SDLD103 - Seismic answer of a system 3 masses and 4 springs multimedia

Summary

The problem consists in analyzing the answer of a mechanical structure of standard beam embed-embedded and not deadened, modelled by a system 3 masses and 4 springs and subjected to an unspecified seismic loading.

One tests the discrete element in traction and rotation, the calculation of the clean modes and the static modes and the calculation of the transitory answer by modal superposition of a structure subjected to a accélérogramme of translation (modeling A) or rotation (modeling B).
In modeling C, one tests macro-order MACR_SPECTRE when the supports see a different excitation with the supports.

The got results are in very good agreement with the results of reference (analytical results).

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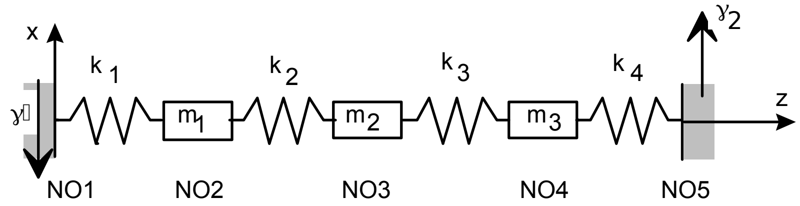
1 Problem of reference

1.1 Geometry

The beam is modelled by a set of 4 springs and of 3 specific masses.



1.2 Material properties

Stiffness of connection: $k=k_1=k_2=k_3=k_4=104\ N/m$;
specific mass: $m=m_1=m_2=m_3=10\ kg$.

1.3 Boundary conditions and loadings

Boundary conditions :
Only authorized displacements are the translations according to the axis x .
Points $NO1$ and $NO5$ are embedded: $dx=dy=dz=drx=dry=drz=0$.
The other points are free in translation according to the direction x : $dy=dz=drx=dry=drz=0$.

Loading :
Points of anchoring $NO1$ and $NO5$ each one are subjected to a transverse acceleration $\gamma_1(t)=a\,t^2$ with $a=2.10^5 m/s^4$ in $NO1$ and $\gamma_2(t)=0\,m/s^2$ in $NO5$.

1.4 Initial conditions

The system is at rest: with $t=0$, $dx(0)=0$, $dx/dt(0)=0$ in any point.

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2 Reference solution

2.1 Method of calculating used for the reference solution

The problem consists in calculating the response of a system to five degrees of freedom subjected to two accelerations $\gamma_1(t)$ and $\gamma_2(t)$ distinct of an unspecified form. It is exposed in detail in the reference [bib2].

One calculates the Eigen frequencies initially f_i, associated clean vectors standardized compared to the modal mass Φ_{Ni} and static modes Ψ system (analytical values). One calculates then the generalized answer of the system multimedia by solving analytically the integral of Duhamel [bib1]. Lastly, one restores on the physical basis the vector of relative displacements (on the active degrees of freedom) X_r, which allows us, after having calculated the vector of displacements of training X_e, to calculate the vector of absolute displacements $X_a = X_r + X_e$.

2.2 Results of reference

- Calculation of the three Eigen frequencies f_i, associated clean vectors standardized compared to the modal mass Φ_{Ni} and of the static modes Ψ system

$$\begin{cases} f_1 = \dfrac{1}{2\pi\sqrt{(2+\sqrt{2})m/2\mathrm{k}}} = 3.85\,Hz \\ f_2 = \dfrac{1}{2\pi\sqrt{m/2\mathrm{k}}} = 7.12\,Hz \\ f_3 = \dfrac{1}{2\pi\sqrt{(2-\sqrt{2})m/2\mathrm{k}}} = 9.30\,Hz \end{cases}, \quad \Phi_N = \frac{1}{2\sqrt{m}}\begin{bmatrix} 1 & -\sqrt{2} & -1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & \sqrt{2} & 1 \end{bmatrix} \text{ and } \Psi = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}.$$

- Calculation of the generalized answer of the multimedia system

The fundamental equation of dynamics, in the relative reference mark on the active degrees of freedom is written:

$M\ddot{X}_r + K X_r = (M\Psi + M_{XS})\ddot{X}_S$ with $\ddot{X}_S = \begin{bmatrix} at^2 \\ 0 \end{bmatrix}$, the vector of the accelerations imposed on the level of the various points of anchoring.

The equation of the movement projected on the basis as of dynamic modes standardized compared to the modal mass Φ_N is written, by considering only the active degrees of freedom:

$$\ddot{q}(t) + K_G\, q(t) = -\Phi_N^T M\,\Psi\,\ddot{X}_S = \frac{a\sqrt{m}\,t^2}{4}\begin{bmatrix} 2+\sqrt{2} \\ \sqrt{2} \\ 2-\sqrt{2} \end{bmatrix}$$

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The answer of this linear system, at one moment t, then consists in calculating the integral of Duhamel:

$$q(t)=-\frac{a\sqrt{m^3}}{4\mathrm{k}}\begin{bmatrix}(3+2\sqrt{2})(t^2+(2+\sqrt{2})(\cos\omega_1 t-1)m/k)\\(t^2+(\cos\omega_2 t-1)m/k)/\sqrt{2}\\(3-\sqrt{2})(t^2+(2-\sqrt{2})(\cos\omega_3 t-1)m/k)\end{bmatrix}$$

- Calculation of displacement relating to the active degrees of freedom: $X_r=\sum_i \Phi_{\mathrm{Ni}} q_i$ that is to say:

$$X_r=-\frac{am}{8\mathrm{k}}\begin{bmatrix}7t^2+\left[(10+7\sqrt{2})\frac{m}{k}(\cos\omega_1 t-1)+(\cos\omega_2 t-1)+(10-7\sqrt{2})(\cos\omega_3 t-1)\right]m/k\\8t^2+\left[(10\sqrt{2}+14)\frac{m}{k}(\cos\omega_1 t-1)+(-10\sqrt{2}+14)(\cos\omega_3 t-1)\right]m/k\\5t^2+\left[(10+7\sqrt{2})\frac{m}{k}(\cos\omega_1 t-1)-(\cos\omega_2 t-1)+(10-7\sqrt{2})(\cos\omega_3 t-1)\right]m/k\end{bmatrix}.$$

- Calculation of displacements of training to the active degrees of freedom: $X_e=\Psi X_S=a\frac{t^4}{48}\begin{bmatrix}3\\2\\1\end{bmatrix}$

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- Calculation of absolute displacements to the active degrees of freedom: $X_a=X_r+X_e$.

2.3 Uncertainty on the solution

No if one calculates the integral of Duhamel analytically [bib1].

2.4 Bibliographical references

1) J.S. PRZEMIENIECKI: Theory of matrix structural analysis. New York, Mac Graw-Hill, 1968, pages 351-357.
2) Fe WAECKEL: Documentations use and validation of the developments carried out to calculate the seismic answer of multimedia structures. HP-52/96/002

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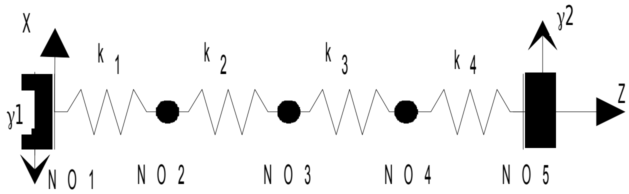
3 Modeling A

3.1 Characteristics of modeling

The elements are modelled by discrete elements with 3 degrees of freedom DIS_T.



The node $NO1$ is subjected to an imposed acceleration $\gamma_1(t)$, the node $NO5$ with $\gamma_2(t)$. One calculates the relative displacement of the nodes $NO2$, $NO3$ and $NO4$ compared to their static deformation, their displacement of training and their absolute displacement.

Temporal integration is carried out with the algorithms of Euler (not of time: 10^{-3} second), of Devogelaere (not of time: 10^{-3} second) and with an algorithm with step of adaptive time of order 2.

3.2 Characteristics of the grid

The grid consists of 5 nodes and 4 discrete elements (DIST_T).

3.3 Sizes tested and results

3.3.1 Relative displacements of the nodes $NO2$, $NO3$ and $NO4$

- Relative displacements of the node $NO2$ with the algorithms of digital of Euler, Devogelaere, adaptive integration of order 2 and Runge-Kutta (32 and 54):

Time (S)	Reference
0.1	– 8,47734E-01
0.3	– 1,55202E+01
0.5	– 4,36449E+01
0.7	– 8,50830E+01
1.0	– 1,74790E+02

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- Relative displacements of the node $NO3$ with the digital algorithm of integration of Euler:

Time (S)	Reference
0.01	9,87666E-10
0.02	2,49501E-07
0.03	6,25468E-06
0.04	6,05829E-05
0.05	3,47191E-04
0.06	1,42349E-03
0.07	4,62144E-03
0.08	1,26245E-02
0.09	3,01825E-02
0.1	– 7,68449E-01
0.3	– 1,76923E+01
0.5	– 4,99310E+01
0.7	– 9,70711E+01
1.0	– 1,99722E+02

- Relative displacements of the node $NO3$ with the algorithms of digital of Devogelaere, adaptive integration of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0.1	– 7,68449E-01
0.3	– 1,76923E+01
0.5	– 4,99310E+01
0.7	– 9,70711E+01
1.0	– 1,99722E+02

- Relative displacements of the node $NO4$ with the digital of Euler, Devogelaere, adaptive algorithm of integration of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0.1	– 4,09632E-01
0.3	– 1,10372E+01
0.5	– 3,12415E+01
0.7	– 6,05833E+01
1.0	– 1,24803E+02

3.3.2 Absolute displacements of the nodes $NO2$, $NO3$ and $NO4$

- Absolute displacements of the node $NO2$ with the digital of Euler, Devogelaere, adaptive algorithm of integration of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0.1	4,02266E-01
0.3	8,57298E+01
0.5	7,37605E+02
0.7	2,91617E+03
1.0	1,23252E+04

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- Absolute displacements of the node $NO3$ with the digital of Euler, Devogelaere, adaptive algorithm of integration of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0.1	6,48847E-02
0.3	4,98077E+01
0.5	4,70902E+02
0.7	1,90376E+03
1.0	8,13361E+03

- Absolute displacements of the node $NO4$ with the digital of Euler, Devogelaere, adaptive algorithm of integration of order 2 and Runge-Kutta (54 and 32):

Time (S)	Reference
0.1	7,03506E-03
0.3	2,27128E+01
0.5	2,29175E+02
0.7	9,39833E+02
1.0	4,04186E+03

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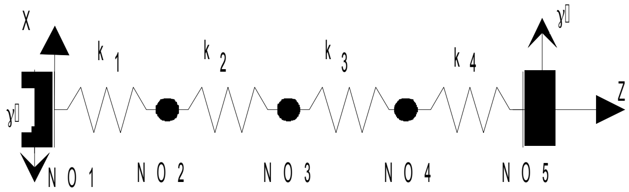
4 Modeling B

It is the same one modeling that the preceding one except for the loading which is a accélérogramme of rotation.

4.1 Characteristics of modeling

The elements are modelled by discrete elements with 3 degrees of freedom `DIS_T`.



The node $NO1$ is subjected to an imposed acceleration $\gamma_1(t)$, the node $NO5$ with $\gamma_2(t)$. One calculates the relative displacement of the nodes $NO2$, $NO3$ and $NO4$ compared to their static deformation, their displacement of training and their absolute displacement.

Temporal integration is carried out with the algorithm of Euler (not of time: 10^{-3} second).

4.2 Characteristics of the grid

The grid consists of 5 nodes and 4 discrete elements (**`DIST_TR`**).

4.3 Sizes tested and results

4.3.1 Relative displacements of the nodes $NO2$, $NO3$ and $NO4$

- Relative displacements of the node $NO2$:

Time (S)	Reference
0.1	– 8,47734E-01
0.3	– 1,55202E+01
0.5	– 4,36449E+01
0.7	– 8,50830E+01
1.0	– 1,74790E+02

- Relative displacements of the node $NO3$:

Time (S)	Reference
0.1	– 7,68449E-01
0.3	– 1,76923E+01
0.5	– 4,99310E+01
0.7	– 9,70711E+01
1.0	– 1,99722E+02

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- Relative displacements of the node $NO4$:

Time (S)	Reference
0.1	– 4,09632E-01
0.3	– 1,10372E+01
0.5	– 3,12415E+01
0.7	– 6,05833E+01
1.0	– 1,24803E+02

4.3.2 Absolute displacements of the nodes $NO2$, $NO3$ and $NO4$

•Absolute displacements of the node $NO2$:

Time (S)	Reference
0.1	4,02266E-01
0.3	8,57298E+01
0.5	7,37605E+02
0.7	2,91617E+03
1.0	1,23252E+04

•Absolute displacements of the node $NO3$:

Time (S)	Reference
0.01	9,87666E-10
0.02	2,49501E-07
0.03	6,25468E-06
0.04	6,05829E-05
0.05	3,47191E-04
0.06	1,42349E-03
0.07	4,62144E-03
0.08	1,26245E-02
0.09	3,01825E-02
0.10	6,48847E-02
0.30	4,98077E+01
0.50	4,70902E+02
0.70	1,90376E+03
1.0	8,13361E+03

•Absolute displacements of the node $NO4$:

Time (S)	Reference
0.1	7,03506E-03
0.3	2,27128E+01
0.5	2,29175E+02
0.7	9,39833E+02
1.0	4,04186E+03

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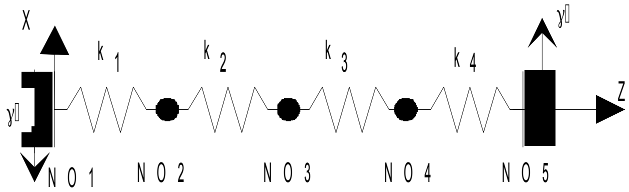
5 Modeling C

It is the same one modeling which the preceding one, the loading is identical to the loading of modeling A.

5.1 Characteristics of modeling

The elements are modelled by discrete elements with 3 degrees of freedom `DIS_T`.



The node $NO1$ is subjected to an imposed acceleration $\gamma_1(t)$, the node $NO5$ with $\gamma_2(t)$. Temporal integration is carried out with the algorithm of Euler (not of time: 10^{-3} second).

One calculates the spectrum envelope of the absolute acceleration of the nodes $NO2$, $NO3$ and $NO4$. This calculation can be done either using a sequence of order RECU_FONCTION and CALC_FONCTION or using the macro-order MACR_SPECTRE which calls on these same orders.

5.2 Characteristics of the grid

The grid consists of 5 nodes and 4 discrete elements (**`DIST_TR`**).

5.3 Sizes tested and results

5.3.1 Absolute acceleration wraps nodes $NO2$, $NO3$ and $NO4$

- Acceleration wraps:

Frequency (Hz)	Reference
0.1	483.65
0.3	3840.04
0.52	9016.62

6 Summary of the results

Results got for modelings A and B with *Code_Aster* are in conformity with the results of reference (the error is in general lower than 0.03%).

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