

## SZLZ105 - Counting of cycles by RAINFLOW and calculation of the damage

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### Summary:

Transitory linear elastic problem quasi-static in mechanics of the structures.

Calculation of the final damage in an element subjected to a cyclic loading, with a linear elastic behavior.

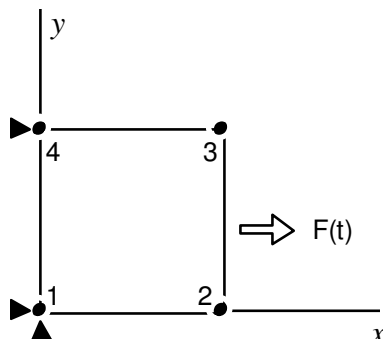
A modeling in plane constraints and a modeling in 3D.

This test validates the method of counting of cycles (RAINFLOW) established in the operator `CALC_FATIGUE` as well as the method of calculating of the damage in imposed constraint (curve of Wöhler) or imposed deformation (curve of Manson-Whetstone sheath). The reference solution is an analytical solution.

It also validates the calculation of the constraints and deformations equivalent using the options `SIEQ_ELGA`, `SIEQ_ELNO`, `EPEQ_ELGA`, `EPEQ_ELNO`, `EPMQ_ELGA` and `EPMQ_ELNO`.

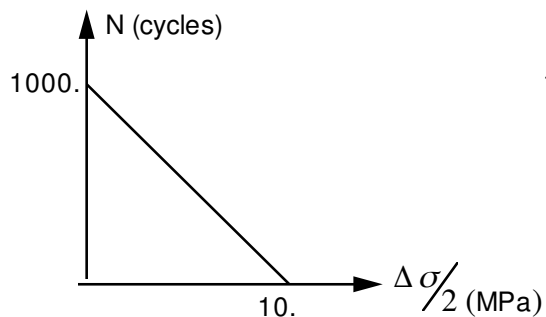
## 1 Problem of reference

### 1.1 Geometry

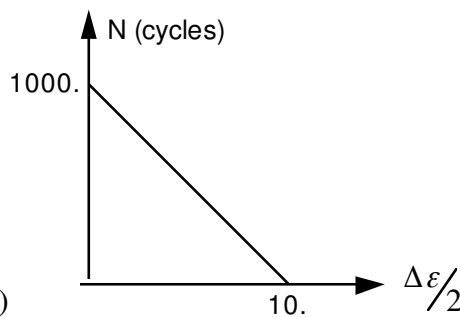


### 1.2 Material properties

Linear elasticity:  $E = 1. MPa$   $\nu = 0.3$



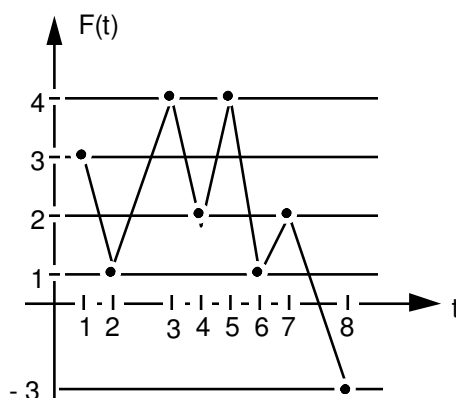
Courbe de Wöhler



Courbe de Manson-Coffin

### 1.3 Boundary conditions and loadings

- Blocked on face 1–4 according to  $X$  - node 1 blocked according to  $Y$ .
- In unit simple traction on the face 2–3.
- Loading  $F(t)$  in teeth of saw (according to the Article of Downing and Socie 1982) [bib1].



### 1.4 Initial conditions

Worthless constraints and deformations.

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

Analytical solution

- calculation of the constraints and deformations. For a loading in simple traction, one obtains a homogeneous state of stress uniaxial in any point:

$$\sigma = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \varepsilon = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

the equivalent sizes are thus  $\begin{cases} \sigma_{VMIS} = |\sigma| = \sigma_{TRESCA} \\ \sigma_{VMIS-SG} = \sigma \end{cases}$

$$\text{and } \begin{cases} \varepsilon_{INVA-2} = \frac{2}{3} |\varepsilon - \gamma| \\ \varepsilon_{INVA-2SG} = \frac{2}{3} |\varepsilon - \gamma| * \text{sign} \left[ \frac{\varepsilon + 2\gamma}{3} \right] \end{cases}$$

- then manual calculation of the cycles by the method of RAINFLOW, as well as amplitudes of loading ( $\frac{\Delta \sigma}{2}$  or  $\frac{\Delta \varepsilon}{2}$ ).

cycles	$\Delta \sigma / 2$	$\Delta \varepsilon_{INVA-2} / 2$
1	1.	0.8667
2	0.5	0.433315
3	1.	0.8667
4	3.5	3.03335

- finally carryforward of these values on the curves of Wöhler or Manson-Whetstone sheath to consider the damage unit at each cycle  $i$ , that is to say  $Du_i = \frac{1}{N_i}$  ( $N_i$  being the number of cycles with rupture for a given amplitude), as well as the cumulated damage  $D = \sum_i Du_i$  (linear rule of office plurality TO MINE).

**Note:**

One will use as equivalent constraint  $\sigma_{VMIS-SG}$  and like equivalent deformation  $\varepsilon_{INVA-2SG} = \frac{2}{3} |\varepsilon - \gamma| \times \text{sign} \left[ \frac{\varepsilon + 2\gamma}{3} \right]$ .

### 2.2 Results of reference

- Being given the values of the parameters of loading used, one obtains simply at the end of the loading (increment 8)  $\sigma = -3$ .  $\varepsilon = -3$ .  $\gamma = 0.9$   $\varepsilon_{INVA-2} = 2.6$ .
- For the calculation of the damage, one obtains:

$$D_{Wöhler} = 4,8133 \cdot 10^{-3} = \sum_{i=1}^4 Du_i$$

$$D_{Manson} = 4,67 \cdot 10^{-3} = \sum_{i=1}^4 Du_i$$

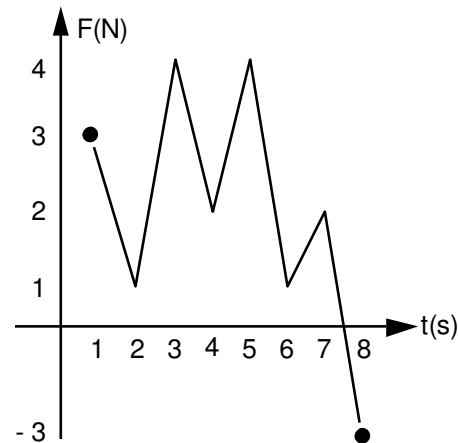
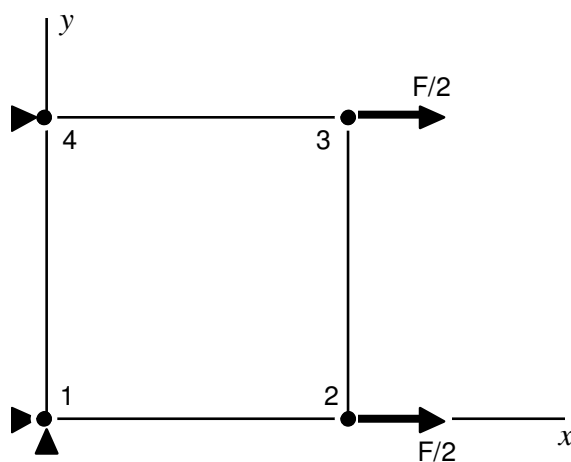
### 2.3 Bibliographical references

- DOWNING and SOCIE, 1982. "Simple Rainflow counting algorithms". Int. J. Tiredness, January 1982 (p. 31).

## 3 Modeling A

### 3.1 Characteristics of modeling

Modeling in plane constraints:



### 3.2 Characteristics of the grid

1 mesh QUAD4.

Square width = 1  
thickness = 1

### 3.3 Sizes tested and results

Identification	Reference
in all nodes at the end of the loading in constraint or deformation	
Wöhler damage	$4.8133 \cdot 10^{-3}$
Damage Manson-Whetstone sheath	$4.6705 \cdot 10^{-3}$
$\sigma$	-3.
$\sigma_{VMIS}$	3.
$\sigma_{TRESCA}$	3.
$\sigma_{VMIS-SG}$	-3.
$\varepsilon$	-3.
$\gamma$	0.9
$\varepsilon_{INVA-2}$	2.6
$\varepsilon_{INVA-2-SG}$	-2.6

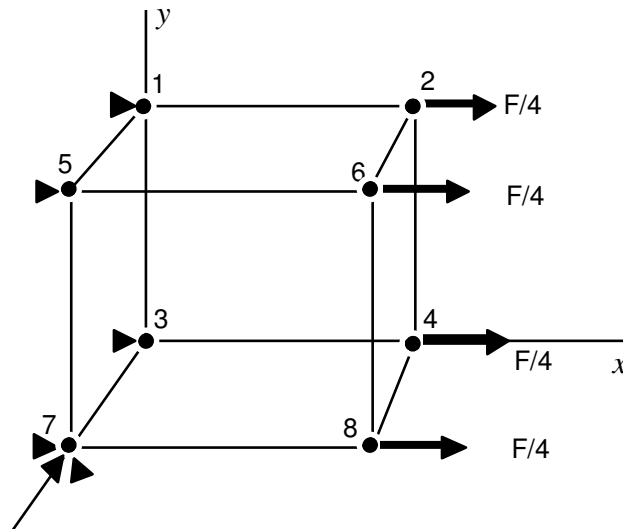
### 3.4 Remarks

Fast test in time calculation.

## 4 Modeling B

### 4.1 Characteristics of modeling

Modeling in 3D :



### 4.2 Characteristics of the grid

1 mesh HEXA8.  
cubic of width = 1

### 4.3 Sizes tested and results

Identification	Reference
in all nodes at the end of the loading	
Wöhler damage	$4.8133 \cdot 10^{-3}$
Damage Manson-Whetstone sheath	$4.6705 \cdot 10^{-3}$
$\sigma$	-3.
$\sigma_{VMIS}$	3.
$\sigma_{TRESCA}$	3.
$\sigma_{VMIS-SG}$	-3.
$\epsilon$	-3.
$\gamma$	0.9
$\epsilon_{INVA-2}$	2.6
$\epsilon_{INVA-2}^{SG}$	-2.6
$(\epsilon - \epsilon^{th})_{INVA-2}$	2.6
$(\epsilon - \epsilon^{th})_{INVA-2}^{SG}$	-2.6

## 4.4 Remarks

Same results and reference that in plane constraints.

## 5 Summary of the results

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This test validates the method and the calculation of the damage of Wöhler and Manson-Whetstone sheath.

Results of *Code\_Aster* are identical to those obtained analytically.