SDLL113 - dynamic Under-structuring transient: beam in simple traction

Summary:

The structure considered is an annular beam of section in simple traction, embedded on a side, and subjected at its end with a force of the Heaviside type. Its transitory dynamic response is calculated by under - structuring.

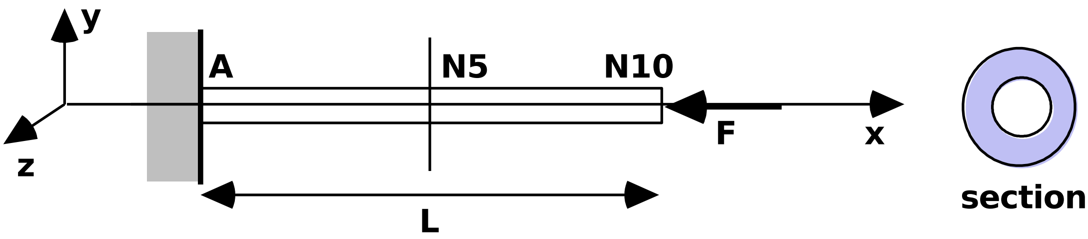
The beam is modelled by elements of the beams type of Timoshenko (linear model). Two modelings are proposed according to whether the beam is deadened or not. Damping tested is of type RAYLEIGH (damping proportional).

The results of reference result from a direct transitory calculation by modal recombination without under - structuring. This test thus makes it possible to validate the computational tools of answer transitory per under-structuring, in the linear case.

The third modeling dealing exactly with the same deadened problem that second was added in order to treat a case of use of a method of condensation dynamic where the resolution of the dynamic problem on physical basis takes place on a mixed model made up of the macronutrient of the right part of the structure and a physical model reduced to its left part. The results same as those of the second modeling like are exactly expected.

1 Problem of reference

1.1 Geometry



Length of the beam: $L=1\,m$

Section: Interior ray $=0.09\,m$
External ray $=0.10\,m$

1.2 Properties of materials

$E=10^{10}\,Pa$ $\nu=0.3$ $\rho=1.10^{4}\,kg/m^{3}$

Modeling a: not of damping
Modeling b: Damping proportional (RAYLEIGH):

$C=\alpha K+\beta M$ with $\alpha=6.5\,10^{-6}\,s$ and $\beta=16.0\,s^{-1}$.

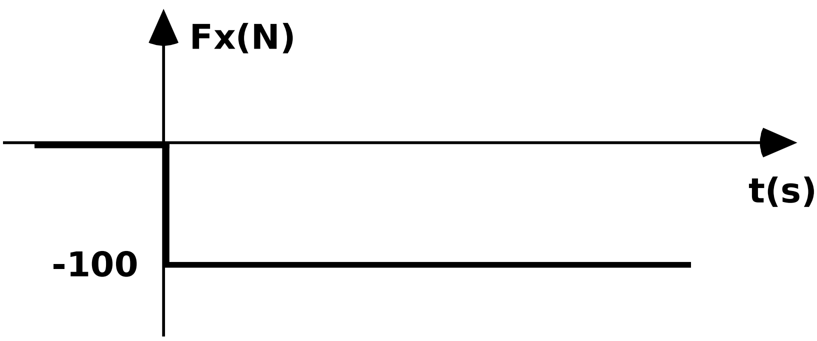
These values correspond to a reduced damping of 1% on the first mode of the structure.

1.3 Boundary conditions and loadings

On all the structure one imposes $DY=DZ=DRX=DRY=DRZ=0$.

At the point A the condition of embedding is imposed $DX=0$.

In $N10$ one applies a constant force as from the moment $t=0$: $F_x=-100\,N$.



1.4 Initial conditions

The structure is initially at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

There exists an analytical solution detailed in the reference [bib1].

The following notations are adopted:

E	:	Young modulus
ρ	:	density
L	:	length of the bar
A	:	section of the bar
N	:	normal effort directed according to the axis X
α, β	:	damping coefficients of Rayleigh

One also poses:

$\omega_n = (2n-1)\frac{\pi}{2}$ where $n = 1,2,3, ...$

$\varepsilon_n = \frac{1}{2}(\alpha\omega_n + \beta/\omega_n)$

Displacement in a point $M(x)$ unspecified is given by:

$$u(x,t) = \frac{Nx}{EA} + \frac{8NL}{\pi^2 EA}\sum_{n=1}^{\infty}(-1)^n \frac{e^{-\omega_n\varepsilon_n t}}{(2n-1)^2}\left\{\cos\left(\sqrt{1-\varepsilon_n^2}\omega_n t\right) + \frac{\varepsilon_n}{\sqrt{1-\varepsilon_n^2}}\sin\left(\sqrt{1-\varepsilon_n^2}\omega_n t\right)\right\}$$

2.2 Results of reference

Values of the fields of displacement, speed and acceleration of the loose lead (node $N10$) are worth at the moment $t = 0.0195\,s$:

	Displacement (m)	**Speed** $(m.s^{-1})$	**Acceleration** $(m.s^{-2})$
Calculation without damping	$-8.3766\text{x}10^{-7}$	$1.6753 \text{ X } 10^{-3}$	0
Calculation with structural damping	$-1.00462\text{x}10^{-6}$	$1.20384\text{x}10^{-3}$	– 1.21564

2.3 Uncertainty on the solution

Analytical solution.

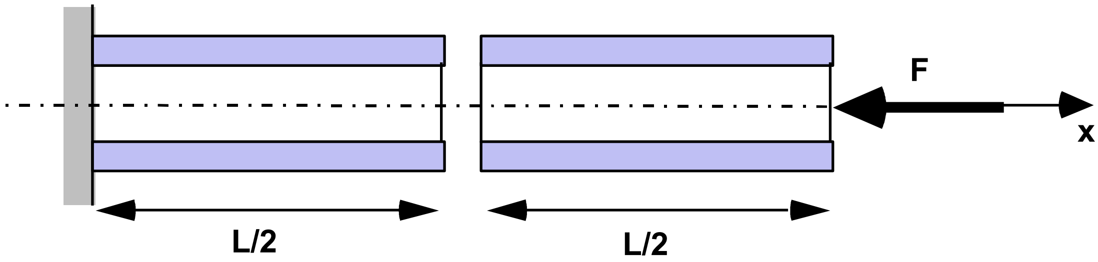
2.4 Bibliographical references

[1] G. ROBERT: Analytical solutions in dynamics of the structures. Report Samtech n°121, March 1996.

3 Modeling A

3.1 Characteristics of modeling

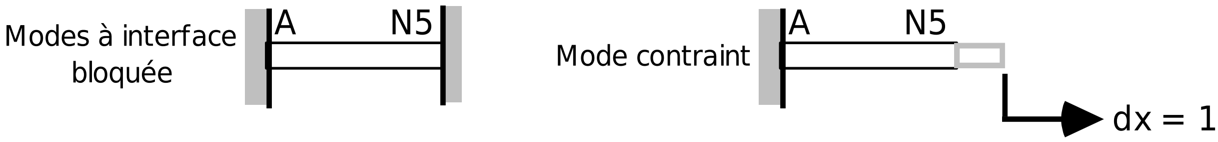
The beam is cut out in two parts of equal size. Each substructure considered is with a grid in segments to which are affected of the elements of the type `POU_D_T`.



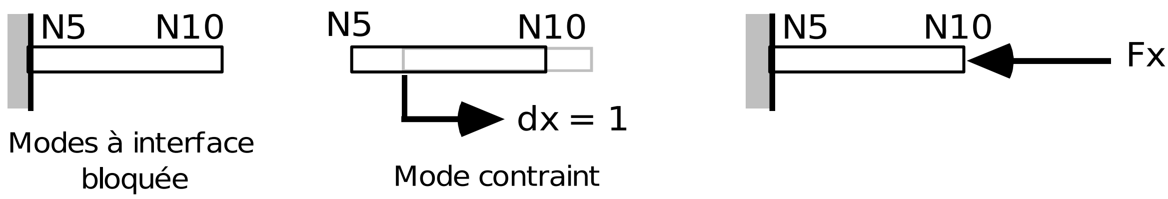
The structure is studied using the method of under-structuring transitory with interfaces of the type “Craig-Bampton” (blocked interfaces).

The modal base used is made up of 4 clean modes for the substructure of left, of 5 clean modes for the substructure of right-hand side to which are added the constrained modes associated with the degrees of freedom with interface (2).

Base projection of the substructure of left:



Base projection of the substructure of right-hand side:



3.2 Characteristics of the grids

The grid of the complete beam to carry out the calculation of reference shows the following characteristics:

file of type grid `Aster (.mail)`
Many nodes = 11
Many meshs = 10 SEG2

The grid of the half-beam to carry out calculation by under-structuring, shows the following characteristics:

file of the type `Ideas (.msup)`
Many nodes = 6
Many meshs = 5 SEG2

3.3 Features tested

One tests the features of dynamic calculation per under-structuring as well as the restitution in physical space.

3.4 Sizes tested and results

The values are restored on a grid skeleton made up of the two substructures. The initial grid which contains 6 nodes is duplicated to create the substructure of right-hand side. The node of end thus corresponds to node 12.

Identification	Reference (beam supplements)	Under-structuring	Difference (%)
Node 12: displacement (m)	– 6.2818E-7	– 6.2818E-7	< 0.1
Node 12: speed $(m.s^{-1})$	2.0957E-3	2.0957E-3	< 0.1
Node 12: acceleration $(m.s^{-2})$	1.1139E+1	1.1139E+1	< 0.1

3.5 Remarks

One can be astonished that the adopted reference corresponds to the complete beam modelled by 10 elements and not to the analytical solution. It is that the development of the solution in series of the clean modes converges very slowly: the modal solution is very far away from the theoretical solution here. The relevant comparison is thus well that selected.

Calculation by modal recombination is carried out on the basis of complete modal structure (11 modes) taking into account the adopted discretization. In the same way, the dimension of the base of projection used for calculation by dynamic under-structuring is of 11 (substructure of left: 4 clean modes + 1 constrained mode; substructure of right-hand side: 5 clean modes + 1 constrained mode). It is thus normal to obtain an excellent agreement between the modeling of the complete beam and that of the beam divided into two substructures.

4 Modeling B

4.1 Characteristics of modeling

The characteristics of this modeling are identical to the preceding one (modeling A). The only difference lies in the fact that the structure is deadened. Damping used is of type proportional:

$$C=\alpha K+\beta M \quad \text{with} \quad \alpha=6.5\text{x}10^{-6} s \quad \text{and} \quad \beta=16.0 s^{-1} .$$

These values correspond to a reduced damping of 1% on the first mode of the structure.

4.2 Characteristics of the grid

The characteristics of the grid are also identical to those of modeling A (cf [§ 3.2]).

4.3 Features tested

One tests the features of dynamic calculation, with catch in depreciation account, by under-structuring as well as the restitution in physical space.

4.4 Sizes tested and results

The results are restored on a grid skeleton made up of the two substructures. The initial grid which contains 6 nodes is thus duplicated to create the substructure of right-hand side. The node of end thus corresponds to node 12.

Identification	Reference (complete model)	Under-structuring	Difference (%)
Node 12 : displacement (m)	– 9.54882E-7	– 9.54882E-7	< 0.1
Node 12 : speed $(m.s^{-1})$	1.22190E-3	1.22190E-3	< 0.1
Node 12 : acceleration $(m.s^{-2})$	– 1.91712E+0	– 1.91712E+0	< 0.1

4.5 Remarks

One can be astonished that the adopted reference corresponds to the complete beam modelled by 10 elements and not to the analytical solution. The important differences between the digital and theoretical solutions are ascribable with the reduced number of elements. The use of 50 elements instead of 10 would have made it possible to approach theoretical acceleration with a margin of 1%. That put except for, one can note that the use of a method of under-structuring provides the same results as those of the complete beam.

Calculation by modal superposition is carried out on the basis of complete modal structure (11 modes). In the same way, the dimension of the base of projection used for calculation by under - dynamic structuring is of 11 (substructure of left: 4 clean modes + 1 constrained mode; substructure of right-hand side: 5 clean modes + 1 constrained mode). It is thus normal to obtain an excellent agreement between the modeling of the complete beam and its modeling in two substructures.

5 Modeling C

5.1 Characteristics of modeling

As in preceding modeling (modeling B), the structure is deadened with the same damping of the type proportional and the same coefficients of Rayleigh (cf [§ 5.1]).
The difference lies inuse of a method of condensation dynamic of the right part of the structure (including this time 6 elements) which is then represented by a dynamic macronutrient of under-structuring. The resolution of the dynamic problem takes place on a mixed model made up of the macronutrient of the right part of the structure where is exerted the force applied and of a physical model reduced to its left part including 4 elements where there is embedding.

5.2 Characteristics of the grid

The grid used is the initial grid complete (cf §1.1) obtained by duplication then assembly of the grid of modeling A what gives 10 elements and 11 nodes.

5.3 Features tested

One tests the features of dynamic calculation with condensation of part of the structure as well as the restitution in physical space.

5.4 Sizes tested and results

The results are restored on the complete initial grid assembling the two substructures of preceding modelings. The node of end corresponds to the node $N10$ (cf appears of the §1.1). They are exactly the same ones as those of modeling B as hoped.

Identification	Reference (complete model)	Dynamic condensation	Difference (%)
Node $N10$: displacement (m)	– 9.54882E-7	– 9.54882E-7	< 0.1
Node $N10$: speed ($m.s^{-1}$)	1.22190E-3	1.22190E-3	< 0.1
Node $N10$: acceleration ($m.s^{-2}$)	– 1.91712E+0	– 1.91712E+0	< 0.1

6 Summary of the results

As well in the case not deadened as in the deadened case, the results got using the complete model and by under-structuring do not present significant variations. The operators of linear transitory calculation by under-structuring are thus validated.

In the deadened case, the agreement between the solutions digital and analytical would have been better by taking more elements (50 instead of 10 for example).

Lastly, let us announce that the results got by *Code_Aster* were compared with results got by the SAMCEF software. They are included in the table below. It is noted that in the case not deadened, the two software provides nearby results, quite as far away from the analytical solution.

Identification	**Case not deadened**		**Deadened case**	
	Code_Aster	The SAMCEF software	*Code_Aster*	The SAMCEF software
$N12$, displacement (m)	– 6.282E-7	– 6.290E-7	– 9.549E-7	– 9.557E-7
$N12$, speed (m/s)	2.096E-3	2.080E-3	1.222E-3	1.222E-3
$N12$, acceleration (m/s^2)	1.114E+1	1.075E+1	– 1.917E+0	– 1.910E+0