SDLD31 - Elementary validation of the diagrams in time in dynamics

Summary:
This CAS-test makes it possible to validate the programming of the diagrams of integration in time in DYNA_NON_LINE and DYNA_TRAN_MODAL.

More precisely, for DYNA_NON_LINE the following implicit schemes are tested:

1) average acceleration (keyword NEWMARK) with resolution in displacement and acceleration;
2) modified average acceleration (keyword HHT with MODI_EQUI='NOT');
3) Complete HHT (keyword HHT with MODI_EQUI='YES');
4) θ- diagram (keyword THETA_SCHEMA with THETA = 0.61) and resolution in displacement and of speed;
5) Krenk (keyword KRENK with KAPPA = 1.22) with resolution in displacement and of speed.

With complete diagram HHT, one tests also the continuations because this diagram requires a particular initialization. In the same way, one validates also the continuations for the average acceleration with resolution in acceleration because it is not tested in other CAS-tests.

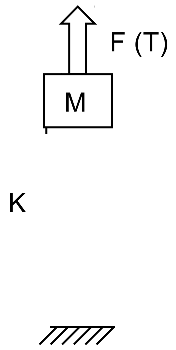
As for DYNA_TRAN_MODAL, the diagrams with step of constant time are tested, i.e.:

1) diagram of order 1 clarifies known as EULER ;
2) diagram NEWMARK implicit of order 2;
3) diagram clarifies order 4 known as DEVOGELAERE

The goal being to study the behavior of the diagrams in time, the selected problem is voluntarily very simple: it is about a linear system 1 degree of freedom mass-arises which is subjected to a sinusoidal force.
The reference solution is obtained by reprogramming of the diagrams of integration in Matlab and by calculation of the analytical solution.

1 Problems of reference

1.1 Geometry



The system with 1 degree of freedom is of the type masses M at the end of a spring of stiffness K directed according to the vertical direction z .

1.2 Properties of material

Stiffness arises K : $36.\pi^2 N/m$
Specific mass M : $1\,kg$
The values are selected so as to have an own pulsation of the system mass-arises ω_0 such as:
$\omega_0 = 6.\pi\, rad/s$ because $\omega_0^2 = K/M$.

1.3 Geometrical characteristics

Displacement is done according to the vertical direction z .

1.4 Boundary conditions and loadings

The base of the spring is embedded, the only degree of freedom is thus following displacement z specific mass M which is fixed at the other end of the spring.

The imposed loading is a sinusoidal effort $F(t)$ vertical imposed on the specific mass M:
$F(t) = \sin(1{,}1\,.\omega_0.t)$.

1.5 Initial conditions

The system is initially at rest.

2 Reference solution

2.1 Method of calculating

By this CAS-test one wants to study the behavior of the various diagrams of integration in implicit times of the operator `DYNA_NON_LINE`. It is not thus a question of seeking most accurately to reproduce possible an analytical solution.

A step of time is thus chosen $dt = 10^{-2} s$, sufficiently small compared to the own pulsation of the system and one will solve the linear transitory problem with the operator `DYNA_NON_LINE`.

For the nondissipative diagram of the average acceleration (keyword `NEWMARK`) it would be possible to calculate the analytical solution to compare itself with it. One tests the resolution in displacement or acceleration which must obviously give the same results.

For the other diagrams which one wishes to test and which are dissipative, obtaining an analytical solution is not very easy.

We thus chose to compare all calculations with a digital solution obtained with the Matlab code. For that, the various diagrams were programmed in Matlab.

One will thus carry out several transitory calculations in only one stage: with the diagram of average acceleration, the diagram of modified average acceleration, complete diagram HHT, it θ - diagram and the diagram of Krenk. For these the last two dissipative diagrams (with the selected values of parameters), one validates the resolution in displacement like of speed.

Then one tests the resumption of calculation with complete diagram HHT, to validate the mechanism of continuation with this diagram (one makes two continuations, the first with $0{,}2\,s$ and the second with $0{,}35\,s$).

2.2 Sizes and results of reference

The comparisons will relate to the displacement and the acceleration of the specific mass M at the following moments: $0{,}5\,s$, $0{,}7\,s$ and $1\,s$.

3 Modeling A - DYNA_NON_LINE

3.1 Sizes tested and results

The sizes tested are displacements and accelerations of the specific mass M .

Diagram in time	Type of the field	Moment	Values of Reference	tolerance
NEWMARK Solved in displacement or acceleration (with and without continuation)	DEPL	0.5 S	1.0804500210685E-02	1.E-5%
		0.7 S	-4.0671779495390E-03	1.E-5%
		1.0 S	-1.3026189840935E-02	1.E-5%
	ACCE	0.5 S	-4.6479181362891E+00	1.E-5%
		0.7 S	2.3748682319566E+00	1.E-5%
		1.0 S	5.5793367773016E+00	1.E-5%
HHT MODI_EQUI=‘NOT‘	DEPL	0.5 S	9.0224842641940E-03	1.E-5%
		0.7 S	-2.0242152707660E-03	1.E-5%
		1.0 S	-9.074477606657 E-03	1.E-5%
	ACCE	0.5 S	-4.0147576088701E+00	1.E-5%
		0.7 S	1.6489918279122E+00	1.E-5%
		1.0 S	4.022313059734449E+00	1.E-5%
HHT MODI_EQUI=‘YES‘ With or without continuations	DEPL	0.5 S	1.0775515187707E-02	1.E-5%
		0.7 S	-4.1787420850760E-03	1.E-5%
		1.0 S	-1.3121050364360E-02	1.E-5%
	ACCE	0.5 S	-4.6864764249454E+00	1.E-5%
		0.7 S	2.7540329873126E+00	1.E-5%
		1.0 S	5.9586276847714E+00	1.E-5%
THETA_SCHEMA Solved in displacement or speed	DEPL	0.5 S	9.4664592252170E-03	1.E-5%
		0.7 S	-2.4964363793720E-03	1.E-5%
		1.0 S	-9.0744776066570E-03	1.E-5%
	ACCE	0.5 S	-4.1642290444260E+00	1.E-5%
		0.7 S	1.6728854044803E+00	1.E-5%
		1.0 S	4.0223130597344E+00	1.E-5%

KRENK Solved in displacement or speed	DEPL	0.5 S	9.5870021341210E-03	1.E-5%
		0.7 S	-2.8112460401650E-03	1.E-5%
		1.0 S	-9.4157749054510E-03	1.E-5%
	ACCE	0.5 S	-4.1725044691246E+00	1.E-5%
		0.7 S	1.8167747070564E+00	1.E-5%
		1.0 S	4.1752706647681E+00	1.E-5%

4 Modeling B - DYNA_VIBRA

In the tests on the diagrams of DYNA_VIBRA one introduces a light viscous damping of for thousand. The good treatment of damping is thus validated. It is also the occasion to validate quadratic modal calculation.

4.1 Analytical solution

William Weaver Jr. Stephen P. Timoshenko and Donovan H. Young provide to chapter 1.9 of "Vibration Problems in Engineering" the solution with the problem of a system mass/arise with viscous damping subjected to a harmonic excitation.

The equation to be solved is an equation of the second order in time on only one degree of freedom in space:

$$\ddot{x}+2\eta\,\dot{x}+\omega_0^2 x=\frac{F}{m}\sin(\omega_e t)$$

where x is the displacement of the mass, $\dot{x}$ its speed and $\ddot{x}$ its acceleration.

$\omega_0^2=\frac{k}{m}$ is the own pulsation of the system, m being its mass and k its stiffness.

η is reduced damping.

Finally F is the amplitude of the force of excitation whereas ω_e is its pulsation.

4.2 Sizes tested and results

Calculation	Type of the field	Moment (or modal method)	Values of Reference	tolerance
CALC_MODES		'SORENSEN'		
	FREQ		3 Hz	1.E-4%
	AMOR_REDUIT		1E-03	1.E-4%
		'TRI_DIAG'		
	FREQ		3 Hz	1.E-4%
	AMOR_REDUIT		1E-03	1.E-4%
EULER (with and without REST_GENE_PHYS)	DEPL	0.5 S	0.010785	1.E-2%
		0.7 S	-3.745074E-03	1.E-1%
		1.0 S	-0.0125639	1.E-1%
NEWMARK	DEPL	0.5 S	0.010785	1.E-2%
		0.7 S	-3.745074E-03	1.E-1%
		1.0 S	-0.0125639	1.E-1%
DEVOG	DEPL	0.5 S	0.010785	1.E-2%
		0.7 S	-3.745074E-03	1.E-1%
		1.0 S	-0.0125639	1.E-1%

5 Summary of the results

This case test makes it possible to validate, into linear, the diagrams in implicit times (average acceleration, modified average acceleration, complete HHT, θ - diagram and Krenk) of the operator DYNA_NON_LINE, in the case of a variable loading imposed, for resolutions in displacement, speed or acceleration according to the cases.

Within this framework one validates also the continuation with complete diagram HHT.

On the same model one validates also the diagrams ofEULER, of NEWMARK and DEVOGELAERE of the operator DYNA_VIBRA.