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# Use of transitory methods of resolution for the strongly nonlinear quasi-static problems

**Summary:**

This document presents the use of transitory methods of resolution (implicit or explicit) for the digital simulation of quasi-static problems presenting of strong non-linearities materials. One only seeks to obtain solutions where the dynamic effects are negligible.

These methods of resolution (based on the use of DYNA_NON_LINE) are conceived like an alternative to the usual quasi-static approaches of *Code_Aster* (based on STAT_NON_LINE) when they prove to be unable to converge in an acceptable time. That can be the case for applications implementing damaging materials.

The methods which are the object of this document are of two types:

1) implicit transitory resolution,
2) resolution clarifies pseudo-dynamics.

The choice will be depend on the nature of the problem to treat, in particular, in link with the scale of time considered. The use of these strategies being more delicate than the quasi-static approach, they are to be held for the most problematic cases.

The reading of U2.06.13 documentation strongly constitutes pre-necessary advised: general advices of use of DYNA_NON_LINE remain relevant.

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# 1 Introduction

Many problems of mechanics require the taking into account, on the level material, of damaging behaviors: structures out of reinforced concrete, models of grounds…
The relation of behavior can then present softening and the known implicit quasi-static methods of resolution have difficulty of converging (the tangent operator of stiffness becomes singular). In certain cases, even the recourse to very powerful strategies as mixed linear research or piloting is insufficient. In order to be able to free itself from these limitations, there exist alternative strategies which is based on methods inspired by the tools of the direct transitory analysis [bib1].
It is specified clearly that it is not question of wanting here to simulate a dynamic response of vibratory type or with wave propagation: one seeks to obtain solutions in slow evolution, therefore in coherence with the assumption of quasi-staticity. The usual dynamic methods must thus be adapted to this framework and the solution thus obtained will have to check the assumptions of sufficiently slow evolution.
Lastly, it is advisable well to keep in mind that these transitory approaches, of share their specificities, must be used as a last resort, when all parades available in `STAT_NON_LINE` failed.
The precondition to the use of the methods presented in this documentation is thus to have already developed and have exhaustively tested the options available in `STAT_NON_LINE` for the application considered. It goes without saying that a good knowledge of `STAT_NON_LINE` and `DYNA_NON_LINE` is also strongly recommended just as the reading of corresponding documentations: R5.03.01, R5.05.05 and especially U2.06.13.
In particular, general advices of use of the operator `DYNA_NON_LINE` given in U2.06.13 documentation remain valid and they thus constitute an essential precondition to the good implementation of the methods which are the object of this documentation.

# 2 Features available into quasi-static and absent in transient

One will approach here, mainly, the problems of the laws endommageantes, in local version. Indeed, nonlocal approaches, like `GRAD_VARI` are not available in dynamics. It will thus be advisable to pay particularly attention to be defined a size of minimal mesh adapted not to observe excessive phenomena of localization.

Then, the methods of the type searches linear (mixed or not) are not authorized in dynamics. This lack is to be relativized, knowing that the attempts at applications of these methods on studies of reinforced concrete structures in dynamics did not put forward significant contribution on convergence, as opposed to what one observes into quasi-static. Let us announce, nevertheless, that no theoretical argument would prohibit the use of these methods in dynamics.

Lastly, techniques of piloting available in `STAT_NON_LINE` (length of arc, for example) are prohibited in dynamics because they then do not have a direction.

# 3 Passage d' a quasi-static calculation with a transitory calculation

Far from limiting itself to replace the term STAT_NON_LINE by DYNA_NON_LINE in a command file (like defining the densities, *has minimum*…), the passage of quasi-static to the transient must be accompanied by a certain number of precautions essential, under penalty of strongly degrading the quality of the digital solution obtained.
These adaptations, described in detail in U2.06.13 documentation, relate to:

- the regularization in time of the boundary conditions,
- the definition of initial conditions which do not introduce digital oscillations.

In complement of these aspects generals, the user will have to pay attention to other more specific adaptations.

- Définition of the densities :
  D‘a physical point of view, it is necessary, obviously, to define the density in any point of the model. If the model understands discrete elements, will have to be attached to them a discrete mass. L' operator of mass assembled must be invertible. Artifices sometimes employed into quasi-static like stiffenings on zones of the model (anchorings…) with materials having very large Young moduli (or very large specific stiffnesses) are to be handled with precaution. Indeed, these very stiff zones will generate in dynamics of the disturbances high frequencies. Moreover, with a diagram in explicit time, these very stiff zones are likely to make fall the step value of time criticizes (*cf.* R5.05.05).
- Définition of the sizes of mesh and the steps of time :
  Comme preliminary to transitory calculation, it is strongly recommended to carry out a modal calculation (with CALC_MODES) to obtain modal information which will make it possible to qualify the quality of the model in dynamics and to adjust certain parameters. The objective not being to return in the details of the modal analysis, one can nevertheless point out some rules.
  Solutions low frequencies are sought, therefore only the first modes are relevant. Their good representation can give indications on the sizes of meshs to be used, besides the considerations already taken into account for preceding quasi-static calculations. Approximately, about ten meshs by smallest wavelength is sufficient.
  The modal analysis as will make it possible to check as the model is free from problems like contributions nondefinite to inertia or the stiffness.

Lastly, the modal analysis is essential for the use of modal damping in DYNA_NON_LINE or to readjust the damping of Rayleigh, as one will see it in what follows.

- Definition of damping :
  L‘user will have to also put the question of L' intrinsic damping to the model which he wants to use.
  In DYNA_NON_LINE, apart from the discrete elements, one can introduce a total of Rayleigh type or modal damping. Since one seeks to simulate slow evolutions, one can be tried to use values of damping higher than for classical dynamic calculations. A compromise however remains to be found, on a case-by-case basis, between a problem insufficiently deadened (which will have oscillations) and a too deadened problem (damping criticizes even supercritical).
  One thus advises to start by implementing a “realistic” damping (thus of value identical so that one meets in transitory dynamics). Then, if this damping is considered to be insufficient, to increase it gradually.
  On various applications [bib1], one could note that a damping of the Rayleigh type, readjusted on a modal damping are equivalent of about a 20%, even 30% was suitable.

# 4 Numeric work implementation

This chapter will approach the choices to be privileged on the level of the digital methods of DYNA_NON_LINE [bib2].

Generally, one advises to follow following logic:

1) once possibilities of quasi-static resolution through exhausted STAT_NON_LINE (including linear research and piloting),
2) to begin the dynamic approaches with an implicit resolution,
3) in the event of failure of the transitory implicit strategies (including by combining various depreciation like Rayleigh and a diagram of the type HHT [bib2] and [bib3]), one rocks into explicit (while having checked as a preliminary that total damping does not depend on the matrix of stiffness).

For each transitory approach, it is advisable to start with less possible damping, therefore, in particular, with diagrams in nondissipative times (like Newmark or centered differences [bib4]).

## 4.1 Discretization in time

Contrary to quasi-static, time has a physical direction. Its discretization in is all the more sensitive.

One can state some rules:

- the evolution of the imposed loadings must be sampled in a sufficiently fine way (between 5 with 10 pas de time per the shortest period of the signals considered),
- the modal behavior of the structure must be well represented (like above, one must have between 5 and 10 pas de time per the weakest period of the modes considered).

Being given the character low frequency of the problems which one wants to tackle here, these two rules are in general not very penalizing. Nevertheless, compared to step of time of preceding quasi-static calculations, the dynamic steps of time can be rather definitely weaker.

Into explicit, it is moreover necessary to observe the condition of Current (CFL *cf.* [bib5], [bib2] and R05.05.05) under penalty of digital divergence. For a diagram of integration of centered the differences type, the critical step of time is worth $2/\omega$ with $\omega$ who is the highest own pulsation of the system.

One can calculate this pulsation with CALC_MODES by choosing the option 'PLUS_GRANDE'.

For more details, the reader will be able to refer to U2.06.13 documentation.

For most structures, the condition of Current is very penalizing: the celerity of the waves being often about a few thousands of m/s, one arrives at steps of time of less than $10^{-5}$ S.

## 4.2 Choice of the diagrams of integration in time

One can classify the implicit schemes in two categories (one puts side, voluntarily, the diagrams order 1 and/or of speed which are more specifically adapted to the very irregular problems):

- average acceleration (NEWMARK [bib4]) of order 2 and which does not bring digital dissipation: to use in first,
- Complete HHT (MODI_EQUI = 'YES' [bib3]) who remains of order 2, contrary to the case of the modified average acceleration (MODI_EQUI = 'NOT', option by default). This diagram is specifically developed to introduce a digital damping high frequency and thus not to disturb the physical answer low frequency. Damping is directly controlled by the parameter ALPHA diagram.

If one observes oscillations high frequencies in solution digital (approximately, of the oscillations the period is about some steps of time), one can choose complete diagram HHT, to start with a value about – 0.1 for the parameter ALPHA. A value of – 0.3 constitutes a high limit still usable.
If one wishes more damping on average frequency, then the diagram of average acceleration modified can be employed.

Into explicit, one has two diagrams:

- centered differences (DIFF_CENT [bib4]) which is nondissipative,
- Tchamwa-Wielgosz (TCHAMWA [bib6]) which is dissipative, in a way comparable to HHT.

Here still, one recommends to start by using a nondissipative diagram.

## 4.3 Models of damping

The order of introduction and use of dissipation in the discretized model is the following:

1) intrinsic dissipation related on the relations of nonlinear behavior, the connections (friction),
2) total dissipation of standard damping structural (Rayleigh or modal),
3) digital dissipation of the diagram in time.

Ideally, the first category should be sufficient, but in practice, for reasons of simplification of the model, it is often essential to add structural damping, the damping of the diagram being the last recourse.
We will approach here only the use of structural damping, within the meaning of Rayleigh, and that related to the diagram.
Just let us point out that the more one will multiply the sources of dissipation, the more their control and their physical interpretation will be difficult.

### 4.3.1 Damping of Rayleigh

This model makes it possible to define the total matrix of damping C as being a linear combination of the matrices of rigidity and mass (to have a diagonal matrix of damping on the basis of usual dynamic mode):

$$\boldsymbol{C} = \alpha \boldsymbol{K} + \beta \boldsymbol{M}$$

U2.06.13 documentation presents it in detail.

The damping coefficients of Rayleigh are defined, on the level of the characteristics of the material (order DEFI_MATERIAU), by the parameters AMOR_ALPHA and AMOR_BETA. Values to force to obtain desired damping  in the interval of the Eigen frequencies f1 and f2 result from the following equations:

Equation 1: $$\alpha = \frac{\xi}{\pi(f1 + f2)}$$

Equation 2: $$\beta = \frac{4\xi\pi \ f1 f2}{f1 + f2}$$

Where f1 and f2 are the two Eigen frequencies limiting the interval of study considered. Within the framework of this document, one seeks solutions low frequencies, therefore the frequencies f1 and f2 will be associated with the first frequencies of the model, whose modes are coherent with the imposed loading.

### 4.3.2 Damping due to the diagram in time [bib2]

Documentations R05.05.05 and U2.06.13 present this aspect. One here will restrict oneself to recall the main tendencies of them.
To summarize, one can recall that, among the implicit schemes:

- the diagram of average acceleration does not dissipate,
- only complete diagram HHT does not disturb the low frequency field,
- for the same value of the parameter ALPHA modified average acceleration introduced much more dissipation than diagram HHT.

In order to put forward the influence of damping high frequency of the implicit schemes, U2.06.13 documentation presents examples of applications.

Lastly, with regard to the explicit diagrams, the pace of the damping of the diagram of Tchamwa is qualitatively close to that of the modified average acceleration.

## 4.4 Adaptation of the explicit methods

The conditional stability of the explicit diagrams returns them very little adapted to the simulation of slow phenomena. The explicit methods of resolution are not used here to collect fast phenomena like the wave propagation, but their use should be perceived as a particular solvor whom one adapts for slow problems.
In order to be able to increase the critical step of time [bib2], one can increase the density of the structure (what cause a drop in the celerity of the waves proportionally to its square root):

$$c_p = \sqrt{E/\rho}$$

It should however be done gradually.
Indeed, two risks exist:

- if the step of time becomes too large, the calculated solution will be able to miss certain phenomena like the appearance of bands of shearing and to go until forking towards a branch very different from the expected answer,
- the increase in the density can be limited by the bad conditioning of the matrix of mass.

As indication, the step of maximum time into explicit (and thus maximum density) can be of the same order of magnitude as the step of time necessaries to implicit transitory calculation, in any case, it must remain lower than the quasi-static step of time. In fact coarse tendencies do not exempt parametric study on the explicit step of time.

If the model has strong heterogeneities of stiffness (definition of several materials), it can be relevant to modify the densities separately, so as to have a relatively homogeneous condition of Current between the zones having different materials.

**Notice important**

*If one imposes boundary conditions in displacement which evolves in the course of time, it should be taken account owing to the fact that these conditions in fact are imposed in acceleration into explicit (because it is the primal unknown factor). That means that one must enter DYNA_NON_LINE the derivative second of the signal in displacement which one wants to impose. This evolution of imposed displacement must thus be derivable at least twice in time…*

To finish, it is recommended to use a matrix of diagonal mass (lumpée), which is obtained by the keyword MASS_DIAG = 'YES' of DYNA_NON_LINE. This option of calculation not being available for all the finite elements, the user can be constrained to use the consistent mass, if necessary, as into implicit.

# 5 Quality control of the calculated solutions

## 5.1 Quantities of interest resulting from the quasi-static one

As for quasi-static calculations, one can classify the relevant quantities for postprocessing in two categories [bib1]:

1) evolutions of the type forces/displacement: quantities allowing to interpret the total answer of the structure,
2) isovaleurs of fields like the damage or the cumulated plastic deformation.

## 5.2 Dynamic complementary sizes to analyze

Besides these analyses, for dynamic calculations, it is essential to check that the assumption of slow evolution is respected. For that, it should be made sure that the inertial forces remain weak in front of the other efforts in the system (external efforts and interiors). A simple way to have an evaluation of the evolution of the inertial forces during calculation consists in observing the field of acceleration in the course of time.

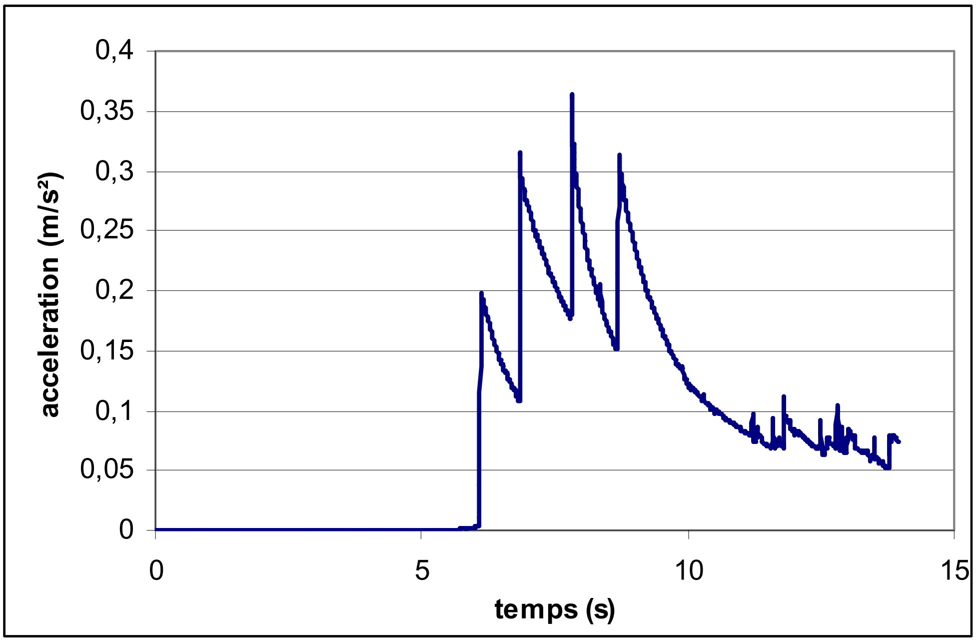
A first simple criterion can be based on a standard (infinite) of acceleration at every moment. In the event of notable and durable increase in acceleration (for example beyond 1 $m/s^2$ on several steps of time of continuation), that means that dynamic phenomena (thus not taken into account by a quasi-static resolution) occur:

- if one is within a physical framework, therefore with a realistic density, the calculated solution is thus subjected to considerable dynamic phenomena;
- if one is within an explicit framework where the density at summer multiplied to increase the step of critical time, then the noted effects of inertia are the sign that the method of resolution is not adapted. It is then imperative to modify the parameters, like lowering the density, modifying damping…

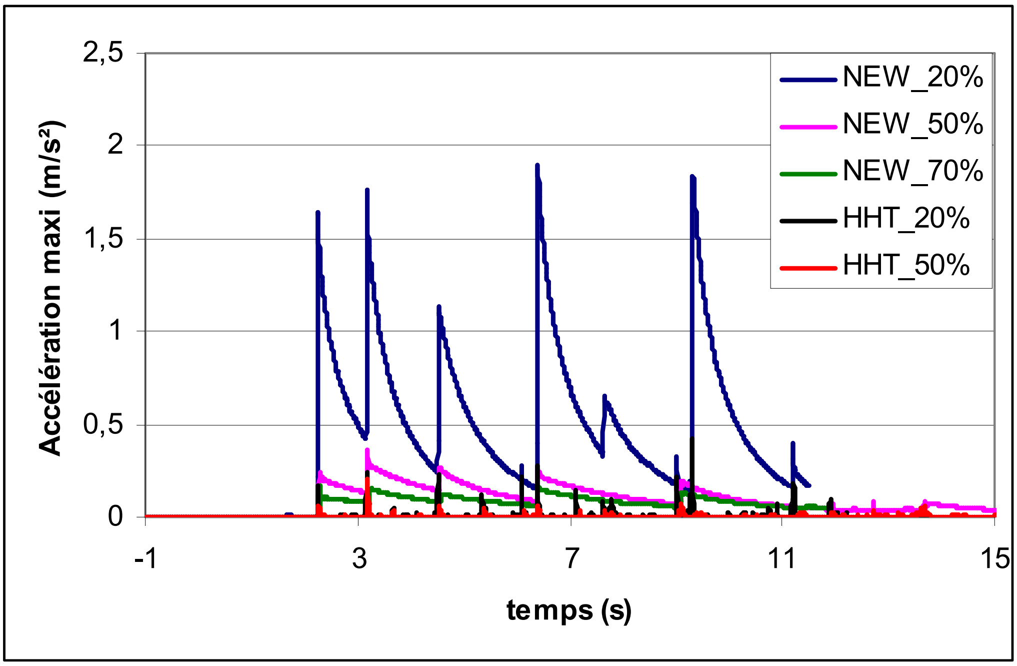
In the first case, one can try to start again a calculation with a step slightly finer and possibly a damping of Rayleigh a little more raised, even with a standard diagram HHT. If, even with all these modifications, the solution still presents effects of inertia, then any quasi-static approach is unsuited.

## 5.3 Examples of analysis

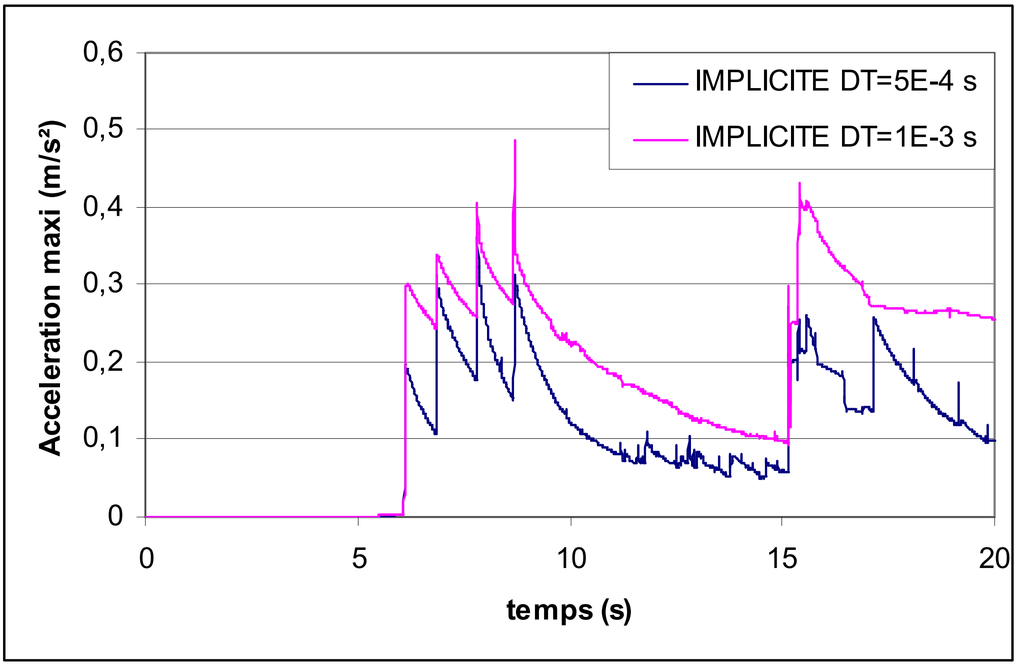
On the graph below (reinforced concrete small structure), one presents a case where acceleration will become considerable, from 6 S. the maximum value remains lower than 0.4 $m/s^2$ and we are in a case with realistic density. Moreover, acceleration will become again weak after 10 S: from all that one can conclude that calculation remains compatible with the assumption of slow evolution.


0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
acceleration (m/s²)
0
5
10
15
temps (s)

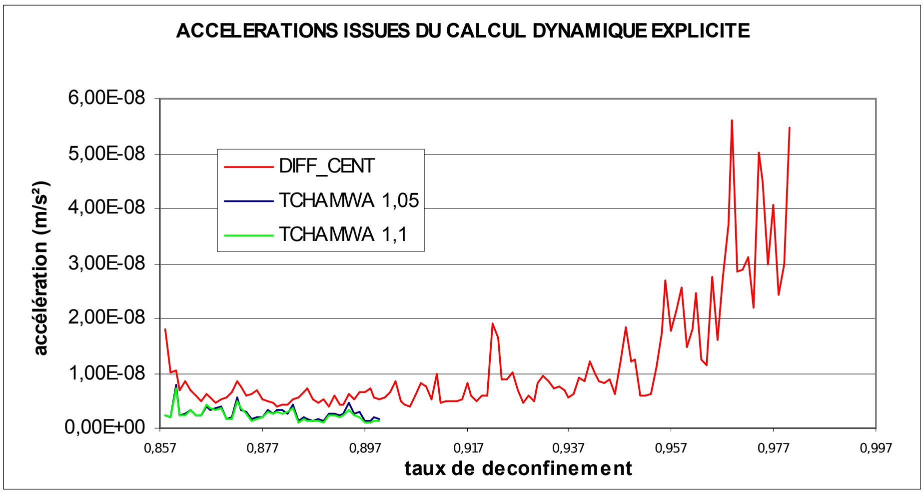
On the following graph, one can quantify the influence of damping (due to Rayleigh and the diagram) on maximum acceleration. All in all, on this example, the use of a diagram of the type HHT with a damping of Rayleigh fixed on a modal value are equivalent of 20% is well adapted (black curve). On the other hand the diagram of average acceleration (Newmark) will make it possible to control acceleration only if one couples it with a damping of very important Rayleigh: 50% of modal damping are equivalent. The solution calculated risk then to be too dissipative.



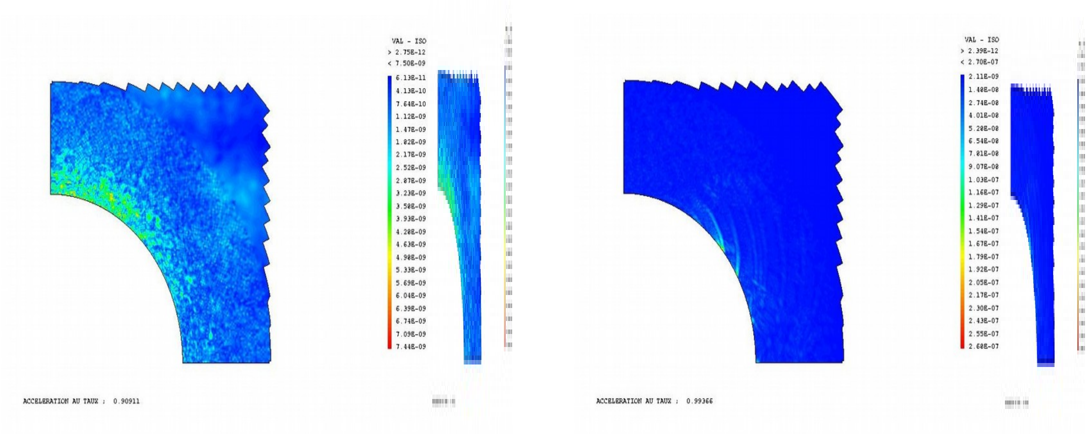

On the following figure, one can judge influence of a parameter of the discretization: the step of time, in term of control of maximum acceleration. The value of $10^{-3}$ S for the step of time is not quite selected and it is then preferable to divide the step of time by two. To analyze this behavior, it is essential to conduct a parametric study on the step of time.



In the case of a resolution clarifies (on an example very different from the precedents: excavation of a circular gallery [bib1]) one compares various diagrams in time: centered differences (thus without induced damping) and the diagram of Tchamwa for two values of the parameter `PHI`. The larger this parameter is, the more one introduces damping high frequency and this damping becomes null for `PHI=1`.




In complement of the preceding curves, it is instructive to visualize some isovaleurs of standard of acceleration (for the same example of excavation into explicit):



On the figure of left, one is in a weak phase of acceleration (non-linearity related to the rate of déconfinement is still low). The pace of the field of acceleration is rather random: one cannot perceive “reason” betraying a real dynamic phenomenon.
On the figure of right-hand side, non-linearity is established and one observes a distribution of very different acceleration: one sees taking shape the pace of bands of shearing on the circumference of the excavation. But even in this case, the maximum values of acceleration remain very low (including by taking account of the factor of increase in the density). The solution obtained explicitly is thus relevant within the meaning of a slow evolution.

# 6 Optimization of the performances

In most case it is recommended to carry out calculation into quasi-static when non-linearities are moderated, then, as soon as the iteration count to convergence for balance increases significantly, to rock in dynamics (implicit, then clarifies if need be). More precisely one advises to rock before the appearance of strong non-linearities, so that dynamic calculation is initiated on a "regular" evolution still enough.

Into quasi-static, it is not rare to have to carry out more than 10 iterations to have convergence within the meaning of the residue in balance. In implicit dynamics this value of 10 iterations constitutes, in general, a good starting value for the parameter `ITER_GLOB_MAXI` of `CONVERGENCE`. If one cannot converge in less than 10 to 20 iterations, it is then preferable to decrease the step of time rather than to increase the authorized maximum number of iterations.

Into explicit, there are no iterations for balance, the cost of calculation of each step of time will be thus constant, whatever the level of non-linearity (except, possibly, the local checking of the behavior). In the case of a quasi-static calculation where non-linearity is growing with time, one can thus find a point of crossing, on the level of the time CPU, for which the effectiveness of the explicit methods (even implicit according to the cases) becomes larger than to continue into quasi-static. The profit brought by a step of larger time into quasi-static is cancelled by the need for making more and more iterations with each step. In the same way, the cost of calculation in explicit constant remainder, one can also evaluate rather precisely the total computing time necessary to the resolution of the complete case, whereas into quasi-static, the iteration count to each step being very variable and also being able to involve an unknown number of subdivision of the step, the total time computing is sometimes very difficult to predict.

The use, even current, of the explicit methods thus seems very tempting within sight of the time CPU which remains controlled. It is however necessary to moderate this optimism while keeping well with the spirit which one deprives of the parapet which is the precise checking of the balance and which, consequently, the quality of the explicit solution obtained must be analyzed with more precautions. The explicit algorithm will not diverge (if the condition of Current is observed), but the solution obtained is not guaranteed by a criterion of checking of balance. In particular a parametric study on the step of time is essential because the pace of the solution can strongly vary when this step becomes too large. This obligation of control of the explicit solution is all the more large as one seeks in more to check the fundamental assumption of slow evolution.

# 7 Conclusion

This document presents the use of transitory dynamic methods for simulation of slow and strongly nonlinear evolutions [bib1]. The dynamic operators thus employed for the resolution can be seen like particular solveurs allowing, in certain cases, to obtain solutions for which quasi-static algorithmy available in *Code_Aster* (around the operator `STAT_NON_LINE`) watch of great difficulties of convergence (within the meaning of the checking of balance).
These dynamic methods are to be used as a last resort because their control is more delicate than the quasi-static framework. It is even more outstanding with the explicit methods since the quality of the solution is not guaranteed by a precise checking of balance at every moment.
The first stage is the adaptation of the model to the dynamic methods. It is mainly a question of making sure of the good regularity of the imposed conditions and the correct definition of the density and total damping (Rayleigh). The methods of piloting, the approaches nonlocal and linear research are not usable in dynamics. Moreover, it is necessary to introduce a model of total damping (Rayleigh or modal) whose level is stronger than in classical dynamics (usually 20% of modal damping are equivalent).
Then, it is recommended to start by using an implicit transitory method (`DYNA_NON_LINE` with a diagram in times of the type `NEWMARK`, or `HHT`). From an optimization point of view of the computing time, one recommends to carry out all the phases slightly nonlinear into quasi-static and not to rock in transient (implicit to start) only during the appearance of notable non-linearities (that can result in a clear increase amongst iterations with convergence, even a subdivision of the step of time).
In the event of failure, including with a complete diagram HHT and a structural damping relatively extremely (until towards 30% of modal damping are equivalent), the user can rock into explicit. Other adaptations should then be made, like increasing the density to obtain a step of time criticizes sufficiently large and modifying damping.
In all the dynamic cases, it is essential to analyze the evolution of acceleration in the course of time, in order to make sure of the validity of the assumption of slow evolution. The effects of inertia must remain weak.

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