WTNP127 – Modeling of a flow of water in a saturated bar, establishment of a permanent mode

Summary:

This CAS-test represents the modeling of a flow of water in a bar subjected to a gradient of pressure. One models here a diphasic flow which one makes degenerate into a monophasic problem. We are interested in this CAS-test in the establishment of the permanent mode which makes it possible analytically to calculate water flows at exit. This CAS-test has as a main objective to validate the calculation of the integral of hydraulic flows on a surface.

1 Problem of reference

The objective of this CAS-test is to test the calculation of the integral of flows on a surface.

1.1 Geometry

One considers a bar of 5m of length and of 1m of top.

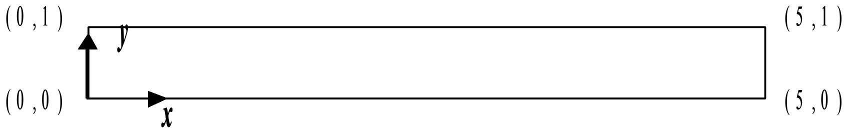

Illustration 1: : Geometry

1.2 Properties of materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

Liquid	Relative permeability	1
	Viscosity $\mu(\ pa.s)$	1
	Module of compressibility	0
	Density of the liquid $\rho\ (\ kg/m^3)$	1
Homogenized parameters	Intrinsic permeability $\text{K\_int}(m^2)$	10^{-13}
	Porosity	$0{,}5$
	Storage	10^{-10}
	Saturation in liquid	1
	Henry $(Pa.mol^{-1}.m^3)$	10^{10}

Table 1.2-1 : Properties of materials

1.3 Boundary conditions and initial

The boundary conditions are conditions of Dirichlet:

On the left part of the field, $P_l(t, x=0, y)=0\,Pa$

On the right part of the field, $P_l(t, x=L, y)=10^4\,Pa$

The initial pressure of liquid is of $P_l(t=0, x, y)=10^4\,Pa$.

1.4 Duration of simulation and not of time

The duration of simulation is of $50000\,s$ and the number of steps of time is of 5.

2 Reference solution

2.1 Method of calculating

That is to say a bar saturated with water (regarded as incompressible). This bar length L and height h has an initial pressure $P_l(x,y,t=0)=P_{ini}$ and is subjected to a gradient of pressure such as $P_l(0,y,t)=P_G$ and $P_l(L,y,t)=P_{ini}$.

This problem of evolution led at the end of a time t_p in a linear permanent state such as $P(x,y,t>t_p)=\frac{P_{ini}-P_G}{L}x+P_G$

The water flow $\boldsymbol{M}_{11}$ (factor of the gradient of pressure) is then constant along the bar. If one integrates it on a vertical cut Γ bar, and of outgoing normal $\boldsymbol{\nu}$, one obtains:

$$\int_\Gamma \boldsymbol{M}_{11}.\boldsymbol{\nu}=h.\rho_l\frac{K_{int}}{\mu_l}.\frac{P_G-P_{ini}}{L}.\boldsymbol{x}.\boldsymbol{\nu}$$

The calculation of this integral will be carried out in this CAS-test on 3 surfaces (or side in $2D$).

2.2 Simplifying assumptions

In order to test the calculation of flows on the hydraulic model most complete possible, one starts from a diphasic modeling which one makes degenerate into monophasic modeling. For that, it is considered that the medium is completely saturated with water and one imposes a worthless gas pressure on all the nodes. The biphasic system is then brought back to solve the following problem:

$$\frac{\partial(\varphi\rho_l)}{\partial t}-div\left(K_{int}\frac{\rho_l k_{rl}}{\mu_l}\nabla P_l\right)=0$$

- The liquid is incompressible: $\rho_l=cst$
- The matrix is compressible and porosity evolves proportionally with the pressure of liquid: $\frac{\partial\varphi}{\partial P_l}=E_m$
- The relative permeability is taken equalizes to 1: $k_{rl}=1$

The conservation equation of the mass for the liquid is thus written:

$$\rho_l E_m\frac{\partial P_l}{\partial t}-div\left(K_{int}\frac{\rho_l}{\mu_l}\nabla P_l\right)=0$$

A really saturated modeling will be also tested (modeling D).

2.3 Uncertainties on the solution

Uncertainties are worthless, because the reference solution is analytical.

3 Modeling A

3.1 Characteristics of modeling

A case here is modelled $2D$. Flows are calculated on 3 sides:

- the left vertical side, noted 'MGAUCHE'
- the vertical side right, noted 'MDROIT'
- a vertical side in the matter, noted 'MILLET'.

One will reorientate the meshs of edge in order to have an outgoing normal. Flow being directed line towards the left in this example will be positive on MGAUCHE and negative on MDROIT.

One will reorientate MILLET so that the normal is outgoing with the right part of the bar resting on MILLET. In this way, flow will be positive on MILLET.

To reorientate the meshs in $2D$, the keyword factor is used ORIE_PEAU_2D order MODI_MAILLAGE. If on the east side interns, it is necessary to specify a group of surface meshs on which it rests to determine the outgoing normal with this group (keyword GROUP_MA_SURF).

To reorientate the meshs in $3D$, the keyword factor is used ORIE_PEAU_3D order MODI_MAILLAGE. If surface is internal, it is necessary to specify a group of voluminal meshs on which it rests to determine the outgoing normal with this group (keyword GROUP_MA_VOLU).

Quantity of water on surfaces Γ thus defined will be thus equal to

$$\int_{\Gamma} \boldsymbol{M}_{11}.\boldsymbol{\nu} = h.\rho \frac{K_{int}}{\mu}.\frac{P_{ini}-P_G}{L} = 2\,10^{-10} kg.s^{-1} .$$

Vapor flows $\boldsymbol{M}_{12}$, of dry air $\boldsymbol{M}_{21}$ and of dissolved air $\boldsymbol{M}_{22}$ are them almost worthless (with simplifications near).

Modeling tested here is D_PLAN_HH2S

3.2 Characteristics of the grid

Many nodes: 805
Many meshs and types: 406 meshs, 206 SEG3 and 200 QUAD8

3.3 Sizes tested and results

Identification	Type of reference	Value	Tolerance ($\%$)
Component $INTE\_FH11$, place $MDROITE$	'AUTRE_ASTER'	-2.0E-10	0.1%
Component $INTE\_FH11$, place $MMIL$	'AUTRE_ASTER'	2.0E-10	0.1%
Component $INTE\_FH11$, place $MGAUCHE$	'AUTRE_ASTER'	2.0E-10	0.1%

One tests hydraulic flow at the first point of Gauss of the mesh M_{401} with the sequence number 5:

Identification	Type of reference	Value	Tolerance ($\%$)
Component $FH11$	'NON_REGRESSION'	2.00133E-10	0.1%
Component $FH22$	'NON_REGRESSION'	1.99933E-11	0.1%

4 Modeling B

4.1 Characteristics of modeling

Even case that previously but in of axisymmetric. The expected results are different because of integration:

$$\int_{\Gamma} \boldsymbol{M}_{11}.\boldsymbol{\nu}=\boldsymbol{M}_{11}.\boldsymbol{\nu}\int_0^1\int_0^1 r\,dr\,d\,\theta=0.5\,\boldsymbol{M}_{11}.\boldsymbol{\nu}$$

Modeling used is AXIS_HH2S.

4.2 Characteristics of the grid

Many nodes: 805
Many meshs and types: 406 meshs, 206 SEG3 and 200 QUAD8

4.3 Sizes tested and results

Identification	Type of reference	Value	Tolerance ($\%$)
Component $INTE\_FH11$, place $MDROITE$	'AUTRE_ASTER'	-1.0E-10	0.1%
Component $INTE\_FH11$, place $MMIL$	'AUTRE_ASTER'	1.0E-10	0.1%
Component $INTE\_FH11$, place $MGAUCHE$	'AUTRE_ASTER'	1.0E-10	0.1%

One tests hydraulic flow at the first point of Gauss of the mesh M_{401} with the sequence number 5:

Identification	Type of reference	Value	Tolerance ($\%$)
Component $FH11$	'NON_REGRESSION'	2.00136E-10	0.1%
Component $FH22$	'NON_REGRESSION'	0.	0.1%

5 Modeling C

5.1 Characteristics of modeling

Even case that modeling A but in of $3D$. The expected results are the same ones.
Modeling used is 3D_HH2S.

5.2 Characteristics of the grid

Many nodes: 1913
Many meshs and types: 206 meshs, 200 HEXA20 and 6 QUAD8

5.3 Sizes tested and results

Identification	Type of reference	Value	Tolerance ($\%$)
Component $INTE\_FH11$, place $MDROITE$	'AUTRE_ASTER'	-2.0E-10	0.1%
Component $INTE\_FH11$, place $MMIL$	'AUTRE_ASTER'	2.0E-10	0.1%
Component $INTE\_FH11$, place $MGAUCHE$	'AUTRE_ASTER'	2.0E-10	0.1%

One tests hydraulic flow at the first point of Gauss of the mesh M_{401} with the sequence number 5:

Identification	Type of reference	Value	Tolerance ($\%$)
Component $FH11$	'NON_REGRESSION'	2.00133E-10	0.1%
Component $FH22$	'NON_REGRESSION'	1.99933E-11	0.1%

6 Modeling D

6.1 Characteristics of modeling

Even case that modeling A but in D_PLAN_HMS (really saturated modeling, gas flows do not exist any more). The grid consists of triangles.

The expected results are the same ones.

6.2 Characteristics of the grid

Many nodes: 625
Many meshs and types: 344 meshs, 60 SEG3 and 284 TRIA6

6.3 Sizes tested and results

Identification	Type of reference	Value	Tolerance ($\%$)
Component $INTE\_FH11$, place $MDROITE$	'AUTRE_ASTER'	-2.0E-10	0.1%
Component $INTE\_FH11$, place $MMIL$	'AUTRE_ASTER'	2.0E-10	0.1%
Component $INTE\_FH11$, place $MGAUCHE$	'AUTRE_ASTER'	2.0E-10	0.1%

7 Modeling E

7.1 Characteristics of modeling

Even case that modeling the preceding one but in D_PLAN_HS (really saturated modeling, gas flows do not exist any more). The grid consists of triangles.
In addition, instead of applying a gradient of pressure, the purpose of one directly applies the flow expected at exit (that is to test the condition in limit in water flow for modeling D_PLAN_HS.).
The expected results are the same ones.

7.2 Characteristics of the grid

Many nodes: 625
Many meshs and types: 344 meshs, 60 SEG3 and 284 TRIA6

7.3 Sizes tested and results

Identification	Type of reference	Value	Tolerance (%)
Component $INTE\_FH11$, place $MDROITE$	'AUTRE_ASTER'	-2.0E-10	0.1%
Component $INTE\_FH11$, place $MMIL$	'AUTRE_ASTER'	2.0E-10	0.1%
Component $INTE\_FH11$, place $MGAUCHE$	'AUTRE_ASTER'	2.0E-10	0.1%

8 Modeling F

8.1 Characteristics of modeling

Even case that modeling D but in D_PLAN_HM_SI (really saturated under-integrated modeling, gas flows do not exist any more). The grid consists of triangles.

The expected results are the same ones.

8.2 Characteristics of the grid

Many nodes: 625
Many meshs and types: 344 meshs, 60 SEG3 and 284 TRIA6

8.3 Sizes tested and results

Identification	Type of reference	Value	Tolerance ($\%$)
Component $INTE\_FH11$, place $MDROITE$	'AUTRE_ASTER'	-2.0E-10	0.1%
Component $INTE\_FH11$, place $MMIL$	'AUTRE_ASTER'	2.0E-10	0.1%
Component $INTE\_FH11$, place $MGAUCHE$	'AUTRE_ASTER'	2.0E-10	0.1%

9 Synthesis

One models in this CAS-test the establishment of a permanent mode for a flow of water in a bar. One validates the calculation of the integral of hydraulic flow on a surface that it is of edge or intern to the field. The validation is made in $2D$ and in $3D$. In all the cases, the got results correspond well to the analytical solution