WTNV146 - Validation of a model of cohesive law for the hydraulic case coupled with XFEM

Summary:

The goal of this test is to make sure of the good performance DU models of cohesive zone of type associated “Mortar” with the hydraulic elements coupled with XFEM.

In this test, one tests the whole of the modes of operation of the model of cohesive zone for elements HM-XFEM. The cohesive law used is `CZM_LIN_MIX` (It is the only available one for elements HM-XFEM). This test is thus purely mechanical, all the degrees of freedom associated with the liquid phase are blocked to zero. Modeling A is two-dimensional while modeling B is three-dimensional.

1 Problem of reference

1.1 Geometry of the problem

It is about a column height $LZ=5m$, length $LX=1m$ and of width $LY=1m$. This column presents in $Z=\frac{LZ}{2}$ a discontinuity of the type interfaces. The column is thus entirely crossed by discontinuity.

One represents on the Figure 1.1-a geometry of the column.

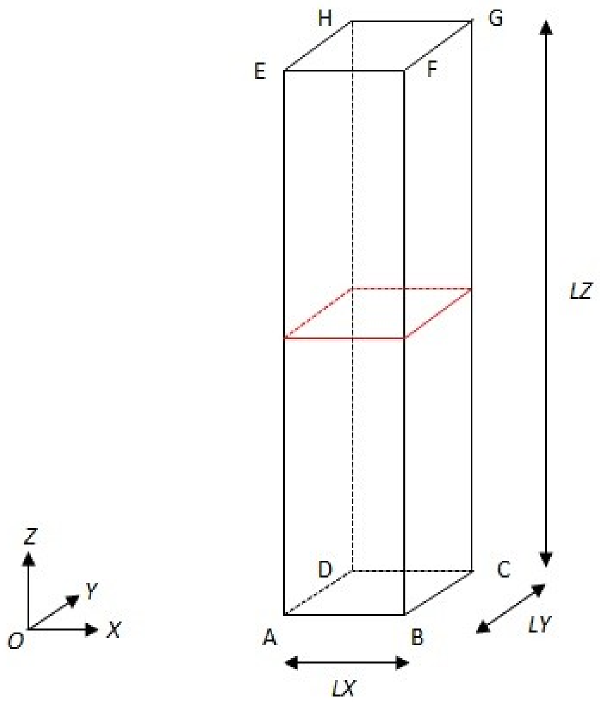


Figure 1.1-a: Geometry of the problem

1.2 Properties materials

Parameters given in the Table 1.2-1 , correspond to the parameters used for modeling in the hydraulic coupled case. The mixing rate used is `'LIQU_SATU'` . The parameters specific to this mixing rate are given but do not have any influence on the solution (because we chose to measure a pressure of pore uniformly worthless in all the field). Only the elastic parameters affect the solution of the pseudo-coupled problem. LE contact used is of type `'MORTAR'`. The law cohesive associated is `'CZM_LIN_MIX'`.

Liquid (water)	Viscosity $\mu_w (en\ Pa.s)$	10^{-3}
	Module of compressibility $\frac{1}{K_w}(en\ Pa^{-1})$	5.10^{-10}
	Density of the liquid $\rho_w(en\ kg/m^3)$	1
Elastic parameters	Young modulus $E(en\ MPa)$	5800
	Poisson's ratio ν	0
	Thermal dilation coefficient $\alpha(en\ K^{-1})$	0
Parameters of coupling	Coefficient of Biot b	1
	Initial homogenized density $r_0(en\ kg/m^3)$	2,5
	Intrinsic permeability $K^{int}(en\ m^2)$	$1{,}01937^{-19}$
Parameters of the cohesive law	Critical stress $\sigma_c(en\ MPa)$	1.1
	Cohesive energy $G_c(en\ Pa.m)$	900
	Coefficient of increase r	10

Table 1.2-1 : Properties of material

In addition, the forces related to gravity (in the conservation equation of the momentum) are neglected. The pressure of pore of reference is taken worthless $p_1^{ref}=0\ MPa$ and the porosity of material is $\varphi = 0{,}15$.

1.3 Boundary conditions

L is appliedbe conditions of Dirichlet following :

- on the face [ABCD], displacements are blocked in Toutes directions ($u_x=0$, $u_y=0$ and $u_z=0$),
- on the face [EFGH], displacements according to y are blocked $u_y=0$ and displacements according to x and z are imposed for each moment of calculation : $u_x=f(t)$, $u_z=g(t)$.

8 different loadings are carried out. The values of the displacements imposed on the higher face for each of the 8 moments of calculation are summarized in the table below:

Moment	$u_x=0$	$u_z=0$
1	0	-0.0001
2	0	0.0001
3	0	0.001
4	0	0.0005
5	0	0.0012
6	0	0.0017
7	0.001	0.0017
8	0	-0.0001

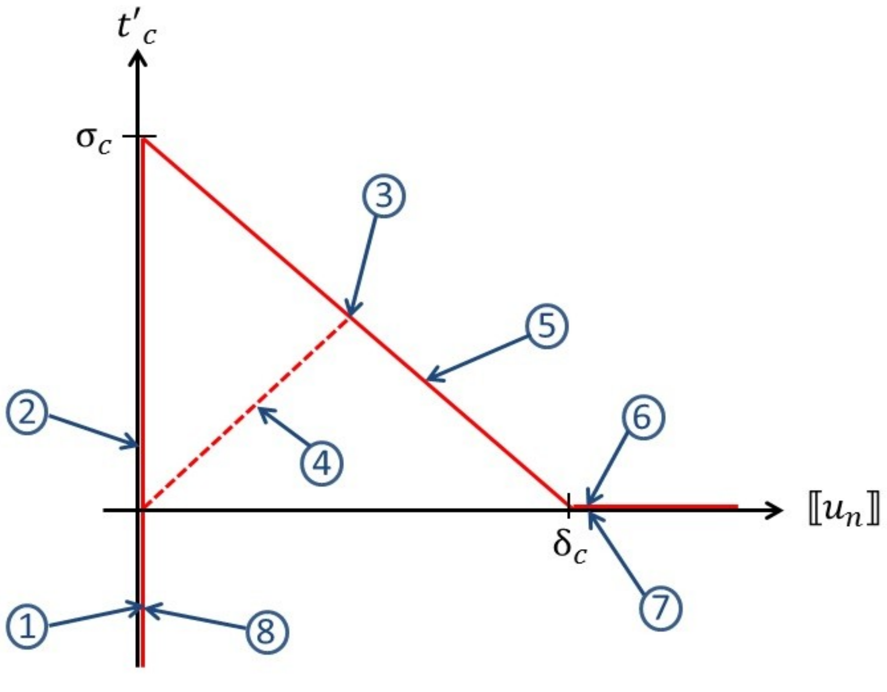
Table 1.3-1 : Displacements imposed on the higher face

2 Reference solution

2.1 Method of calculating

It is about an analytical solution. The normal cohesive constraint $t_{c,n}$ Saccording to the jump of normal displacement $[u_n]$ for the law `'CZM_LIN_MIX'` is represented on the Figure 2.1-a. For each of the 8 moments of calculation, one represents on this same Figure the position in which one is. The jump of displacement criticizes which corresponds to the disappearance of the efforts of cohesion is $\delta_c=\frac{2G_c}{\sigma_c}=0{,}0016363636\,m$

Figure 2.1-a: Normal cohesive constraint according to the jump of normal displacement for the law `'CZM_LIN_MIX'`



By neglecting gravity, the equation of balance total is written (in total constraints):

$$\mathbf{Div}(\boldsymbol{\sigma})=\mathbf{0}$$

In the case of a coupled modeling, the tensor of the total constraints is written:

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}'-p_1\mathbf{1}$$

$\boldsymbol{\sigma}'$ is the tensor of the constraints in the skeleton and p_1 and pressure of pore in the solid mass. The Fish module ν being null, and being in the elastic case, one has $\boldsymbol{\sigma}'=E\boldsymbol{\epsilon}$.

However $\forall \boldsymbol{x}, p_1(\boldsymbol{x})=0$ thus finally $\mathbf{Div}(\boldsymbol{\epsilon})=\mathbf{0}$

Being given imposed displacements and the nullity of the Poisson's ratio ν , the problem is one-way in the direction z . In the solid matrix, the fields of displacements according to z check:

$$\frac{d^2u_z}{dz^2}=0$$

Moment 1

The column is in compression, the lips of the cohesive interface are in contact. The jump of displacement $[u_z]$ is thus null. Consequently $\epsilon_{zz}=g(1)/LZ$ and $\sigma_{zz}=E\epsilon_{zz}=E*g(1)/LZ$. The normal cohesive constraint $t_{c,n}$ is equal to the constraint σ_{zz} .

Moment 2

The column is in traction. The assumption is made that the cohesive interface is in situation of adherence. This assumption is checked if $t_{c,n}\leq\sigma_c$. IF the cohesive interface is adherent, then the lips of the cohesive interface are in contact. The jump of displacement $[u_z]$ is thus null. Consequently $\epsilon_{zz}=g(2)/LZ$ and $\sigma_{zz}=E\epsilon_{zz}=E*g(2)/LZ$. The normal cohesive constraint $t_{c,n}$ is then equalize with the constraint $\sigma_{zz}=0{,}116\,MPa\leq\sigma_c$. The assumption carried out initially is thus validated.

Moment 3

The column is in traction. The assumption is made that the cohesive interface is in situation of adherence. This assumption is checked if $t_{c,n}\leq\sigma_c$. IF the cohesive interface is adherentE, then the lips of the cohesive interface are in contact. The jump of displacement $[u_z]$ is thus null. Consequently $\epsilon_{zz}=g(3)/LZ$ and $\sigma_{zz}=E\epsilon_{zz}=E*g(3)/LZ$. The normal cohesive constraint $t_{c,n}$ is then equalize with the constraint $\sigma_{zz}=1{,}16\,MPa>\sigma_c$. The assumption carried out initially is thus false.
The assumption is thus made that the cohesive interface is in mode of damage. This assumption is checked if $0<[u_n]\leq\delta_c$. The cohesive constraint $t_{c,n}$ is then connected to the jump of normal displacement $[u_z]$ by the relation $t_{c,n}=\sigma_c*(1-\frac{[u_z]}{\delta_c})$. In addition, the total elongation of the column is $g(3)=[u_z]+LZ*\epsilon_{zz}$. Lastly, the normal cohesive constraint is equal to the vertical constraint $t_{c,n}=E*\epsilon_{zz}$. Finally:

$$t_{c,n}=\frac{E*\sigma_c*(1-\frac{g(3)}{\delta_c})}{E-\frac{LZ*\sigma_c}{\delta_c}}$$

$$[u_z]=\frac{-LZ*\sigma_c+E*g(3)}{E-\frac{LZ*\sigma_c}{\delta_c}}$$

NRumériquement, one finds, $[u_z]=0{,}0001230068\,m<\delta_c$. The assumption carried out initially is validated.

Moment 4

The column is always in traction, but a less important traction thatat the previous moment. One is thus in elastic situation of return in the cohesive zone (discharge during the process of damage). Normal cohesive traction is then given by $t_{c,n}=\frac{t_{c,n}(3)*[u_z]}{[u_z(3)]}$ and total elongation column is $g(4)=[u_z]+LZ*\epsilon_{zz}$. Lastly, the normal cohesive constraint is equal to the vertical constraint $t_{c,n}=E*\epsilon_{zz}$. Finally:

$$[u_z]=\frac{E*g(4)*[u_z(3)]}{LZ*t_{c,n}(3)+E*[u_z(3)]}$$

$$t_{c,n}=\frac{t_{c,n}(3)*E*g(4)}{LZ*t_{c,n}(3)+E*[u_z(3)]}$$

Moment 5

The column is again in traction, on a level not yet reached . The assumption is thus made that the cohesive interface is in mode of damage. This assumption is checked if $0<[u_n]\leq\delta_c$. The cohesive constraint $t_{c,n}$ is then connected to the jump of normal displacement $[u_z]$ by the relation $t_{c,n}=\sigma_c*(1-\frac{[u_z]}{\delta_c})$. In addition, the total elongation of the column is $g(5)=[u_z]+LZ*\epsilon_{zz}$. Lastly, the normal cohesive constraint is equal to the vertical constraint $t_{c,n}=E*\epsilon_{zz}$. Finally:

$$t_{c,n}=\frac{E*\sigma_c*(1-\frac{g(5)}{\delta_c})}{E-\frac{LZ\,\sigma_c}{\delta_c}}$$

$$[u_z]=\frac{-LZ\,\sigma_c+E*g(5)}{E-\frac{LZ\,\sigma_c}{\delta_c}}$$

NR_umériquement, one finds,_ $[u_z]=0{,}000059863325\,m<\delta_c$. The assumption carried out initially is validated._

Moment 6

The column is always in traction, on a level not yet reached . The assumption is thus made that the cohesive interface is in mode of damage. This assumption is checked if $0<[u_n]\leq\delta_c$. The cohesive constraint $t_{c,n}$ is then connected to the jump of normal displacement $[u_z]$ by the relation $t_{c,n}=\sigma_c*(1-\frac{[u_z]}{\delta_c})$. In addition, the total elongation of the column is $g(6)=[u_z]+LZ*\epsilon_{zz}$. Lastly, the normal cohesive constraint is equal to the vertical constraint $t_{c,n}=E*\epsilon_{zz}$. Finally:

$$t_{c,n}=\frac{E*\sigma_c*(1-\frac{g(6)}{\delta_c})}{E-\frac{LZ\,\sigma_c}{\delta_c}}$$

$$[u_z]=\frac{-LZ\,\sigma_c+E*g(6)}{E-\frac{LZ\,\sigma_c}{\delta_c}}$$

NRumériquement, one finds,_ $[u_z]=0{,}001787699\,m>\delta_c$. The assumption carried out initially is erroneous. The cohesive interface is not any more in the mode of damage, it is broken. The cohesive forces are thus worthless_ $t_{c,n}=0$ and the jump of displacement is_ $[u_z]=g(6)$.

Moment 7

During this step of time, the vertical elongation of the column remains fixed. On the other hand, one applies a side displacement to the higher face of the column in order to check than the cohesive efforts remain worthless in the event of shearing in the broken cohesive interface. One has as follows:

$$[u_z]=g(7)$$

$$[u_x]=f(7)$$

$$t_{c,n}=0$$

$$\boldsymbol{t_{c,s}=0}$$

Moment 8

Lastly, one compresses the column to check that the contact on the lips of the cohesive interface applies well even when the cohesive zone was broken. The loading is the same one as at moment 1. One has $\epsilon_{zz}=g(8)/LZ$ and $\sigma_{zz}=E\,\epsilon_{zz}=E*g(8)/LZ$. The normal cohesive constraint $t_{c,n}$ is equal to the constraint σ_{zz} .

2.2 Sizes and results of reference

One tests the value of normal cohesive traction on the level of the cohesive interface and tangential cohesive traction (according to x) at every moment. To test all the nodes of the cohesive interface at the same time it is tested `MIN` and it `MAX.`

	Traction cohesive normal (MPa)	Traction cohesive tangential (MPa)
Moment 1	-0.116	0
Moment 2	0,116	0
Moment 3	1.0173120729	0
Moment 4	0.50865603645	0
Moment 5	0.69758542141	0
Moment 6	0	0
Moment 7	0	0
Moment 8	-0.116	0

2.3 Uncertainties on the solution

The no solution is analytical.

2.4 Bibliographical references

[1] Reference material R7.02.18 (Hydraulic elements coupled with the wide finite element method).

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling D_PLAN_HM using quadratic elements HM-XFEM. The bar on which one carries out modeling is divided into 5 QUAD8. The interface is nonwith a grid and cuts the central element. Thus there are 3 classical elements HM-XFEM and 2 elements HM. As indicated on the Figure 3.1-a, 3 elements XFEM undergo under cutting under triangles (to carry out the integration of Gauss-Legendre on both sides of lips of the interface, but these triangular subelements are not elements of the grid).

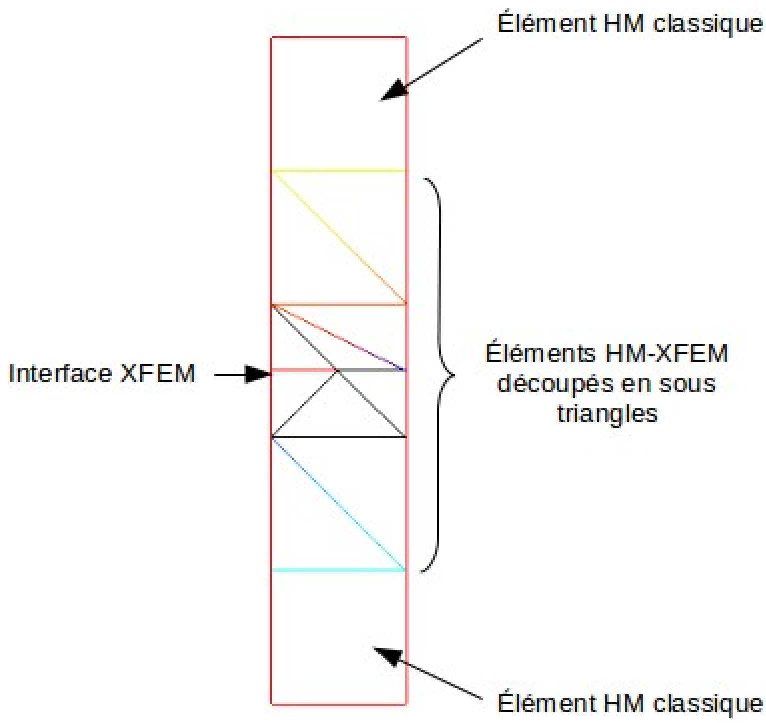


Figure 3.1-a: Characteristics of modeling

3.2 Characteristics of the grid

The grid consists of 5 meshs quadratic quadrangles (QUAD8).

3.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the table below for each of the 8 moments of calculation. To test all the nodes of the bar at the same time, it is calculated MIN and it MAX.

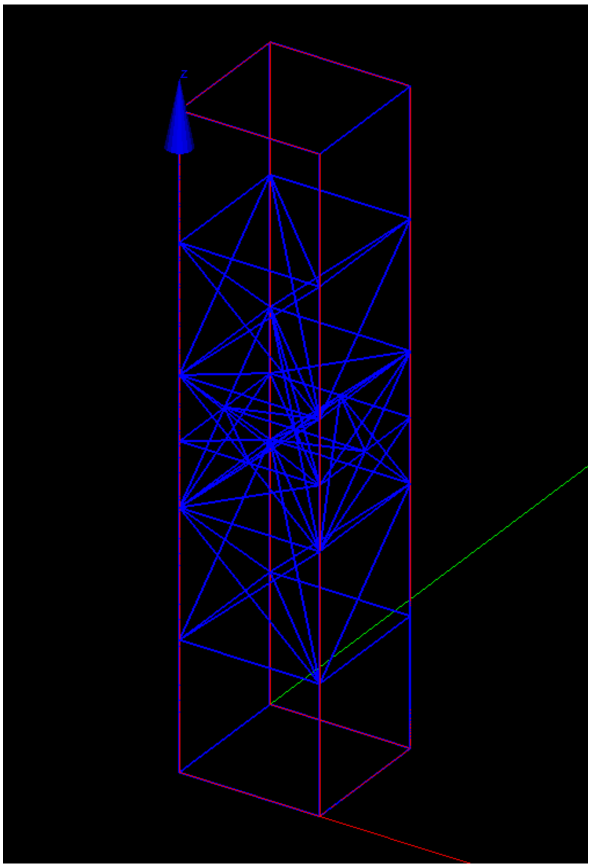
Sizes tested	Type of reference	Value of reference	Tolerance (%)
LAGS_C MIN (moment 1)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_C MAX (moment 1)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_F1 MIN (moment 1)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 1)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 2)	'ANALYTICAL'	0.116 MPa	0.0001
LAGS_C MAX (moment 2)	'ANALYTICAL'	0.116 MPa	0.0001
LAGS_F1 MIN (moment 2)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 2)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 3)	'ANALYTICAL'	1.0173120729 MPa	0.0001
LAGS_C MAX (moment 3)	'ANALYTICAL'	1.0173120729 MPa	0.0001
LAGS_F1 MIN (moment 3)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 3)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 4)	'ANALYTICAL'	0.50865603645 MPa	0.0001
LAGS_C MAX (moment 4)	'ANALYTICAL'	0.50865603645 MPa	0.0001
LAGS_F1 MIN (moment 4)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 4)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 5)	'ANALYTICAL'	0.69758542141 MPa	0.0001
LAGS_C MAX (moment 5)	'ANALYTICAL'	0.69758542141 MPa	0.0001
LAGS_F1 MIN (moment 5)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 5)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MAX (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MIN (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MAX (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MIN (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 8)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_C MAX (moment 8)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_F1 MIN (moment 8)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 8)	'ANALYTICAL'	0 MPa	0.0001

4 Modeling B

4.1 Characteristics of modeling

It is about a modeling 3D_HM using quadratic elements HM-XFEM. The column on which one carries out modeling is divided into 5 HEXA20. The interface is nonwith a grid and cuts the central element. Thus there are 3 classical elements HM-XFEM and 2 elements HM (two hexahedrons which form the ends of the column). As indicated on the Figure 4.1-a, 3 elements XFEM undergo under cutting under tetrahedrons (to carry out the integration of Gauss-Legendre on both sides of lips of the interface, but these subelements tetrahedrons are not elements of the grid).

Figure 4.1-a: Characteristics of modeling



4.2 Characteristics of the grid

The grid consists of 5 meshs hexaédric quadratic (HEXA20).

4.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the table below for each of the 8 moments of calculation. To test all the nodes of the bar at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
LAGS_C MIN (moment 1)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_C MAX (moment 1)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_F1 MIN (moment 1)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 1)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 2)	'ANALYTICAL'	0.116 MPa	0.0001
LAGS_C MAX (moment 2)	'ANALYTICAL'	0.116 MPa	0.0001
LAGS_F1 MIN (moment 2)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 2)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 3)	'ANALYTICAL'	1.0173120729 MPa	0.0001
LAGS_C MAX (moment 3)	'ANALYTICAL'	1.0173120729 MPa	0.0001
LAGS_F1 MIN (moment 3)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 3)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 4)	'ANALYTICAL'	0.50865603645 MPa	0.0001
LAGS_C MAX (moment 4)	'ANALYTICAL'	0.50865603645 MPa	0.0001
LAGS_F1 MIN (moment 4)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 4)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 5)	'ANALYTICAL'	0.69758542141 MPa	0.0001
LAGS_C MAX (moment 5)	'ANALYTICAL'	0.69758542141 MPa	0.0001
LAGS_F1 MIN (moment 5)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 5)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MAX (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MIN (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 6)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MAX (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MIN (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 7)	'ANALYTICAL'	0 MPa	0.0001
LAGS_C MIN (moment 8)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_C MAX (moment 8)	'ANALYTICAL'	-0.116 MPa	0.0001
LAGS_F1 MIN (moment 8)	'ANALYTICAL'	0 MPa	0.0001
LAGS_F1 MAX (moment 8)	'ANALYTICAL'	0 MPa	0.0001

5 Summary of the results

This test makes it possible to validate it cohesive model of type MORTAR and the cohesive law CZM_LIN_MIX for cohesive elements HM-XFEM.