

## SDLL02 - Beam hurled, embed-free, folded up on it even

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### Summary:

This two-dimensional problem consists in searching the frequencies and the modes of vibration of a mechanical structure, made up of a hurled, embedded beam free and folded up on itself.

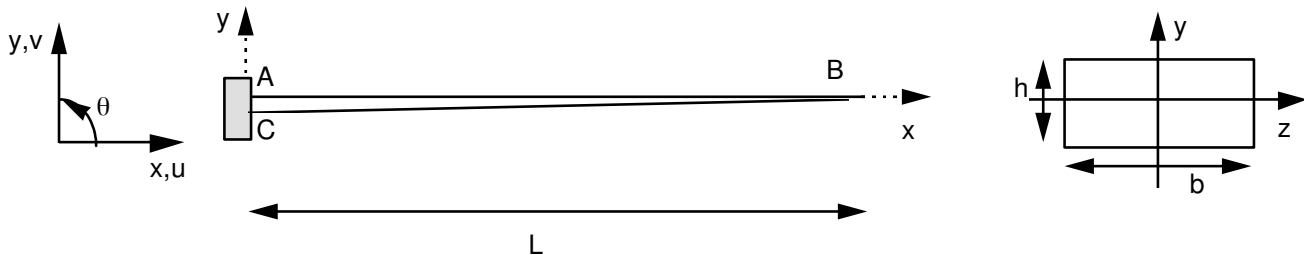
The posed problem does not have physical meaning. It on the other hand makes it possible to validate the research of the Eigen frequencies of inflection multiples and the research of the double modes in a subspace of order 2.

In this test, three different modelings are carried out:

- in the first modeling, the boundary conditions are imposed using parameters of Lagrange (order `AFFE_CHAR_MECA`) and the clean values and vectors are calculated by the method of Lanczos (order `CALC_MODES, METHODE='TRI_DIAG'` under the keyword factor `SOLVEUR_MODAL`),
- in the second modeling, the boundary conditions are imposed by removing degrees of freedom in the matrices of mass and stiffness (order `AFFE_CHAR_CINE`) and the clean values and vectors are calculated by the method of Bathe and Wilson (order `CALC_MODES, METHODE='JACOBI'` under the keyword factor `SOLVEUR_MODAL`),
- in the third modeling, the behavior of modeling is checked `COQUE_C_PLAN` in dynamics. The eigenvalues and the clean modes are calculated with the method of SORENSEN (order `CALC_MODES, METHODE='SORENSEN'` under the keyword factor `SOLVEUR_MODAL`).

## 1 Problem of reference

### 1.1 Geometry



The geometrical characteristics of the beam constituting the mechanical model are the following ones:

Length:  $L = 0.5 \text{ m}$

Rectangular cross-section:

Height:  $h = 0.005 \text{ m}$   
 Width:  $b = 0.050 \text{ m}$   
 Surface:  $A = 2.5 \cdot 10^{-4} \text{ m}^2$   
 Moment of inertia:  $I_z = 5.208 \cdot 10^{-10} \text{ m}^4$

The coordinates (in meters) of the points characteristic of the whole of the beams are:

	A	B	C
x	0.	0.5	0.
y	0.	0.	0.

### 1.2 Material properties

The properties of material constituting the beam are:

$$E = 2.1 \cdot 10^{11} \text{ Pa}$$

$$\nu = 0.3$$

$$\rho = 7800. \text{ kg/m}^3$$

### 1.3 Boundary conditions and loadings

The boundary condition which characterizes this problem is the embedding of the point A and is written:

$$u = v = 0. , \theta = 0 .$$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

The reference solution is that given in card SDLL02/89 of the guide VPCS which presents the method of calculating in the following way:

By the method of stiffness dynamic, one shows that the folded up beam admits double frequencies, solution of:

$$\cos(\lambda) = 0 \quad \Rightarrow \quad \lambda_i = (2i-1) \frac{\pi}{2}$$

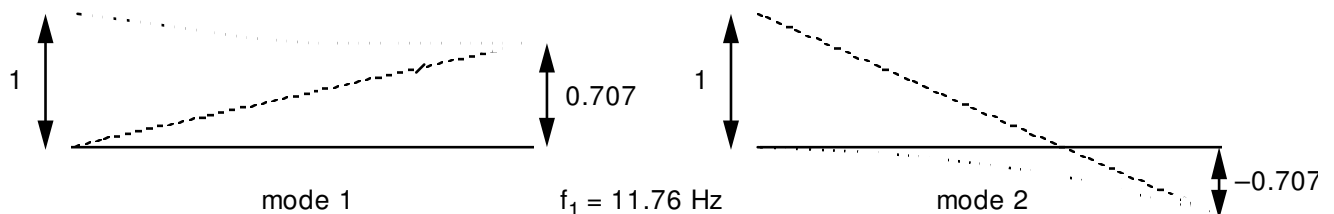
$$f_i = \frac{1}{2\pi} \frac{\lambda_i^2}{L^2} \sqrt{\frac{EI_z}{\rho A}} \quad i = 1, 2, \dots$$

For a rectangular section, one obtains:

$$f_i = (2i-1)^2 \pi \frac{R}{8L^2} \sqrt{\frac{E}{12\rho}} \quad i = 1, 2, \dots$$

This formulation neglects the deformations of shearing action and inertia of rotation (beam of Euler-Bernoulli).

For the clean modes, the forms are given in guide VPCS. They are normalized to 1 or -1 at the point of greater amplitude. There are results only for modes 1, 2, 3, 4, 7 and 8. For example, the forms of the first two clean modes are the following ones:



Note:

In Code\_Aster, when an eigenvalue is multiple, the clean modes associated with this eigenvalue, even if they are normalized and orthogonal two to two, are, a priori, unforeseeable. One does not know, for the moment, to test the form of a multiple mode.

### 2.2 Results of reference

The results of reference are the first eight Eigen frequencies.

### 2.3 Uncertainty on the solution

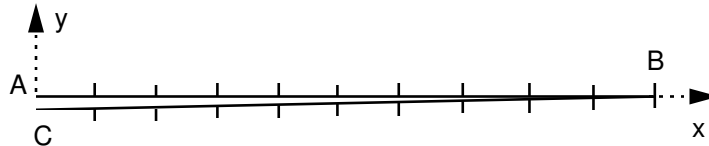
There is no uncertainty on the solution because it is analytical.

### 2.4 Bibliographical references

- 1) PIRANDA J.: Course and Directed Work of Vibrations of the Structures - Mechanical Option - École Nationale Supérieure of Mechanics and Micromechanics - Laboratory of Mechanics Applied - Besancon (France (1983).)

## 3 Modeling A

### 3.1 Characteristics of modeling



The beam in 20 meshes was cut out `SEG2` (10 for part AB and 10 for the part `BC`).

The modeling used for the beams is that of Euler Bernoulli (`POU_D_E`).

Two-dimensional solutions are sought. One can thus block for all the nodes displacement `DZ` and rotations `DRX` and `DRY`.

The end of the beam (not `A`) is embedded from where in this point:

$$DX = DY = 0. \quad DRZ = 0.$$

### 3.2 Characteristics of the grid

The grid contains 21 nodes and 20 meshes of the type `SEG2`.

The points characteristic of the grid are the following:

$$\text{Not } A=A \quad \text{Not } B=B \quad \text{Not } C=C$$

### 3.3 Sizes tested and results

For the frequencies of vibration of the structure, there are the following results:

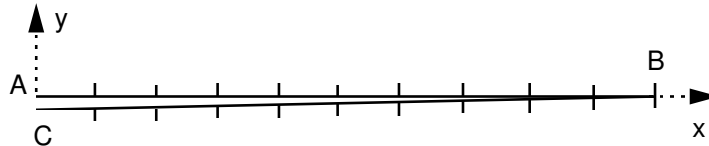
Identification	Reference
Frequency 1	11.76
Frequency 2	11.76
Frequency 3	105.88
Frequency 4	105.88
Frequency 5	294.10
Frequency 6	294.10
Frequency 7	576.44
Frequency 8	576.44

### 3.4 Remarks

For the Eigen frequencies, the got results are very good (error < 0.1%).

## 4 Modeling B

### 4.1 Characteristics of modeling



The beam in 20 meshes was cut out `SEG2` (10 for the part  $AB$  and 10 for the part  $BC$ ).

The modeling used for the beams is that of Euler Bernouilli (`POU_D_E`).

Two-dimensional solutions are sought. One can thus block for all the nodes displacement  $DZ$  and rotations  $DRX$  and  $DRY$ .

The end of the beam (not  $A$ ) is embedded from where in this point:

$$DX = DY = 0. \quad DRZ = 0.$$

### 4.2 Characteristics of the grid

The grid contains 21 nodes and 20 meshes of the type `SEG2`.

The points characteristic of the grid are the following:

$$\text{Not } A=A \quad \text{Not } B=B \quad \text{Not } C=C$$

### 4.3 Sizes tested and results

For the frequencies of vibration of the structure, there are the following results:

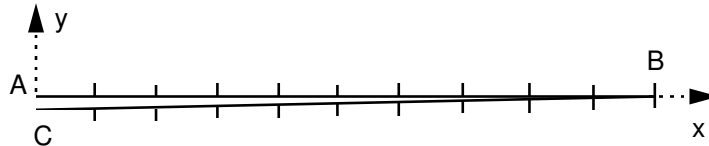
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Frequency 1	11.76
Frequency 2	11.76
Frequency 3	105.88
Frequency 4	105.88
Frequency 5	294.10
Frequency 6	294.10
Frequency 7	576.44
Frequency 8	576.44

### 4.4 Remarks

For the Eigen frequencies, the got results are very good (error < 0.1%).

## 5 Modeling C

### 5.1 Characteristics of modeling



The beam in 20 meshes was cut out SEG3 (10 for part AB and 10 for the part BC).

Modeling used is COQUE\_C\_PLAN.

The end of the beam (not A) is embedded from where in this point:

$$DX = DY = DRZ = 0.$$

### 5.2 Characteristics of the grid

The grid contains 41 nodes and 20 meshes of the type SEG3.

The points characteristic of the grid are the following:

$$\text{Not } A = A \quad \text{Not } B = B \quad \text{Not } C = C$$

### 5.3 Sizes tested and results

For the frequencies of vibration of the structure, there are the following results:

Identification	Reference
Frequency 1	11.76
Frequency 2	11.76
Frequency 3	105.88
Frequency 4	105.88
Frequency 5	294.10
Frequency 6	294.10
Frequency 7	576.44
Frequency 8	576.44

### 5.4 Remarks

In this CAS-test, where the results are independent of the Young modulus, it is not necessary to modify the Young modulus retained for modeling, as in the case of the linear static analysis, to take account of the real width of the beam.

## 6 Summary of the results

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- Modelings A and B of the Beam type:  
With the problem is dealt with a very good precision on the first eight frequencies (tolerance  $< 0.1\%$ ) for two modelings tested.
- Modeling COQUE\_C\_PLAN  
The precision on the results is good for the first three frequencies, the error is about  $0.6\%$ . It is degraded as the frequency increases, the error passes from the 4<sup>ième</sup> frequency to the 8<sup>ième</sup> of  $1.5\%$  with  $6.4\%$ . More the frequency is high plus the difference between the double frequencies is important.