock as well as possible. One chooses to solve on the interval $[0\,s\,;0{,}00032\,s]$.
The diagram of temporal integration selected is:
diagram of Newmark, in average acceleration (values by default) and $\Delta t=10^{-7}s$.

4.2 Sizes tested and results

4.2.1 Oscillatory modes

One puts for information some values of Eigen frequencies obtained with the software Circus [bib7], for which the coefficient of reduced section is obtained according to [bib8]: 0.5011, i.e. a value weaker than that selected in the reference (bib3) from where lower frequencies. To confront itself with the reference [bib3], one puts also the values of the predictions of Circus with the coefficient of reduced section equal to $0{,}530659727$ of [bib3]. It is noted that the results are then identical to 10^{-4} near; the value of the coefficient of section reduced to shearing plays an important role on this example.

Eigen frequencies of beam of Timoshenko (in Hz)

Type of mode	reference	nature	*Code_Aster*	*Circus* Batoz coeff	*Circus* coeff [bib3]
Inflection 1	269.932	analytical	269,261	268.3	
Inflection 2	1077.199	analytical	1059.305		
Inflection 3	2270.705	analytical	2226.699		
Inflection 4	3249.207	analytical	3173.253		
Inflection 5	4649.212	analytical	4554.463	4557.2	4649.6
Inflection 5 (a)	4002.830	analytical		3903.0	4003.2
Traction 1	1263.497	analytical	1263.497	1263.48	
Traction 2	3790.490	analytical	3790.494		
Traction 3	6317.484	analytical	6317.500		
Traction 4	8844.477	analytical	8844.522		
Torsion 1	786.619	analytical	786,619	786.62	
Torsion 2	2359.856	analytical	2359.858		
Torsion 3	3933.094	analytical	3933.104		
Torsion 4	5506.331	analytical	5506.359		

4.2.2 Transitory answer

Displacements of the end - not B (in *m*), rotations of the end - not B

Moment (S)	Component	reference	nature
0.00010	DX	$2.5947\ 10^{-10}$	analytical
0.00015	DX	$3.8921\ 10^{-10}$	analytical
0.00020	DX	$5.1895\ 10^{-10}$	analytical
0.00010	DRX	$1.7329\ 10^{-8}$	analytical
0.00020	DRX	$3.4659\ 10^{-8}$	analytical

Reactions to embedding - not A (in N), moments with embedding - not A (in $N.m$)

Moment (S)	Component	reference	nature
0.00010	DX	0,00	analytical
0.00015	DX	0,00	analytical
0.00020	DX	-2,00	analytical
0.00010	DRX	0,00	analytical
0.00020	DRX	0,00	analytical
0.00032	DRX	-2,00	analytical

5 Modeling C

5.1 Characteristics of modeling

One adopts a modeling by elements of beam-pipe TUYAU_3M.

Grid of the model calculated with *Code_Aster* consists of 1001 nodes and 500 meshs SEG3. Is needed a rather fine grid, because the inertial forces give solutions which are not in the base of the functions of the shape of the elements of beam, to see [bib4]. It is the same grid as in modeling A.

Two models are created: where the pipe is blocked in B on only the 6 degrees of freedom of beam; a second where the pipe is dependent (by one LIAISON_DDL) with a discrete element DIS_TR (on a mesh not POI1) with 6 degrees of freedom, which is completely fixed to him.

One chooses a tight temporal discretization from $t=0\,s$, in order to collect the initial shock as well as possible. One chooses to solve on the interval $[0\,s\,;0{,}00032\,s]$.
The diagram of temporal integration selected is:
diagram of Newmark, in average acceleration (values by default) and $\Delta t=10^{-7}s$.

5.2 Sizes tested and results

5.2.1 Oscillatory modes

Eigen frequencies of "beam" and "hull" (in Hz), case TUYAU_3M

Type of mode	reference	nature
Inflection 1	269.932	analytical
Inflection 2	1077.199	analytical
Inflection 3	2270.705	analytical
Inflection 4	3249.207	analytical
Inflection 5	4649.212	analytical
Traction 1	1263.497	analytical
Traction 2	3790.490	analytical
Traction 3	6317.484	analytical
Traction 4	8844.477	analytical
Torsion 1	786.619	analytical
Torsion 2	2359.856	analytical
Torsion 3	3933.094	analytical
Torsion 4	5506.331	analytical
Coque Rayleigh 2	78.22613	analytical
Coque Rayleigh 3	221.2569	analytical
Coque Rayleigh 4	424.2407	analytical
Coque Rayleigh 5	686.0885	analytical
Coque Love 2	179.7433	analytical
Coque Love 3	708.6504	analytical
Coque Love 4	1762.019	analytical
Coque Love 5	3514.927	analytical

5.2.2 Transitory answer

Displacements of the end - not B (in m), rotations of the end - not B , case TUYAU_3M

Moment (S)	Component	reference	nature
0.00010	DX	$2.5947\ 10^{-10}$	analytical
0.00015	DX	$3.8921\ 10^{-10}$	analytical

0.00020	DX	$5.1895\ 10^{-10}$	analytical
0.00005	DRX	$8.6648\ 10^{-9}$	analytical
0.00010	DRX	$1.7330\ 10^{-8}$	analytical
0.00020	DRX	$3.4659\ 10^{-8}$	analytical

Reactions to embedding - not A (in N), moments with embedding - not A (in Nm), case TUYAU_3M

Moment (S)	Component	reference	nature
0.00010	DX	0.00	analytical
0.00015	DX	0.00	analytical
0.00020	DX	-2.00	analytical
0.00032	DRX	-2.00	analytical

Lastly, at the loose lead point B, values of REAC_NODA are about 10^{-12}, and even definitely lower, at the same moments, which is quite close to the expected zero value.

6 Modeling D

6.1 Characteristics of modeling

One adopts a modeling by elements of beam-pipe TUYAU_6M with 3 nodes.

Grid of the model calculated with *Code_Aster* consists of 1001 nodes and 500 meshs SEG3. Is needed a rather fine grid, because the inertial forces give solutions which are not in the base of the functions of the shape of the elements of beam, to see [bib4]. It is the same grid as in modeling A.

Two models are created: where the pipe is blocked in A on only the 6 degrees of freedom of beam; a second where the pipe is dependent (by one LIAISON_DDL) with a discrete element DIS_TR (on a mesh not POI1) with 6 degrees of freedom, which is completely fixed to him.

One chooses a tight temporal discretization from $t=0\,s$, in order to collect the initial shock as well as possible. One chooses to solve on the interval $[0\,s\,;0{,}00032\,s]$.
The diagram of temporal integration selected is:
diagram of Newmark, in average acceleration (values by default) and $\Delta t=10^{-7}s$.

6.2 Sizes tested and results

6.2.1 Oscillatory modes

Eigen frequencies of "beam" and "hull" (in Hz), case TUYAU_6M

Type of mode	reference	nature
Inflection 1	269.932	analytical
Inflection 2	1077.199	analytical
Inflection 3	2270.705	analytical
Inflection 4	3249.207	analytical
Inflection 5	4649.212	analytical
Traction 1	1263.497	analytical
Traction 2	3790.490	analytical
Traction 3	6317.484	analytical
Traction 4	8844.477	analytical
Torsion 1	786.619	analytical
Torsion 2	2359.856	analytical
Torsion 3	3933.094	analytical
Torsion 4	5506.331	analytical
Coque Rayleigh 2	78.22613	analytical
Coque Rayleigh 3	221.2569	analytical
Coque Rayleigh 4	424.2407	analytical
Coque Rayleigh 5	686.0885	analytical
Coque Love 2	179.7433	analytical
Coque Love 3	708.6504	analytical
Coque Love 4	1762.019	analytical
Coque Love 5	3514.927	analytical

6.2.2 Transitory answer

Displacements of the end - not B (in m), rotations of the end - not B, case TUYAU_6M

Moment (S)	Component	reference	nature
0.00010	DX	$2.5947\ 10^{-10}$	analytical
0.00015	DX	$3.8921\ 10^{-10}$	analytical

0.00020	DX	$5.1895\ 10^{-10}$	analytical
0.00005	DRX	$8.6648\ 10^{-9}$	analytical
0.00010	DRX	$1.7330\ 10^{-8}$	analytical
0.00020	DRX	$3.4659\ 10^{-8}$	analytical

Reactions to embedding - not A (in N), moments with embedding - not A (in Nm), case TUYAU_6M

Moment (S)	Component	reference	nature
0.00010	DX	0.00	analytical
0.00015	DX	0.00	analytical
0.00020	DX	-2.00	analytical
0.00032	DRX	-2.00	analytical

Lastly, at the end - not B , values of REAC_NODA are about 10^{-12} , and even definitely lower, at the same moments, which is quite close to the expected zero value.

7 Modeling E

7.1 Characteristics of modeling

Modeling is made by elements of bar (BAR).
Grid of the model *Code_Aster* consists of 1001 nodes and 1000 meshs SEG2. Is needed a very fine grid, because the inertial forces give solutions which are not in the base of the functions of the shape of the elements of beam, to see [bib4]. One changed the orientation of the grid compared to modeling A.

One chooses a tight temporal discretization from $t=0\,s$, in order to collect the initial shock as well as possible. One chooses to solve on the interval $[0\,s\,;0{,}00032\,s]$.
The diagram of temporal integration selected is that of Newmark, in average acceleration (values by default) and $\Delta t=10^{-7}s$.

7.2 Sizes tested and results

Calculation of the oscillatory modes:
Eigen frequencies of beam of Euler (in Hz)

Type of mode	reference	Nature	Tolerance
Traction 1	1263.5	analytical	1.0E-03
Traction 2	3790.49	analytical	1.0E-03
Traction 3	6317.48	analytical	1.0E-03
Traction 4	8844.48	analytical	1.0E-03

Calculation with DYNA_VIBRA :
Displacements of the end - not B (in m), rotations of the end - not B

Moment (S)	Component	reference	Nature	Tolerance
0.00010	DX	1.8347E-10	analytical	1.0E-03
0.00010	DY	1.8347E-10	analytical	1.0E-03
0.00015	DX	2.7521E-10	analytical	1.0E-03
0.00015	DY	2.7521E – 10	analytical	1.0E-03
0.00020	DX	3.6695E-10	analytical	1.0E-03
0.00020	DY	3.6695E – 10	analytical	1.0E-03

Reactions to embedding - not A (in N), moments with embedding - not A (in $N.m$)

Moment (S)	Component	reference	Nature	Tolerance
0.00010	DX	0.00	analytical	1.0E-03
0.00010	DY	0.00	analytical	1.0E-03
0.00015	DX	0.00	analytical	1.0E-03
0.00015	DY	0.00	analytical	1.0E-03
0.00020	DX	-1.414213	analytical	5.0%
0.00020	DY	-1.414213	analytical	5.0%

Calculation with DYNA_NON_LINE :
Displacements of the end - not B (in m), rotations of the end - not B

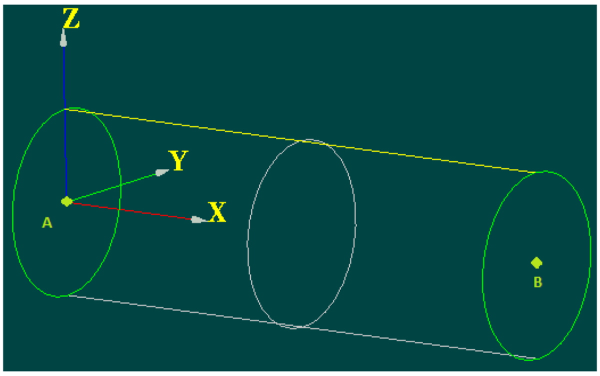
Moment (S)	Component	Reference	Nature	Tolerance
0.00010	DX	1.8347E-10	analytical	1.0E-03
0.00010	DY	1.8347E-10	analytical	1.0E-03
0.00015	DX	2.7521E-10	analytical	1.0E-03
0.00015	DY	2.7521E – 10	analytical	1.0E-03
0.00020	DX	3.6695E-10	analytical	1.0E-03
0.00020	DY	3.6695E – 10	analytical	1.0E-03

Reactions to embedding - not A (in N), moments with embedding - not A (in $N.m$)

Moment (S)	Component	reference	nature	Tolerance
0.00010	DX	0.00	analytical	1.0E-03
0.00010	DY	0.00	analytical	1.0E-03
0.00015	DX	0.00	analytical	1.0E-03
0.00015	DY	0.00	analytical	1.0E-03
0.00020	DX	-1.414213	analytical	5.0%
0.00020	DY	-1.414213	analytical	5.0%

8 Modeling F

Modeling is carried out with elements of hull DKT.



pipe is discretized in the following way:

- 25 meshs in the circumferential direction,
- 500 meshs in the direction length.

What gives a grid made up of:

- 12 550 nodes,
- 12 500 meshs QUAD4.

In this model the nodes located in section A are dependent (LIAISON_ELEM) with a discrete element DIS_TR (mesh of the type not POI1 located in A) with 6 degrees of freedom, which is completely fixed to him.

8.1 Sizes tested and results

Oscillatory modes:

Type of mode	Value of reference	Type of reference	Tolerance %
Traction 1	1263.497	analytical	0.3
Traction 2	3790.490	analytical	0.65
Traction 3	6317.484	analytical	0.1
Traction 4	8844.477	analytical	0.3
Inflection 1	310,133	analytical	9.5
Inflection 2	1943.568	analytical	0.55
Inflection 3	5442.048	analytical	0.3
Inflection 4	10664.242	analytical	0.25
Inflection 4	17628.755	analytical	0.2
Torsion 1	786,619	analytical	0.25
Torsion 2	2359.856	analytical	0.25
Torsion 3	3933.094	analytical	0.25
Torsion 4	5506.331	analytical	0.25

9 Summary of the results

A relatively fine grid is necessary to get precise results (wave propagation).

It is noted that the model of beam of Timoshenko is more precise in inflection, but that the value of the coefficient of section reduced to shearing plays an important role on this example. Software of frequential dynamics of pipings *Circus* give values very close to the frequencies determined by *Code_Aster* for the models of beams of Euler and Timoshenko.

Fast software of dynamics *EuroPlexus* give values very close compared to those to *Code_Aster* for the elements of beams of Euler.

The model by finite elements of hull makes it possible to define the field of validity of modelings in elements of beam and elements pipes.