SDLD106 – System masses spring with damping under harmonic oscillation

Summary

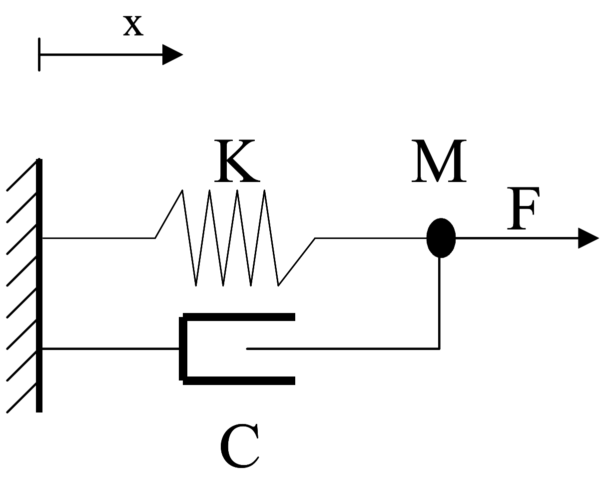
This case test makes it possible to validate the elementary matrix of a discrete element. One calculates the Eigen frequency of an oscillator to a degree of freedom (system masses – arises), the transitory answer due to a sinewave excitation and the forced answer due to a harmonic excitation.

The results of reference are got analytically.

1 Problem of reference

1.1 Geometry

The diagram of the system is presented on the following figure:



1.2 Properties of material

The properties of material are the following ones:

stiffness $K:4\pi^2 N/m$
mass $M:100\,kg$
damping $C=0.4\pi\, Ns/m$

1.3 Boundary conditions and loadings

The spring is embedded with the one of its ends and subjected to a force F at the loose lead. The specific mass can move only according to the direction x .

1.4 Initial conditions

At the initial moment, the mass is motionless and it is with its position of balance.

$x(0)=0\,m$
$v(0)=0\,m/s$

2 Reference solution

2.1 Method of calculating

One proposes to calculate the Eigen frequency of the oscillator, the answer due to a sinewave excitation and the answer due to a harmonic excitation.

2.1.1 Calculation of the Eigen frequency

For the validation of the calculation of the Eigen frequency, we consider the system without damping. Thus, the displacement of the loose lead of the spring is governed by the following relation:

$$M\ddot{x}+Kx=F \quad (1)$$

The Eigen frequency of this oscillator is:

$$f_0=\frac{1}{2\pi}\sqrt{\frac{K}{M}}=\frac{\omega_0}{2\pi}$$ (f_0 : Eigen frequency, ω_0 : own pulsation)

2.1.2 Calculation of the transitory answer

For the validation of the calculation of the transitory answer, we consider the system without damping. With the initial condition $x(0)=0$ and $\dot{x}(0)=0$, if a sinusoidal force is applied $F(t)=F\sin(\Omega t)$, at the loose lead of the spring, the solution of the differential equation (1) is:

$$x(t)=\frac{F\left(\sin\Omega t-\frac{\Omega}{\omega_0}\sin\omega_0 t\right)}{M(\omega_0^2-\Omega^2)}$$

For this calculation of answer to a sinewave excitation, we chose: $\Omega=1\,rd/s$ and $F=1\,N$.

2.1.3 Calculation of the harmonic answer

One then proposes to calculate the answer of the deadened oscillator due to a harmonic excitation.

The displacement of the loose lead of the spring is governed by the following relation:

$$M\ddot{x}+C\dot{x}+Kx=F \quad (2)$$

By applying a sinusoidal force $F(t)=F\sin(\Omega t)$, at the loose lead of the spring, and by adopting the complex notation, one obtains the forced answer: $\hat{x}(\Omega)=\frac{F}{K-\Omega^2 M+jC}$

For this calculation of the harmonic forced answer, we chose: $0.5\,\omega_0\leq\Omega\leq1.5\,\omega_0$

3 Modeling A

3.1 Characteristics of modeling

The oscillator is modelled using elements `2D_DIS_TR`.

3.2 Characteristics of the grid

Number and type of meshs: 1 element of the type `SEG2` and an element of the type `POI1`.

3.3 Sizes tested and results

Eigen frequency:

Size tested	**Reference**	**Tolerance**
$f_0=\frac{\omega_0}{2\pi}$	$0.1\,Hz$	1.E-4

Sinusoidal answer to the moment $t_1=1\,s$:

Size tested	**Reference**	**Tolerance**
$x(t_1=1\,s)$	$1.55346\,10^{-3}\,m$	5.E-4

Harmonic answer to the frequency $\Omega_1=1.5\,\omega_0$:

Size tested	**Reference**	**Tolerance**
$\hat{x}(\Omega)=1.5\omega_0$	$-2.022511\,10^{-2}\ \ -5.15690\,10^{-4}\,i\,m$	5.E-3

The three tests are doubled tests of not-regression with a tolerance of 1.E-6.

4 Summary of the results

The got results are in concord with the theoretical solution.