
Constraints, efforts, forces and deformations

Summary:

This document defines the sizes characterizing the constraints, the forces and the deformations inside a structure in a calculation by finite elements in displacement and how that is translated in *Code_Aster*. The expression of these sizes is given for the finite elements of mechanics: continuous medium 2D or 3D, hulls and beams.

1 Statics

1.1 Constraints

The postulate of Cauchy is that the efforts of contacts exerted in a point by part of a continuous medium on another depends only on the normal on the surface in this point delimiting the parts.

In accordance with this postulate, one calls vector forced, for the nonmicropolar mediums, $\mathbf{F}(\mathbf{n})$ the vector which characterizes the forces of contact exerted through an element of surface dS of normal \mathbf{n} on part of a continuous medium [bib1].

It is shown [bib3], then, that the dependence in a point fixed of \mathbf{F} compared to the normal \mathbf{n} is linear and that there exists a tensor which one calls tensor of the constraints σ such as:

$$\mathbf{F}(\mathbf{n}) = \sigma \mathbf{n}$$

The unit of the constraints in international system is it $\text{N} \cdot \text{m}^{-2} \equiv \text{Pa}$.

For the whole of the structure "the state of constraint" is characterized by a field of tensor of the constraints which one more simply indicates by stress field.

1.2 Effort

With regard to the structures of beams or hulls, contrary to the case of the continuous medium, it should be noted that:

- only normal directions \mathbf{n} cuts according to tangent space with the variety are possible,
- the characteristic sizes are obtained by integration in the section or the thickness of the sizes defined for the continuous mediums.

1.2.1 Case of the discrete ones

The discrete ones are finite elements which can not have of a physical size. They are represented by their matrix of stiffness. The efforts are obtained by the multiplication of this matrix by the vector displacement:

$$\begin{pmatrix} F \\ M \end{pmatrix} = [k] \cdot \begin{pmatrix} D \\ R \end{pmatrix}$$

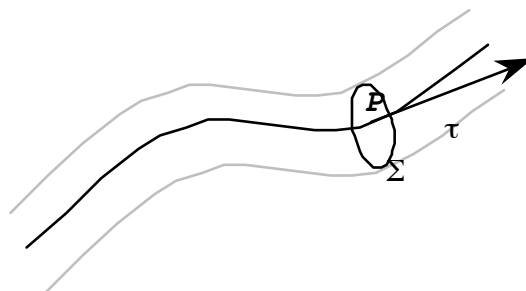
1.2.2 Case of the beams

One calls effort, the end cells (F, M) in P , geometrical centre of inertia of the cross-section Σ , torque resulting from the forces of contact exerted on the section [bib2].

With the preceding notations:

$$F = \int_{\Sigma} \mathbf{F}(\boldsymbol{\tau}) ds \quad (N)$$

$$M_p = \int_{\Sigma} \mathbf{PM} \wedge \mathbf{F}(\boldsymbol{\tau}) ds \quad (N \cdot m)$$



The force F breaks up into a normal effort N and of the efforts cutting-edges T in the plan of the section while moment M being exerted at the point P breaks up into one bending moment and torque at the point P . The elements of skeleton of fuel assembly (R3.08.08) also carry the moment of the grids M_g , of formulation equivalent to M .

For the beams whose cross-section is not regarded as rigid these end cells are not sufficient: for example, for the beams taking of account the warping of the sections one is brought to consider an additional size of effort due to warping (bimoment).

The multifibre beams (with local behavior 1D, connecting constraints to deformations, in a certain number of points of the section) and the pipes (local behavior in plane constraints) provide at the same time the end cells of the beams but also a stress field for each fibre or sector.

1.2.3 Case of the hulls

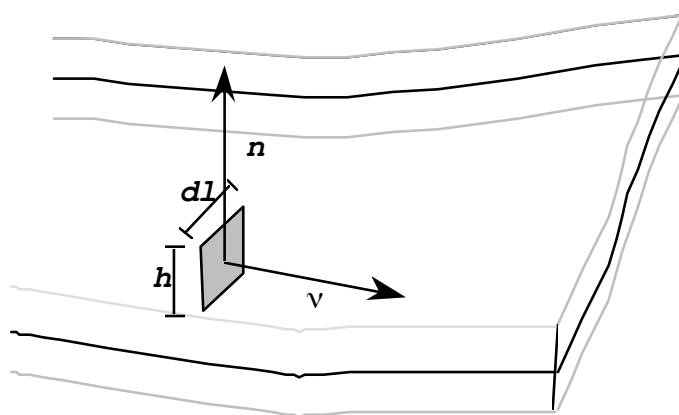
That is to say a point P of a surface medium S of thickness h , that is to say an element length dl on S , that is to say n the normal directing the hull in this point.

Are the end cells (F, M) in this point of a torque resulting from the forces exerted through an element of surface $dS = h dl$ of normal n tangent with S on part of S .

With the preceding notations:

$$F(P) = \int_{-h/2}^{+h/2} \mathbf{F}(v) dh \quad (N)$$

$$M(P) = \int_{-h/2}^{+h/2} \mathbf{PM} \wedge \mathbf{F}(v) dh \quad (N.m)$$



It is clear that M is in the tangent plan with S in P .

That is to say $N(P)$ the projection of $F(P)$ on the tangent level with S in P and is, $T(P)$ its normal component with this tangent plan.

In the same way that for the continuous mediums, one shows that there exist two symmetrical tensors \mathbf{N} and \mathbf{M} and a vector Q , defined in the tangent plan in S , such as:

$$\mathbf{F} = N v$$

$$T = Q \cdot v$$

$$\mathbf{M} = n \wedge M v$$

(N, M, Q) the efforts at the point are called P :

- the tensor N characterize the membrane efforts,
- the tensor M bending moments,
- the vector Q the efforts cutting-edges.

Note:

- There are no universal conventions on the denomination and the signs of these tensors. In particular, the tensor of the bending moments is sometimes taken with a sign opposite in the teaching and in practice of the French engineers of the civil engineer. Our convention is used in the great codes of finite elements and makes it possible to have the same sign for a beam and a plate such as $\tau = \nu$.
- For nonlinear materials, the law of behavior is evaluated in several points thickness but the equilibrium equations always relate to the fields of effort. It is not necessary to go down again to the constraints to define "the state of stress".

Links with the stress field

Under these conditions is a reference mark whose third component is carried by \mathbf{n} , one has ($\alpha, \beta = 1$ ou 2):

$$\begin{aligned} N_{\alpha\beta} &= N_{\beta\alpha} = \int_{-h/2}^{+h/2} \sigma_{\alpha\beta} dh \\ M_{\alpha\beta} &= M_{\beta\alpha} = \int_{-h/2}^{+h/2} x_3 \sigma_{\alpha\beta} dh \\ Q_{\alpha} &= \int_{-h/2}^{+h/2} \sigma_{\alpha 3} dh \end{aligned}$$

1.3 Nodal forces

One calls equivalent nodal force or more simply nodal force, a vector F who is the representative of a linear form W (generally dependent on an energy) acting on fields of displacement $u(x)$ discretized by finite elements.

Fields of displacements $u(x)$ express themselves starting from its nodal values which form a vector q and of the functions of form $\Phi_i(x)$ by:

$$u(x) = \sum_i q_i \Phi_i(x)$$

Under these conditions:

$$w(u) = \sum_i q_i F_i$$

Note:

- The concept of node here is very general and wants to say, in fact, carrying degree of freedom (that it is of Lagrange or Hermit).
- The concept of displacement is also very general and includes the concept of generalized displacement including of the translations and rotations.

1.4 Representation of the fields

There are several ways of representing the fields in a modeling by finite elements:

- for the continuous-current fields on all the field, one uses the values with the nodes (`CHAM_NO` of Code_Aster)

$$u(x) = \sum_i u_i \Phi_i(x)$$

one speaks then about displacements to the nodes, constraints to the nodes or efforts to the nodes,

Note:

The effort or stress fields are generally calculated at the points of Gauss, if they continuously are represented it is only at ends of visualization.

Pour the other fields, one uses the values in certain points characteristic of the elements (points of Gauss or nodes).

One speaks then about constraints by elements to the nodes or efforts by elements to the nodes, or about constraints at the points of Gauss or efforts at the points of Gauss.

1.5 Sizes associated in Code_Aster

1.5.1 SIEF_R

Size SIEF_R represent "the state of stress" of the structure, it thus contains, at least, the components:

- stress fields of the continuous mediums (in total reference mark):
SIXX SIYY SIZZ SIXY SIXZ SIYZ
- fields of efforts of beam and discrete (in reference mark "user" with the beam, discrete):
NR VY VZ MT MFY MFZ, MGX, MGY, MGZ
- for the beams with warping, it is necessary to add bimoment (necessarily in reference mark "user" with fibre):
BX
- fields of efforts of hull (necessarily in reference mark "user" on the surface):
NXX NYX NXY MXX MYX MXY QX QY

Moreover, it is sometimes convenient to be able to directly exploit the fields of efforts of beam and discrete in the total reference mark:

FX FY FZ MX MY MZ

It is also interesting to represent the components of a stress field on the elements of beams or hulls in the reference mark "user". For that, one will use the same components as in total reference mark, although confusion is possible.

1.5.2 FORC_F and FORC_R

These sizes represent the forces applied to the structure to an interface.

For:

- a continuous medium it is thus a vector of force,
- a beam, a torque of forces,
- a hull, a torque of forces.

This size must thus have the following components:

- for a continuous medium:
FX FY FZ
- more for the beams and the hulls:
MX MY MZ

1.5.3 DEPL_R

Since in **Code_acefor the third time**,

- a field can be attached only to only one size,
- that the methods finite elements mixed (mixing unknown of standard displacement and unknown factors of nodal the forces type) are not excluded,
- that the dualisation of the boundary conditions results in having for unknown a comprising vector of the variables of Lagrange which are nodal forces with the direction where it higher was specified,
- that it is necessary to be able to carry out any type of linear combination on the nodal forces,
- that the classification of the unknown factors must be the same one as that of the second members,

nodal forces (dual within the meaning of energy W nodal displacements) have necessarily the same components as displacements namely:

DX DY DZ DRX DRY MARTINI DRZ, DRGX, DRGY, DRGZ

more, for the beams with warping, the degree of freedom associated with bimoment: GRX.

1.6 Options of calculation

1.6.1 Calculation of the state of stress

1.6.1.1 Field SIEF_ELGA

It is the field representative of the state of constraint and making it possible to continue calculations (geometrical rigidity, nodal forces, etc). It is expressed at the points of Gauss (and is possibly under-points for the elements of structure). The prefix of this field is SIEF, because according to the elements, it contains constraints or efforts.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D, 2D, Coques1D PIPE Multifibre beams	Beams: POU_D_T POU_D_E POU_D_TG POU_D_T_GD Discrete	Plates: DKT DST Q4G Q4GG COQUE_3D
SIEF_ELGA	SIEF_ELGA	starting from a field of displacement in linear elasticity	σ	(F, M) in reference mark "user"	σ in reference mark "user" *
RAPH_MECA FULL_MECA	SIEF_ELGA	into nonlinear	σ	(F, M) in reference mark "user"	σ in reference mark "user" *

(*) for the elements of plate and hull, the reference mark "user" is that definite starting from the data of the user (keyword ANGL_REP or VECTOR in AFFE_CARA_ELEM/COQUE).

These options thus calculate:

- the stress field for the elements of continuous mediums 2D and 3D, and the elements with local behavior: COQUE_3D, plates, hulls 1D (COQUE_AXIS, COQUE_D_PLAN, COQUE_C_PLAN), multifibre pipes, beams, in each "under-point" of integration (layers in the thickness of the hulls, fibres, sectors angular and position in the thickness for the pipes). The reference mark "user" of the plates and hulls can be specific to each element.
- the field of efforts for the beams (torque).

1.6.1.2 Field SIGM_ELGA

It is the field representative of the state of constraint at the points of Gauss (or possibly at the under-points for the elements of structure). The prefix of this field is SIGM because this field contains only constraints. It is an extraction of the constraints contained in the field SIEF_ELGA.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D, 2D, Coques1D PIPE Multifibre beams	Beams: POU_D_T POU_D_E POU_D_TG POU_D_T_GD Discrete	Plates: DKT DST Q4G COQUE_3D
SIGM_ELGA	SIGM_ELGA	starting from the field SIEF_ELGA, extraction of the constraints	σ	non-available	σ in reference mark "user" *

1.6.2 Other representations of the state of stress

1.6.2.1 Field SIEF_ELNO and SIEF_NOEU

They are fields representative of the state of constraint at ends of exploitation (impression or postprocessing of visualization) to the nodes by element (or possibly at the under-points for the elements of structure) and to the nodes of the element. According to the elements, they contain constraints or efforts.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D, 2D, Coques1D PIPE Multifibre beams	Beams: POU_D_T POU_D_E POU_D_TG POU_D_T_GD Discrete	Plates: DKT DST Q4G COQUE_3D
SIEF_ELNO	SIEF_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	σ	(F, M) in reference mark "user"	σ in reference mark "user" *
*	SIEF_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	σ	(F, M) in reference mark "user"	σ in reference mark "user" *

1.6.2.2 Field SIGM_ELNO and SIGM_NOEU

It is about a field representative of the state of constraint at ends of exploitation (impression or postprocessing of visualization) to the nodes by element (or possibly under-points for the elements of structure) and to the nodes of the element. The prefix of this field is SIGM because this field contains only constraints.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D, 2D, Coques1D PIPE Multifibre beams	Beams: POU_D_T POU_D_E POU_D_TG POU_D_T_GD Discrete	Plates: DKT DST Q4G COQUE_3D
SIGM_ELNO	SIGM_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	σ	non-available	σ in reference mark "user"
*	SIGM_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	σ	non-available	σ in reference mark "user"

Note:

- In this case, confusion is possible between the components in reference mark user and those in total reference mark which bear the same name.
 - The 6 components delivered in the local reference marks by the beams and the hulls contain possibly worthless terms according to the models used. For the most standard models:
 - three worthless terms for the beams,
 - two worthless terms for the hulls.
- Thus, the stress field will be complete and, especially, it could be enriched each time modeling requires it (beam with shearing, hull with pinching, etc...)

1.6.2.3 Field EFGE_ELGA, EFGE_ELNO and EFGE_NOEU

They are fields containing the efforts on the elements of beam or hull at ends exploitation (impression or postprocessing of visualization) at the points Gauss, the nodes by elements and the nodes.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D, 2Ds	Beams, pipes, beam multi - fibres, Discrete	Hulls, plates
EFGE_ELGA	EFGE_ELGA	by integration of the constraints	non-available	(F, M) in reference mark "user"	(N, M, V) in reference mark "user"
EFGE_ELNO	EFGE_ELNO	starting from a field of displacement in linear elasticity	non-available	(F, M) in reference mark "user"	(N, M, V) in reference mark "user"
EFGE_ELNO	EFGE_ELNO	by integration of the constraints into non-linear	non-available	(F, M) in reference mark "user"	(N, M, V) in reference mark "user"
*	EFGE_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	non-available	(F, M) in reference mark "user"	(N, M, V) in reference mark "user"

1.6.3 Calculation of the nodal forces and the generalized reactions

The generalized nodal forces are calculated starting from the state of stress, only one option is envisaged:

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	Solid elements (3D, 2D)	Beam, elements discrete	Hull
FORC_NODA	idem	starting from the field SIEF_ELGA	Forces	Forces and moments	Forces and moments

Nodal forces (dual within the meaning of energy W nodal displacements) have the same components as displacements namely:

DX DY DZ DRX DRY MARTINI DRZ

The option REAC_NODA of the operator CALC_CHAMP carry out a call to FORC_NODA and withdrawn:

- the loading in statics,
- the loading, the inertial forces and viscous in dynamics (in the facts, the viscous contribution in dynamics is currently neglected in CALC_CHAMP).

For the solid elements, them `FORC_NODA` in general have the dimension of a force. It is about a field on the nodes of the grid where the value in a node is obtained starting from the constraints calculated on the convergent elements with this node, thus their values thus vary when the grid changes. In the absence of loading distributed, balance imposes their nullity in an interior node, while they correspond to the reaction on the supports where a kinematic relation is imposed (case of an imposed displacement).

In the case of hulls, components `DX`, `DY` and `DZ` they give `FORC_NODA` (of dimension of a force) in the total reference mark of the grid. These components are built with the normal efforts and cutting-edges in the hull. Components `DRX`, `DRY` `MARTINI` and `DRZ` they give `FORC_NODA` (of one moment dimension) in the total reference mark of the grid, built with the bending moments in the hull.

2 Kinematics

2.1 Deformations

2.1.1 Continuous medium

In this case, displacements of the structure are represented by a field of vector u with three components in general.

The deformation (on the assumption of the small disturbances) is defined by the tensor of deformation ε by (option EPSEI_ELGA and EPSEI_ELNO):

$$\varepsilon_{ij}(u) = \frac{1}{2}(u_{i,j} + u_{j,i})$$

One can want to calculate the “mechanical” deformation, i.e. by cutting off thermal dilations (options EPME_ELGA and EPME_ELNO):

$$\varepsilon_{ij}^m(u) = \frac{1}{2}(u_{i,j} + u_{j,i}) - \varepsilon^{th}$$

For non-linear calculations, it is sometimes interesting to know the plastic deformation (options EPSP_ELGA and EPSP_ELNO) noted ε^{pl} .

In the case of great displacements, the deformations of Green-Lagrange are (options EPSG_ELGA and EPSG_ELNO):

$$E_{ij}(u) = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j})$$

To which one can want to cut off the thermal deformations (options EPMG_ELGA and EPMG_ELNO):

$$E_{ij}^m(u) = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) - \varepsilon^{th}$$

For the assumption of the plane deformations (D_PLAN), it is important to note that the condition of plane deformation is written on the deformation *total* :

$$\varepsilon_{33}(u) = 0$$

If the total deflection is written as the sum of a purely mechanical deformation and deformation coming from the variables of order (like the thermal deformation of dilation):

$$\varepsilon_{33}(u) = \varepsilon_{33}^m + \varepsilon_{33}^{th} = 0$$

Then it comes naturally that in plane deformations, the deformation *mechanics* except plan is not worthless in the presence of a thermal dilation.

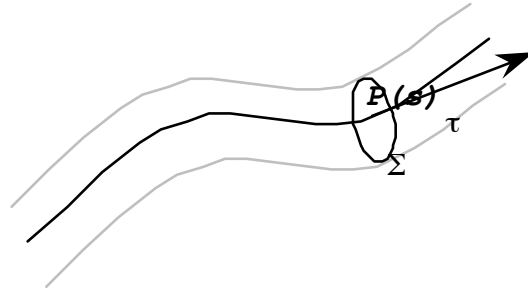
$$\varepsilon_{33}^m(u) \neq 0$$

2.1.2 Case of the beams

In the theories of traditional beams, each point P beam represents a cross-section. They are thus the end cells of the torque $(T(s), \Omega(s))$ of displacement of the presumed rigid cross-section which characterizes the displacement of the point P with the curvilinear X-coordinate s . T is the translation of the centre of inertia of the section, $\Omega(s)$ the vector rotation of the section in this point. The application of the theorem of virtual work (cf [bib2]) naturally led to define as deformation the torque (ε, χ) derived from $(T(s), \Omega(s))$ compared to the curvilinear X-coordinate s :

$$\varepsilon = \frac{dT}{ds} + \tau \wedge \Omega$$

$$\chi = \frac{d\Omega}{ds}$$



Let us pose then:

$$\varepsilon = \varepsilon_L \tau + \gamma_T$$

$$\chi = \gamma_t \tau + \mathbf{K}$$

ε_L is the longitudinal deflection,

γ_T is the vector of the deformations of distortion (no one on the assumption of Navier-Bernoulli),

γ_t is the deformation of torsion of the section,

\mathbf{K} is the deformation of inflection.

Note:

For modelings of beam with taking into account of warping, kinematics is more complicated to describe, but they lead however to concepts close to those presented above.

2.1.3 Case of the hulls

We will limit ourselves here to the cases of the plates. Indeed, in the case general of the hulls:

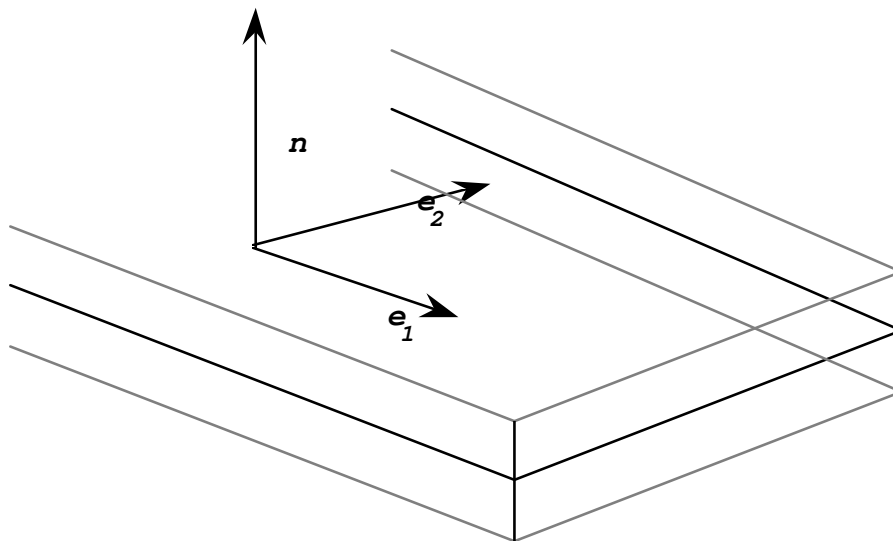
- space derivations use too complicated mathematical concepts for the framework of this document, [R3.07.04],
- the hulls are very often modelled by elements of assembled plates.

In this case, in fact only the material normals are supposed to be rigid. The displacement of these normals is thus represented by the end cells of a torque (T, Ω) . T is the translation of the point located on the average layer, Ω the vector rotation of the normal in this point.

It is clear that the normal component of Ω is worthless (in the case of nonmicropolar mediums). One introduces, the vector \mathbf{I} in the tangent plan defined by:

$$\mathbf{I} = \Omega \wedge \mathbf{n}$$

where n is the normal vector directing surface.



Maybe, decomposition:

$$T = w \mathbf{n} + \mathbf{u}_T$$

\mathbf{u}_T is tangent displacement,

w is the arrow.

In the same way that for the beams, the application of the theorem of virtual work (cf [bib2]) led to define as deformation the unit formed by the tensors E and K and the vector γ , all these sizes being defined in the tangent plan by:

$$E_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta} + u_{\beta,\alpha})$$

$$K_{\alpha\beta} = \frac{1}{2}(l_{\alpha,\beta} + l_{\beta,\alpha})$$

$$\gamma_\alpha = l_\alpha + w_{,\alpha}$$

The deformation is thus defined by 7 realities.

$E_{\alpha\beta}$ are the membrane deformations,

$K_{\alpha\beta}$ are the opposite of the curves of the deformed average layer,

γ_α is the vector of deformation of distortion.

Note:

There still, there is no universal convention and the disparity of conventions is even larger than for the tensors of efforts.

Link with the three-dimensional field of deformation

Under these conditions, one a:

$$\varepsilon_{\alpha\beta} = E_{\alpha\beta} + x_3 K_{\alpha\beta}$$

$$\varepsilon_{\alpha 3} = \gamma_\alpha$$

$$\varepsilon_{33} = 0$$

2.2 Sizes associated in Code_Aster

2.2.1 DEPL_R and DEPL_C

Sizes `DEPL_R` and `DEPL_C` as components the degrees of freedom of modeling by finite elements have and thus do not have necessarily only the components of the fields of displacement which are:

DX DY DZ

with which it is necessary to associate for the beams or the hulls:

DRX DRY MARTINI DRZ

For the hulls, we need the three components of the vector of rotation, because the equation with the finite elements can be expressed only in one total Cartesian reference mark.

2.2.2 EPSI_R

Size `EPSI_R` represent the structural deformations, therefore it must have, at least, the components:

- fields of deformations ε continuous mediums (in total reference mark):
EPXX EPYY EPZZ EPXY EPXZ EPYZ
- fields of deformations of beam (in reference mark "user" with the beam):
EPXX GAXY GAXZ KY KZ GAT
- fields of deformations of hull (necessarily in reference mark "user" on the surface)
GAY EXX EYY EXY KXX KYY KXY GAX

2.3 Options of calculation

2.3.1 Fields `EPSI_ELGA`, `EPME_ELGA`, `EPSG_ELGA`, `EPMG_ELGA` and `EPSP_ELGA`

They are fields containing the deformations at the points of Gauss and possibly at the under-points elements.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D	Pipes, Multifibre beams	Hulls, plates (except DKTG and Q4GG)
<code>EPSI_ELGA</code>	<code>EPSI_ELGA</code>	starting from a field of displacement in small deformations	ε	ε in reference mark "user" 6 components	ε in reference mark "user"
<code>EPSG_ELGA</code>	<code>EPSG_ELGA</code>	Tensor of Green-Lagrange starting from a field of displacement	E	non-available	non-available
<code>EPME_ELGA</code>	<code>EPME_ELGA</code>	starting from a field of displacement and a field of temperature in small deformations	ε^m	ε^m in reference mark "user" 6 components	non-available
<code>EPMG_ELGA</code>	<code>EPMG_ELGA</code>	Tensor of Green-Lagrange starting from a field of displacement and a field of temperature	E^m	non-available	non-available

EPSP_ELGA	EPSP_ELGA	starting from a field of displacement, of a stress field, of a field of temperature in small deformations	ε^{pl}	ε^{pl} in reference mark "user" 6 components	non-available
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2.3.2 Fields EPSI_ELNO, EPME_ELNO, EPSG_ELNO, EPMG_ELNO and EPSP_ELNO

They are fields containing the deformations whatever the modeling at ends of exploitation (impression or postprocessing of visualization) to the nodes and possibly at the under-points of the elements.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D	Pipes, Beams multi_fibres	Hulls, plates (except DKTG and Q4GG)
EPSI_ELNO	EPSI_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	ε	{ EN locates "user"	ε EN locates "user"
EPSG_ELNO	EPSG_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	E	non-available	non-available
EPME_ELNO	EPME_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	ε^m	ε^m . EN locates "user"	non-available
EPMG_ELNO	EPMG_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	E^m	non-available	non-available
EPSP_ELNO	EPSP_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	ε^{pl}	ε^{pl} . EN locates "user"	non-available
*	EPSI_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	ε	ε EN locates "user"	ε EN locates "user"

*	EPSP_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	E	non-available	non-available
*	EPME_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	ε^m	ε^m . EN locates "user"	non-available
*	EPMG_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	E^m	non-available	non-available
*	EPSP_NOEU	by arithmetic mean with the nodes of the quantities to the nodes by element	ε^{pl}	ε^{pl} . EN locates "user"	non-available

2.3.3 Fields DEGE_ELGA and DEGE_ELNO

They are fields containing the deformations generalized on the elements of beam or hull at ends of exploitation (impression or postprocessing of visualization) at the points of Gauss or the nodes of the structure.

Option of calculation	Reference symbol of concept RESULT	Calculation carried out	3D	Multifibre beams, beams	Plates, Coques1D
DEGE_ELGA	DEGE_ELGA	starting from a field of displacement in small deformations	non-available	non-available	(E, K, γ) in reference mark "user"
DEGE_ELNO	DEGE_ELNO	by extrapolation with the nodes of the quantities at the points of Gauss	non-available	(ε, χ) in reference mark "user"	(E, K, γ) in reference mark "user"

3 Bibliography

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- 3) C.TRUEDELL, W.NOLL: Encyclopedia of Physics volume III/3 - The non-linear Field Theories of Mechanics Springer-Verlag, 1965.