SDLD104 - Extrapolation of local measurements on a complete model (discrete)

Summary:

It is about a test of linear dynamics discrete.

The goal is to test the order PROJ_MESU_MODAL in the case of a discrete system. This order makes it possible to project experimental dynamic transitory answers in a certain number of points on a modal basis of a digital modeling.

This test contains 2 modelings where projection is done on a concept of the type [mode_meca], the difference being given by the way in which this concept is manufactured.

For 2 modelings, provided experimental measurements are identical and make it possible to test the research of the nodes in opposite, the taking into account of a local orientation and the treatment of a sampling in constant time or not, for measurements in displacement.

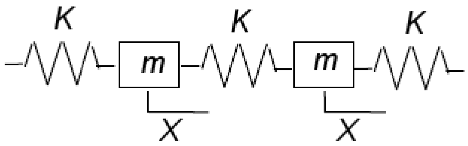
In both cases, the reference solution is analytically given (by Maple); projection is carried out in the favorable configuration where the number of modes is equal to the number of measurements.
The answers in displacement obtained after projection are identical to the displacements of reference provided in data.

The values speeds and the accelerations deduced from the displacements obtained after projection are close to those obtained analytically. The weak noted variations are due to the errors of approximation generated by the determination via a linear diagram in time of speeds and accelerations.

1 Problem of reference

1.1 Description of the system

We consider the system represented by the diagram below:



1.2 Masses and rigidity

The three springs are of identical rigidity: $k=1000\,N/m$.
The two masses are equal to $m=10\,kg$.

1.3 Boundary conditions and loading

The two ends are embedded.
The loading is a thrust load in traction applied to the mass $m1$, sinusoidal according to time, of pulsation ω .

2 Reference solutions

2.1 Method of calculating used for the reference solution

The analytical solution of this problem is presented below.

- Modes and frequencies of vibration:
  The following system characterizes the dynamics of the masses:

$$\begin{cases} m\ddot{x}_1 + 2k\,x_1 - k\,x_2 = 0 \\ m\ddot{x}_2 + 2k\,x_2 - k\,x_1 = 0 \end{cases} \quad \text{éq 2.1-1}$$

  What is equivalent to the following system:

$$\begin{cases} m(\ddot{x}_1 + \ddot{x}_2) + k(x_1 + x_2) = 0 \\ m(\ddot{x}_1 - \ddot{x}_2) + 3k(x_1 - x_2) = 0 \end{cases} \quad \text{éq 2.1-2}$$

  The 2 Eigen frequencies of the system are thus given by:

$$\omega_1 = \sqrt{\frac{k}{m}} \quad \text{and} \quad \omega_2 = \sqrt{\frac{3k}{m}} \quad \text{éq 2.1-3}$$

  and the associated modal deformations are:

$$\Phi_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \Phi_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{éq 2.1-4}$$

  The generalized matrices are:

$$\begin{aligned} \bar{M} &= \Phi^T M \Phi = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2m & 0 \\ 0 & 2m \end{pmatrix} \\ \bar{K} &= \Phi^T K \Phi = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2k & 0 \\ 0 & 6k \end{pmatrix} \end{aligned} \quad \text{éq 2.1-5}$$

- Transitory answer:
  The sinusoidal effort is applied to the first mass: $F = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(\omega t)$

The checked dynamic system is the following:

$$M\ddot{X}+KX=F \quad \text{éq 2.1-6}$$

While projecting on the basis of clean mode, we obtain:

$$\Phi^T M\Phi\ddot{\eta}+\Phi^T K\Phi\eta=\Phi^T F \quad \text{éq 2.1-7}$$

That is to say:

$$\begin{pmatrix}2m & 0\\ 0 & 2m\end{pmatrix}\begin{pmatrix}\ddot{\eta}_1\\ \ddot{\eta}_2\end{pmatrix}+\begin{pmatrix}2k & 0\\ 0 & 6k\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}\sin(\omega t) \quad \text{éq 2.1-8}$$

We thus end to the following uncoupled system:

$$\begin{cases} m\ddot{\eta}_1+k\eta_1=\frac{1}{2}\sin(\omega t)\\ m\ddot{\eta}_2+3k\eta_2=\frac{1}{2}\sin(\omega t)\end{cases} \quad \text{éq 2.1-9}$$

The solution of this system is given by:

$$\begin{cases} \eta_1(t)=A_1\cos(\omega_1 t)+B_1\sin(\omega_1 t)+\frac{\sin(\omega t)}{2m(\omega_1^2-\omega^2)}\\ \eta_2(t)=A_2\cos(\omega_2 t)+B_2\sin(\omega_2 t)+\frac{\sin(\omega t)}{2m(\omega_2^2-\omega^2)}\end{cases} \quad \text{éq 2.1-10}$$

Displacements in physical space are obtained by the formula of Ritz:

$$X=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\Phi\eta=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}=\begin{pmatrix}\eta_1+\eta_2\\ \eta_1-\eta_2\end{pmatrix} \quad \text{éq 2.1-11}$$

One from of deduced the expressions from $x_1(t)$ and $x_2(t)$:

$$\begin{cases} x_1(t)=A_1\cos(\omega_1 t)+B_1\sin(\omega_1 t)+A_2\cos(\omega_2 t)+B_2\sin(\omega_2 t)+\frac{\sin(\omega t)}{2m}\left(\frac{1}{\omega_1^2-\omega^2}+\frac{1}{\omega_2^2-\omega^2}\right)\\ x_2(t)=A_1\cos(\omega_1 t)+B_1\sin(\omega_1 t)-A_2\cos(\omega_2 t)+B_2\sin(\omega_2 t)+\frac{\sin(\omega t)}{2m}\left(\frac{1}{\omega_1^2-\omega^2}-\frac{1}{\omega_2^2-\omega^2}\right)\end{cases}$$

éq 2.1-12

At the initial moment, the system is at rest, from where final expressions of $x_1(t)$ and $x_2(t)$:

$$\begin{cases} x_1(t)=\dfrac{1}{2\mathrm{m}}\left[\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_1}\sin(\omega_1 t)}{\omega_1^2-\omega^2}+\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_2}\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \\ x_2(t)=\dfrac{1}{2\mathrm{m}}\left[\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_1}\sin(\omega_1 t)}{\omega_1^2-\omega^2}-\dfrac{\sin(\omega t)-\dfrac{\omega}{\omega_2}\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \end{cases}$$ éq 2.1-13

Speeds of the two masses are calculated by deriving displacements compared to time:

$$\begin{cases} \dot{x}_1(t)=\dfrac{\omega}{2\mathrm{m}}\left[\dfrac{\cos(\omega t)-\cos(\omega_1 t)}{\omega_1^2-\omega^2}+\dfrac{\cos(\omega t)-\cos(\omega_2 t)}{\omega_2^2-\omega^2}\right] \\ \dot{x}_2(t)=\dfrac{\omega}{2\mathrm{m}}\left[\dfrac{\cos(\omega t)-\cos(\omega_1 t)}{\omega_1^2-\omega^2}-\dfrac{\cos(\omega t)-\cos(\omega_2 t)}{\omega_2^2-\omega^2}\right] \end{cases}$$ éq 2.1-14

Accelerations of the two masses are calculated by deriving speeds compared to time:

$$\begin{cases} \ddot{x}_1(t)=\dfrac{\omega}{2\mathrm{m}}\left[\dfrac{\omega\sin(\omega t)-\omega_1\sin(\omega_1 t)}{\omega_1^2-\omega^2}+\dfrac{\omega\sin(\omega t)-\omega_2\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \\ \ddot{x}_2(t)=\dfrac{\omega}{2\mathrm{m}}\left[\dfrac{\omega\sin(\omega t)-\omega_1\sin(\omega_1 t)}{\omega_1^2-\omega^2}-\dfrac{\omega\sin(\omega t)-\omega_2\sin(\omega_2 t)}{\omega_2^2-\omega^2}\right] \end{cases}$$ éq 2.1-15

2.2 Results of reference

The comparison of the results relates to displacements, speeds and accelerations along the axis of the two masses, at five different moments.

2.3 Uncertainty on the solution

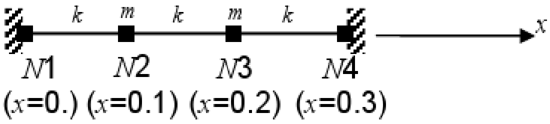
The reference solution is exact.

The discrete model represents perfectly the problem arising (the modal base is complete; there is thus no approximation related to a possible modal truncation). The number of modes of the base of modal projection is equal to the number of measurements, therefore the solution of the inversion is exact (in opposition to an approximate solution of a generalized opposite problem). If the research of the nodes in opposite is good, the displacements obtained after projection must be in perfect adequacy with the experimental values. Speeds and accelerations are determined by derivation of the modal contributions identified via a diagram of linear approximation in time, thus being able to generate some errors.

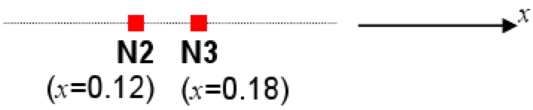
3 Modeling A

3.1 Characteristics of modeling and the grids

Digital grid:
The digital grid is carried out directly with the format ASTER. It comprises 4 nodes and 3 discrete meshs.



Experimental grid:
The grid of measurement understands only 2 specific elements and 2 nodes:



3.2 Characteristics of measurements

Provided experimental measurements are:

- With the node $N3$:
  The data are the axial displacements, multiplied by $-1/\sqrt{2}$, and applied in the direction $-X$. The local orientation indicated in the command file is $(45.0.0.)$
  The sampling of time is constant: initial time is $0\,s$, the step of time is $10^{-3}\,s$ and the number of moments is 1001 (either until a final time of $1\,s$).

- With the node $N2$:
  The data are the axial displacements, applied in the direction x .
  The sampling of time is variable: every moment is indicated of $0\,s$ with $1\,s$, by step of $10^{-3}\,s$ (1001 moments on the whole).

  The values result from the analytical calculation carried out with Maple.

3.3 Characteristics of the modal base

The two only modes are stored in a concept of the type mode_meca created by the order CALC_MODES. Their Eigen frequencies are identical to the analytical Eigen frequencies.

3.4 Values tested

Identification		Reference	*Code_Aster*	difference
	with T = 0.1 s	$1{,}745\ 10^{-4}$	$1{,}745\ 10^{-4}$	0.01%
	with T = 0.3	$6{,}797\ 10^{-4}$	$6{,}797\ 10^{-4}$	0.01%

		S			
DEPL_X (m)	with the node $N2$ (mass 1)	with T = 0.5 S	– 1,217 10^{-3}	– 1,217 10^{-3}	0.01%
		with T = 0.7 S	5,214 10^{-4}	5,214 10^{-4}	– 0.01%
		with T = 0.9 S	9,031 10^{-4}	9,031 10^{-4}	0.00%
DEPL_X (m)	with the node $N3$ (mass 2)	with T = 0.1 S	9,154 10^{-6}	9,154 10^{-6}	0.00%
		with T = 0.3 S	6,414 10^{-4}	6,414 10^{-4}	0.00%
		with T = 0.5 S	– 8,636 10^{-4}	– 8,636 10^{-4}	0.00%
		with T = 0.7 S	– 1,107 10^{-4}	– 1,107 10^{-4}	0.03%
		with T = 0.9 S	1,633 10^{-3}	1,633 10^{-3}	0.02%
VITE_X (m/s)	with the node $N2$ (mass 1)	with T = 0.1 S	4,586 10^{-3}	4,616 10^{-3}	0.65%
		with T = 0.3 S	– 7,598 10^{-3}	– 7,663 10^{-3}	0.85%
		with T = 0.5 S	– 1,581 10^{-4}	– 8,000 10^{-5}	7.81 10^{-5} m/s
		with T = 0.7 S	9,382 10^{-3}	9,354 10^{-3}	– 0.30%
		with T = 0.9 S	– 7,481 10^{-3}	– 7,537 10^{-3}	0.75%
VITE_X (m/s)	with the node $N3$ (mass 2)	with T = 0.1 S	4,328 10^{-4}	4,405 10^{-4}	1.79%
		with T = 0.3 S	3,671 10^{-3}	3,640 10^{-3}	– 0.84%
		with T = 0.5 S	– 1,539 10^{-2}	– 1,536 10^{-2}	– 0.20%
		with T = 0.7 S	2,453 10^{-2}	2,457 10^{-2}	0.15%
		with T = 0.9 S	– 1,899 10^{-2}	– 1,912 10^{-2}	0.68%
ACCE_X (m/s^2)	with the node $N2$ (mass 1)	with T = 0.1 S	6,112 10^{-2}	6,100 10^{-2}	– 0.20%
		with T = 0.3 S	– 1,306 10^{-1}	– 1,300 10^{-1}	– 0.46%
		with T = 0.5 S	1,571 10^{-1}	1,600 10^{-1}	1.85%
		with T = 0.7 S	– 5,657 10^{-2}	– 5,800 10^{-2}	2.53%
		with T = 0.9 S	– 1,124 10^{-1}	– 1,130 10^{-1}	0.53%
ACCE_X (m/s^2)	with the node $N3$ (mass 2)	with T = 0.1 S	1,562 10^{-2}	1,618 10^{-2}	3.58%
		with T = 0.3 S	– 6,031 10^{-2}	– 6,223 10^{-2}	3.18%
		with T = 0.5 S	5,102 10^{-2}	5,374 10^{-2}	5.33%
		with T = 0.7	7,428 10^{-2}	7,043 10^{-2}	– 5.19%

S			
with T = 0.9 S	– 2,364 10^{-1}	– 2,263 10^{-1}	– 4.28%

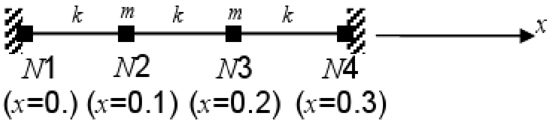
Note:

Speed with the node $N2$ at the moment $t=0.5\,s$ being relatively close to zero, the comparison is carried out for this case in absolute value.

4 Modeling B

4.1 Characteristics of modeling and the grids

Digital grid:
The digital grid is carried out directly with the format ASTER. It comprises 4 nodes and 3 discrete meshs.



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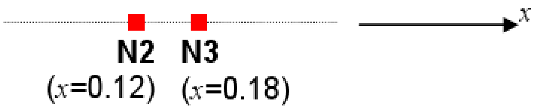

4.2 Characteristic of measurements

Provided experimental measurements are:

- With the node $N3$:
  The data are the axial displacements, multiplied by $-1/\sqrt{2}$, and applied in the direction $-x$. The local orientation indicated in the command file is $(45.0.0.)$
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- With the node $N2$:
  The data are the axial displacements, applied in the direction X.
  The sampling of time is variable: every moment is indicated of $0\,s$ with $1\,s$, by step of $10^{-3}\,s$ (1001 moments on the whole).

  The values result from the analytical calculation carried out with Maple.

4.3 Characteristics of the modal base

The two only modes are stored in a concept of the type mode_meca, created by the order DEFI_BASE_MODALE. The interface, of Craig-Bampton type, is placed on the degree of freedom in following displacement x node $N2$ (corresponding to the mass $m1$). The modal base thus contains a dynamic mode (with $N2$ blocked) and a static mode.

4.4 Values tested

Identification	Reference	*Code_Aster*	difference
with $T = 0.1$	1,745 10^{-4}	1,745 10^{-4}	0.01%

		s			
		with T = 0.3 s	6,797 10^{-4}	6,797 10^{-4}	0.01%
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		with T = 0.3 s	– 6,031 10^{-2}	– 6,223 10^{-2}	3.18%
ACCE_X	with the node	with T = 0.5	5,102 10^{-2}	5,374 10^{-2}	5.33%

	$N3$	S			
(m/s^2)	(mass 2)	with T = 0.7 S	7,428 10^{-2}	7,043 10^{-2}	– 5.19%
		with T = 0.9 S	– 2,364 10^{-1}	– 2,263 10^{-1}	– 4.28%

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Speed with the node $N2$ at the moment $t=0.5\,s$ being relatively close to zero, the comparison is carried out for this case in absolute value.

5 Summary of the results

For two modelings, the answers in displacement obtained after projection are identical to the displacements of reference calculated analytically with Maple and provided in data.

The values speeds and the accelerations deduced from the displacements obtained after projection are close to those obtained analytically. The weak noted variations are due to the errors of approximation generated by the determination by a linear diagram in time of speeds and accelerations.