SDLD34 – To release of a simple mass/arises

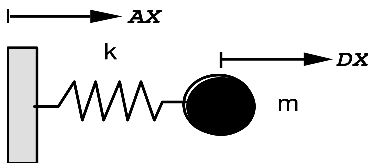
Summary:

A simple oscillator, made up of a mass connected to a support by a spring, is subjected to releasing on the basis of the tended spring. It is checked that *Code_Aster* calculate well the oscillatory answer to the initial conditions without external forces.

One tests the features of linear transitory calculation on physical basis and modal operator `DYNA_VIBRA`.

1 Problem of reference

1.1 Geometry



One is interested in the movement of the mass m .

1.2 Material properties

Specific mass: $m=1\,kg$
Elastic spring: $k=\pi^2 N/m$

Case 1 : conservative system (without damping)
Case 2 : dissipative system $c=0{,}2\pi\ N.s/m$

1.3 Boundary conditions and loadings

The problem is unidimensional in the direction x , and with a degree of freedom: the displacement of the mass m .

The mass is left free, without external excitation.

Initially it is except balance: the spring is tended with an elongation of 1 meter.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is analytical. In the absence of damping, it is a simple sinusoid of which the period is equal to the own pulsation of the oscillator, $\omega_0=\sqrt{\frac{k}{m}}$, and whose amplitude is initial lengthening (x_0) spring. The position $x(t)$ mass is given by the equation:

$$x(t)=x_0\cos(\omega_0 t) \qquad (1)$$

The speed of the mass is thus:

$$v(t)=-\omega_0 x_0\sin(\omega_0 t) \qquad (2)$$

In the presence of a viscous damping $(c_{[N.s/m]})$, the oscillations become deadened and the position $x(t)$ is written :

$$x(t)=x_0 e^{-\zeta\omega_0 t}[\cos(\omega t)+(\frac{\zeta}{\sqrt{1-\zeta^2}})\sin(\omega t)] \qquad (3)$$

where ζ is the reduced damping given by $\zeta=\frac{c}{2\omega_0 m}$. ζ is considered to be lower than 1 to preserve the oscillations. The pulsation is given by the formula $\omega=\omega_0\sqrt{(1-\zeta^2)}$. It is thus different from the own pulsation (ω_0) system.

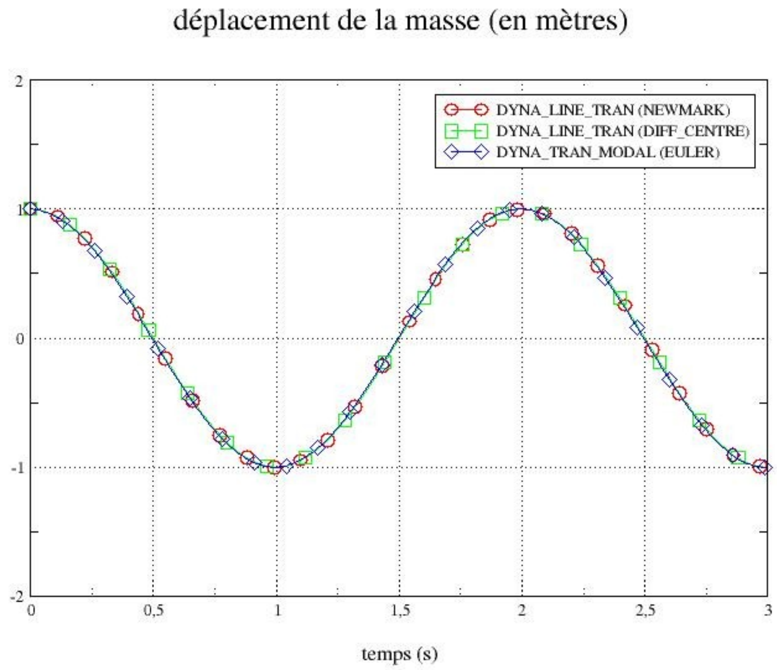
2.2 Results

Case 1 : conservative system (without damping)

For this system, the own pulsation $\omega_0=\pi\, rad/s$. The Eigen frequency is thus $f_0=\omega_0/2\pi=0,5\, Hz$.

Displacement (in m) and the speed (in m/s) of the mass, given respectively by Eqs.1 and 2 are:

$$x(t)=\cos(\pi t) \quad \text{and} \quad v(t)=-\pi\sin(\pi t)$$



Case 2 : viscous dissipative system

Reduced damping is of $\zeta=0,1$. The pulsation is $\omega=0,995\pi\, rad/s$ and the frequency is thus $f=\omega/2\pi=0,4975\, Hz$.

Displacement (in m) can then be calculated according to Eq.3.

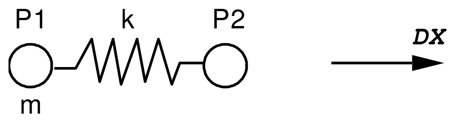
2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

Discrete element in translation of the type `DIS_T`



Characteristics of the elements:

With the nodes $P1$ and $P2$: matrices of masses of the type M_T_D_N with $m=100\,kg$.
Enter $P1$ and $P2$: a matrix of rigidity of the type K_T_D_L with $K_x=10^6\,N/m$

Boundary conditions:

All the degrees of freedom are blocked except the degree of freedom DX node $P2$.

3.2 Characteristics of the grid

Many nodes: 2

Many meshs and types: 1 SEG2, 2 POI1

3.3 Features tested

One tests the features of linear transitory calculation on physical basis and modal operator DYNA_VIBRA.

3.4 Sizes tested and results

Dynamic response

One tests the position of the mass at the end of one period, i.e. 2 seconds. Moreover, one tests the value of the modal participation of mode 1. As it is about a single mode and that he is normalized according to the node which carries the mass, the modal participation is identical to displacement.

Identification	Reference	Tolerance
DYNA_VIBRA/base physical (NEWMARK)	$1\,m$	1.E- 4%
DYNA_VIBRA/base physical (DIFF_CENTRE)	$1\,m$	1.E- 4%
DYNA_VIBRA/base_modale (EULER)	$1\,m$	0.01%
DYNA_VIBRA (modal participation)	$1\,m$	0.01%

One tests also the value the speed (in m/s) of the mass with T = 1.5 S, i.e. when it passes by the static position of balance $(x=0)$.

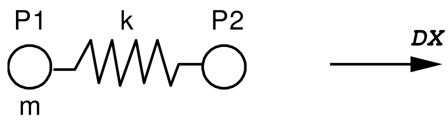
DYNA_VIBRA/base physical (NEWMARK)	π	1.E- 4%
DYNA_VIBRA/base_modale (EULER)	π	0.1%

4 Modeling B

4.1 Characteristics of modeling

Modeling A is taken again, but by adding a damping to the system masses/arises.

Discrete element in translation of the type DIS_T



Characteristics of the elements:

With the nodes $P1$ and $P2$: matrices of masses of the type M_T_D_N with $m=100\,kg$.
Enter $P1$ and $P2$: a matrix of rigidity of the type K_T_D_L with $K_x=10^6\,N/m$

Boundary conditions:

All the degrees of freedom are blocked except the degree of freedom DX node $P2$.

Damping: one adds to the system a reduced damping of $0{,}1$.
It is introduced into the case test, that is to say in a usual way by the key word AMOR_REDUIT, that is to say, to validate the functionality RELA_EFFO_VITE, by a linear relation between the speed of the mass/arises and a force applied to the node $P2$.

4.2 Characteristics of the grid

Many nodes: 2

Many meshs and types: 1 SEG2, 2 POI1

4.3 Features tested

One tests in particular, in an elementary way, in this modeling the functionality RELA_EFFO_VITE of the operator DYNA_VIBRA (BASE_CALCUL=' GENE'). By his use, one can introduce a nonlinear behavior depend on the speed of a point. Here one validates in a simple way this relation in the linear case by comparing it with a behavior of modal damping (which, in the case with only one mode, returns to a viscous damping).

4.4 Sizes tested and results

Identification	Reference	Tolerance
DYNA_VIBRA (BASE_CALCUL=' GENE') AMOR_REDUIT	$0{,}53\,\mathrm{m}$	1%
DYNA_VIBRA (BASE_CALCUL=' GENE') RELA_EFFO_VITE	$0{,}53\,\mathrm{m}$	1 %
DYNA_VIBRA (BASE_CALCUL=' GENE') AMOR_REDUIT	0.531338 (not regression)	1.E-4%
DYNA_VIBRA (BASE_CALCUL=' GENE') RELA_EFFO_VITE	0.531338 (not regression)	1.E-4%

5 Summary of the results

The results are satisfactory. The relative error corresponds to the digital error related to integration in time. The initial conditions are well taken into account. One concludes from it that *Code_Aster* correctly simulate to release in linear dynamics.

This test is also a functional validation of recovery in the form of function of the evolution in time of the participation of a mode, as well as a validation, on the case of a linear relation, functionality `RELA_EFFO_VITE`.