Structural analysis in fatigue vibratory

1 Goal

The purpose of this document is to describe the implementation of a structural analysis in fatigue vibratory. It is more precisely a question of considering the amplitude maximum of vibration acceptable of a structure subjected to a static loading (known) and to a dynamic loading (unknown).

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2 Industrial context

The purpose of this document is to describe the implementation of a structural analysis in fatigue vibratory. One is interested more particularly in the studies on the revolving machines (wings of turbines for example). These components are requested by a static loading (centrifugal load related to the rotation of the machine) and by a dynamic loading (vibrations induced by the flow of the fluid). The lifetime of the structure depends at the same time on the dynamic part (amplitude of the variations of the constraints) and on the static part (average constraint).

The static loading is generally well-known; the dynamic part is as for it difficult to measure as well as to calculate. One cannot thus implement directly the classical tools for calculation of the lifetime, based on the evolution of the constraints in the course of time (confer notices below).

Contrary, it can be interesting to consider the amplitude of maximum variation acceptable by the structure, for one or more clean modes of vibration. This amplitude can then be compared with the vibrations measured on site (measurements BVM for example).

The approach of such a study is the following one:

- calculation of the constraint related to the static loading;
- calculation of the constraint associated with the clean mode considered;
- application of the criteria of tiredness to calculate, in each node of the grid, the acceptable maximum vibration to have an unlimited endurance of the structure.

The last stage can be done with the operator `CALC_FATIGUE` (`TYPE_CALCUL = 'FATIGUE_VIBR'`). The principle of calculation, its implementation in Code_Aster and an industrial example are described in the following chapters.

Remarks :

A certain number of assumptions are introduced for calculation, cf following paragraph. It is a question here of estimating, in a conservative way, an acceptable vibratory level. If 'temporal evolution of the constraints in the structure is known, it is necessary to use the classical features of the operators `CALC_FATIGUE` or `POST_FATIGUE` (with uniaxial or multiaxial criteria, methods of counting of cycles,…).
Apart from the turbines, methodology presented here could also apply to the vibratory analysis of lines of piping (the average constraint corresponding then to the pressure interns), or for the wind mills.
Calculation requires to know as a preliminary stress the rupture and the limit of endurance of material.

3 Presentation of the method of calculating

3.1 Tally of calculation and assumptions

One places oneself within the framework of a decomposition of the constraint in the structure on a modal basis:

$$\sigma_{total}(t)=\sigma_{stat}+\sum_{i=1}^{\infty}\alpha_i\sigma_{mod}^{i}\cos(\omega_i t+\phi_i) \ ,$$

while noting σ_{stat} the constraint related to the static loading and σ_{mod}^{i} constraints associated with clean modes structure. ω_i and ϕ_i are respectively the pulsation (known) and the dephasing (unknown) of mode **I**.
Classical calculation would consist in evaluating the damage, the modal contributions and dephasings being known. Here, reciprocally, one seeks to consider the contributions modal maximum associated with an endurance unlimited with the structure.

It is not possible to identify in a general way the maximum contribution of each mode. One is thus led to introduce three simplifying assumptions: uniaxial criterion of tiredness; imposition a priori of the relative weight of the modes; maximization of the amplitude.

Uniaxial criterion of tiredness: the use of a uniaxial criterion of tiredness (method of Wöhler) amounts supposing that the principal directions of the static loading and the dynamic loading are them same. This assumption seems licit for the usual structures concerned (wings, lines of piping,…) ; it induces a conservatism undoubtedly excessive in the case general. Subsequently, the notation σ will correspond to the standard of the constraint (von Mises or Tresca).
It should be noted that a multiaxial approach (with criteria of the type Crossland or Dang Van) would be also possible. These criteria require however to know, besides the limit of endurance in traction of material, the limit of endurance in pure shearing. This data is often inalienable for materials of the power stations, and the functionality would have been then not easily usable.

Relative weight of the modes: it is supposed that the relative weight of the various clean modes considered is known. This relative weight can for example be estimated starting from measurements on site. ON considers also only one limited number of clean modes N (in practice, one will generally have $N<4$ or 5). In other words: $\sigma_{total}(t)=\sigma_{stat}+\alpha\sum_{i=1}^{N}\beta_i\sigma^i_{mod}\cos(\omega_i t+\phi_i)$.
The coefficient α is the parameter which one seeks to calculate.

Maximization of the amplitude: in the expression of the constraint above, dephasings ϕ_i are unknown. The alternate constraint S_{alt} , definite like the half-amplitude of variation of the constraint, is then calculated as follows: $S_{alt}=\alpha\sum_{i=1}^{N}\beta_i\sigma^i_{mod}$. This definition of S_{alt} is conservative.

3.2 Calculation of the acceptable vibrations

Under the assumptions introduced above, calculation is summarized to identify the coefficient α associated with an unlimited endurance of the structure.
The static stress σ_{stat} corresponds here to a average constraint, generally nonworthless. It is necessary to take into account this constraint in the curve of tiredness of Wöhler. This taking into account is done classically with the aidE of the diagram of Haigh [R7.04.01], either with the line of Goodman, or with the parabola To stack.
While noting S_l limit of endurance and S_u limit with the rupture material, one a:

- with the line of Goodman:

$$S_{alt}^{\max}=S_l\left(1-\frac{\sigma_{stat}}{S_u}\right) \text{, that is to say } \alpha=S_l\left(1-\frac{\sigma_{stat}}{S_u}\right)/\sum_{i=1}^{N}\beta_i\sigma^i_{mod} .$$

- with the parabola To stack:

$$S_{alt}^{\max}=S_l\left(1-\frac{\sigma^2_{stat}}{S^2_u}\right) \text{, that is to say } \alpha=S_l\left(1-\frac{\sigma^2_{stat}}{S_u^{\ 2}}\right)/\sum_{i=1}^{N}\beta_i\sigma^i_{mod} .$$

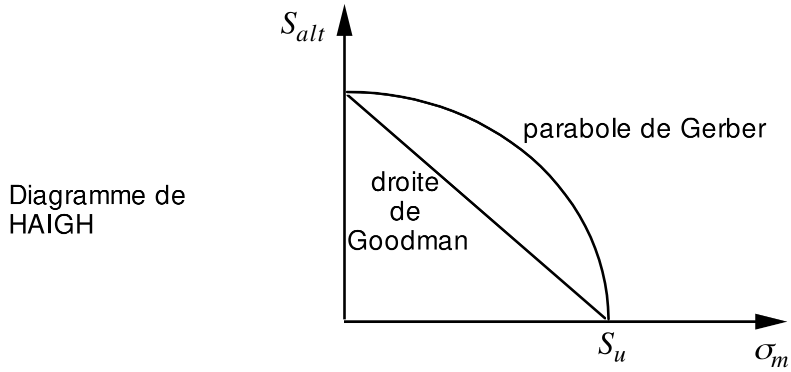

Figure 3.2-1: Diagram of Haigh

This calculation is done for all the nodes or points of Gauss of the structure. Lbe zones where α is weakest correspond to the zones which limit the lifetime of the structure.
To pass from the coefficient α with the acceptable amplitude of vibration, an additional operation is to be realized. This operation is described in the § 4.4 .

4 Implementation of calculation

4.1 Calculation of the static stresses and dynamic

The estimate of the acceptable vibratory level requires the first calculation of the static stresses and dynamic:

- the result corresponding to the static loading (centrifugal load, pressure,…) can be calculated with `MECA_STATIQUE` or `STAT_NON_LINE` . Only one step of time must be present in the structure of data result. The calculation of the constraints σ_{stat} starting from the field of displacement is done using the order `CALC_CHAMP` (option `SIEQ_ELGA` or `SIEQ_ELNO` according to whether one wishes to calculate the damage with the nodes or the points of Gauss).
- The calculation of N clean modes considered can be done with the order `CALC_MODES`. The calculation of the modal constraints σ_{mod}^{i} is done using the order `CALC_CHAMP` (option `SIEQ_ELGA` or `SIEQ_ELNO` according to whether one wishes to calculate the damage with the nodes or the points of Gauss).

4.2 Definition of the properties material

Two parameters material are necessary for calculation:

- Lvalue of the limit to the rupture of material has S_u . This parameter must be introduced into the operator `DEFI_MATERIAU` [U4.43.01] (keyword factor `RCCM`, operand `Known`).
- limit of endurance of material S_l . This parameter corresponds to the first point of the curve of Wöhler (operator `DEFI_MATERIAU` , keyword `TIREDNESS` , operand `WOHLER`).

4.3 Call to `CALC_FATIGUE`

The use of `CALC_FATIGUE` (`TYPE_CALCUL = 'FATIGUE_VIBR'`) require to inform the following parameters :

- static stresses and modal beforehand calculated;
- the properties material;
- the list of the modes to be considered (`NUME_MODE`) and their relative weight (`FACT_PARTICI`), corresponding to the coefficients $(\beta_i)_{1\le i\le N}$;
- the choice of the mode of taking into account of the average constraints (To stack or Goodman: operand `CORR_SIGM_MOYE`) ;

- the choice of the place of calculation of the damage: with the nodes (`OPTION= 'DOMA_ELNO_SIGM'`) or at the points of Gauss (`OPTION= 'DOMA_ELGA_SIGM'`).

4.4 Interpretation of the results

At exit of the operator `CALC_FATIGUE` , one has a field (with the nodes or the points of Gauss) of the vacceptable alor of α : zones where α is weakest correspond to the zones which limit the lifetime of the structure.

To pass from the coefficient α with the acceptable amplitude of vibration, an additional operation is to be realized. Let us suppose for example that one is interested in the amplitude of displacement in a given point, that one will note $\partial \tilde{u}$. This point can correspond for example to the position of a sensor, where at the zone of amplitude of maximum vibration.

One notes $\tilde{u}_{mod}^{i}$ displacement at the point of interest associated with mode I. The acceptable amplitude of vibration at the point of interest is then:

$$\partial \tilde{u}=min(\alpha)\sum_{i=1}^{N}\beta_i \tilde{u}_{mod}^{i}$$

This calculation is illustrated in the example below.

The minimal value of α can be obtained in three different ways:

- maybe by post-treating in Aster the field result (`POST_RELEVE_T, OPERATION=' EXTREMA'`);
- maybe by visualizing the field result;
- maybe in the file message. Information is printed as follows:

```
-----------------------------------------------------------

 Amplitude of maximum vibration acceptable by the structure:  1.318438
 ---------------------------------------------------------------
```

4.5 Remarks and advices

Some remarks can be formulated on this functionality:

- The result of calculation is necessarily very sensitive to the quality of the calculation of the constraints. The user must thus make sure that the smoothness of its grid is sufficient. The use of quadratic elements is essential.
- In the same way, it is advised to do the calculation at the points of Gauss, the interpolation for the passage points of Gauss – nodes causing inaccuracy.
- The critical zones generally correspond to the geometrical singularities. The user will have to check if the singularity corresponds to a physical reality or a too coarse discretization. It should be noted that the use of mending of meshes (software Lobster, operator `MACR_ADAP_MAIL`) does not allow to converge towards a nonworthless stable solution if the initial geometry is discretized too coarsely.

5 Example

5.1 Description of the calculation case

Postprocessing in fatigue vibratory will be illustrated on the case of the wings of bladings 5-12 of the low pressure turbines of the power stations REFERENCE MARK. The wings are fixed at the disc by fingers and pins. The diameter external of the bladings is about 2 meters.

The properties materials retained for this study are the following ones:

- limit of endurance: $S_l=400\,MPa$

- limit with the rupture: $S_u=800\,MPa$.

The static loading corresponds to the rotation of the wheel to $1500\,tours/min$. Three of the first modes correspond respectively to the tangential inflection ($f=100\,Hz$), with the axial inflection ($f=180\ Hz$) and with torsion ($f=360\,Hz$)

Note:

The properties material and the loadings used here do not correspond to the real case.

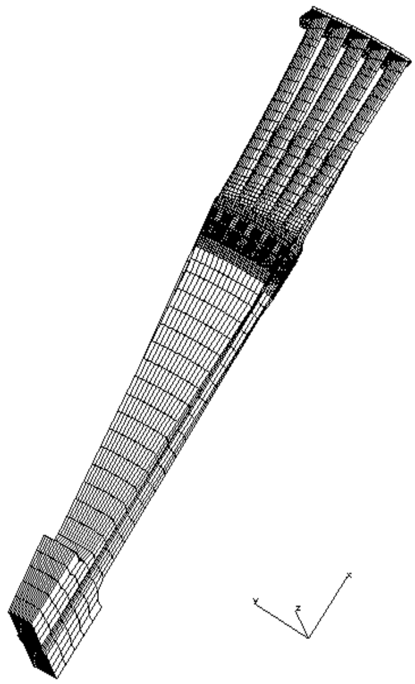

Figure 5.1-1: Grid of a package of wing of the blading 5-12

5.2 Got results

One is interested here in the maximum amplitude of acceptable vibration at the top of wing (group of nodes 'NO_CAPT' , corresponding to the position on site of sensor BVM), so that the lifetime in the first finger of the first wing of the package of blading is infinite (group of meshs called '**DOI_1_1**').

- Results resulting from CALC_FATIGUE :

The call to CALC_FATIGUE, for a calculation of the damage at the points of Gauss for the first mode of the structure, is done as follows:

```
DMG_MOD1= CALC_FATIGUE (TYPE_CALCUL = 'FATIGUE_VIBR',
                        OPTION      = 'DOMA_ELGA_SIGM',
                        HISTORY    = _F (RESULT = PRECONT,
                                         MODE_MECA = MODE,
                                         NUME_MODE=1,
                                         FACT_PARTICI = 1,),
                        TOO BAD     = 'WOHLER',
                        CORR_SIGM_MOYE = 'GOODMAN',
                        MATER       = SUBDUE,
                       )
```

Then, to extract the coefficient α minimal on the zone of interest:

```
IMPR_RESU (FORMAT=' RESULTAT',
             RESU=_F (CHAM_GD=DMG_MOD1, GROUP_MA=' DOI_1_1'),);
```

The got results, for the first three modes of the structure and for a combination between the first two modes are given in the following table.

	Mode 1 (tangential inflection)	Mode 2 (axial inflection)	Mode 3 (torsion)	Mode 1 + 0.5 Mode 2
Maximum static stress	685 MPa			
Maximum modal constraint	13 MPa	51 MPa	27 MPa	
α_{Gerber}^{min}	1,8	0,8	1,3	1,5
$\alpha_{Goodman}^{min}$	3,3	1,5	2,3	2,8

- Use of the results :

An additional operation is to be realized to pass from the coefficient α with the acceptable amplitude of vibration in a given node. The node considered here corresponds to the position of one of the incremental position sensors .

This operation can for example be made by simple loops python:

```
# Parameters of calculation (the same ones as in CALC_FATIGUE)
fact_partici = [1. , 0.5]
nume_mode = [1.2]
nbmode = len (nume_mode)

# EXTRACTION OF DISPLACEMENTS OF THE TABLE
TDEP = POST_RELEVE_T (ACTION=_F (     GROUP_NO=' NO_CAPT',
                                    INTITULE=' Depl. modal',
                                    RESULTAT=MODE,
                                    NOM_CHAM=' DEPL',
                                    TOUT_CMP=' OUI',
                                    OPERATION=' EXTRACTION',),);
DX = [Nun] *nbmode
DY = [Nun] *nbmode
DZ = [Nun] *nbmode
tdepl = TDEP.EXTR_TABLE ()
for I in arranges (nbmode):
  dmod = tdepl.NUMÉRIQUE_MODE==numérique_mode [I]
  DX [I] = dmod ['DX'] .values () ['DX']
  DY [I] = dmod ['DY'] .values () ['DY']
  DZ [I] = dmod ['DZ'] .values () ['DZ']

# EXTRACTION OF COEFFICIENT AMIN
TDMG12=POST_RELEVE_T (ACTION=_F (  GROUP_NO=' ELTS',
                                  INTITULE=' (MODE 1) + 0.5 * (MODE 2)
',
                                 CHAM_GD=D_MOD12,
                                 TOUT_CMP=' OUI',
                                 OPERATION=' EXTREMA',),);
AMIN = TDMG12.EXTR_TABLE ()
```

```
AMIN = AMIN.EXTREMA==' MIN'
AMIN = AMIN ['VALE'] .VALUES () ['VALE']

# CALCULATION OF ACCEPTABLE MAXIMUM DISPLACEMENT
DXmax = 0.
DYmax = 0.
DZmax = 0.
for I in arranges (nbmode):
  DXmax = DXmax + AMIN [0] * fact_partici [I] * ABS (DX [I] [0])
  DYmax = DYmax + AMIN [0] * fact_partici [I] * ABS (DY [I] [0])
  DZmax = DZmax + AMIN [0] * fact_partici [I] * ABS (DZ [I] [0])
Dtot = sqrt (DXmax*DXmax + DYmax*DYmax +DZmax*DZmax) .
```

This example can be found in the case test SDLV129A. The results for the present study are given in the following table.

Case	DX (mm)	DY (mm)	DZ (mm)	α_{Gerber}^{min}	DX^{max} (mm)	DY^{max} (mm)	DZ^{max} (mm)	D^{max} (mm)
Mode 1	-0,02	0,07	0,99	1,8	0,04	0,11	1,78	1,79
Mode 2	0,07	0,98	0,21	0,8	0,06	0,78	0,77	0,8
Mode 3	0,02	-0,18	-0,82	1,3	0,03	0,23	1,07	1,09
Mode 1 + 0.5 Mode 2	0,05	0,56	1,1	1,5	0,08	0,83	1,64	1,84