HSNS101 - Square plate in traction and temperature variables. Plane constraints method OF BORST

Summary:

This test, inspired by that proposed by the IPSI for the Phi2As day of March 30th, 2000 (test HSNV124) makes it possible to validate the method of Borst to treat the condition of plane constraints for an unspecified nonlinear behavior. In particular, one tests here the method for the behaviors VMIS_CINE_LINE, VMIS_CIN1_CHAB and VMIS_CIN2_CHAB, by comparison with VMIS_ECMI_LINE, which uses a method of direct integration of the plane constraints. All the parameters of these various models are adjusted in order to reproduce in fact the same behavior.

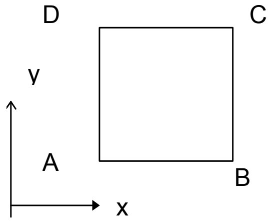
Four modelings make it possible to validate the plane constraints (method of Borst).

The results are compared with those of test HSNV124 (analytical reference).

1 Problem of reference

1.1 Geometry

Element of volume materialized by a square plate $ABCD$ on unit side ($1\,mm$):



1.2 Properties of materials

$E=2.10^5\,MPa$, $\nu=0.3$, $\alpha=2.10^5\,°C^{-1}$

The material is elastoplastic with a linear kinematic work hardening:

$\sigma=\pm\sigma_y(T)+C(T)\varepsilon^p$
$SIGY=200.-1.7.T$ (in Mpa)
Module of work hardening D_SIGM_EPSI = $C(T)=1000+2990.T$ (in MPa)

1.3 Boundary conditions and loadings

Such that the stress and strain state are uniform in the element of volume:

Not A blocked in x and y .
$DY=0$ on AB

Force distributed on CD : Fy
Uniform temperature T on $ABCD$. The temperature of reference is worth $0\,°C$.

Fy and T vary according to time in the following way:

moment t	0	1s	2s
$Fy(t)$	0	$210\,MPa$	$210\,MPa$
$T(t)$	0	$0\,°C$	$100\,°C$

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution for the plastic deformation, identical to test HSNV124B.
For displacements (elements of structure): Comparison with the displacements obtained with *Code_Aster* for modeling A.

2.2 Results of reference

Evolution of the plastic deformation according to y and of following displacement y (plastic deformation not being accessible for the elements of structure).

Plastic deformation:

$t=1\text{s}$ **(** $T=0°C$ **)**: $\varepsilon^p=1\%$

Heating: Constant plastic deformation until $t=356/316=1.12658\text{s}$ ($T=12.658°C$):

Then, the plastic deformation decreases to reach with $t=2\text{s}$: $\varepsilon^p=0.08\%$

One tests also the components `SIYY` (constant) and `SIZZ` (worthless, if calculation is well carried out in plane constraints).

$T(s)$	Plastic deformation `EPYY`	`DY` (mm)	`SIYY` (Mpa)	`SIZZ` (Mpa)
1	0.01	$1.105\,10^{-2}$	210	0
1.1	0.01	$1.115\,10^{-2}$	210	0
2	8.10^{-4}	$2.85\,10^{-3}$	210	0

2.3 Precision on the results of reference

Analytical

2.4 References bibliographical

1) IPSI: day of Phi2AS study on the nonlinear behaviors of materials of March 30th, 2000

3 Modeling A

3.1 Characteristics of modeling

Modeling C_PLAN. Behaviour with linear kinematic work hardening is modelled in four ways:

- maybe using the behavior VMIS_CINE_LINE, while taking:
  D_SIGM_EPSI = $E.C(T)/(E+C(T))$ with $C(T)=(1000+2990.\mathrm{T})$
- maybe using the behavior VMIS_ECMI_LINE, while taking:
  D_SIGM_EPSI = $E.C(T)/(E+C(T))$ and the constant of Prager $PRAG=2/3\,C(T)$
- maybe using the behavior VMIS_CIN1_CHAB, by keeping only linear kinematic work hardening: It is enough to take then: $R_0=R_I=SIGY$, $b=0$, $C_I=C(T)$, $G_0=0$
- maybe using the behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:
  $R_0=R_I=SIGY$, $b=0$, $C1_I=C2_I=C(T)/2$, $G1_0=G2_0=0$

Temporal discretization: 1 pas de time enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ and 40 pas de time enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

3.2 Characteristics of the grid

The grid comprises a mesh QUAD4

3.3 Sizes tested and results

Behavior	Moment	Deformation and forced	Reference	*Aster*	% difference
VMIS_CINE_LINE	1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210	210	0
	1	SIZZ	0.	5.e-7	5.e-7
VMIS_ECMI_LINE	T1 = 1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210	210	0
	1	SIZZ	0.	0.	0.
VMIS_CIN1_CHAB	T1 = 1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210	210	0
	1	SIZZ	0.	5.e-7	5.e-7
VMIS_CIN2_CHAB	T1 = 1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210	210	0
	1	SIZZ	0.	2.8.e-5	2.8.e-5

4 Modeling B

4.1 Characteristics of modeling

Modeling DKT. Thickness unit (to find the same reference solutions as the case C_PLAN). Behaviour with linear kinematic work hardening is modelled in four ways:

- maybe using the behavior VMIS_CINE_LINE, while taking:
  D_SIGM_EPSI = $E.C(T)/(E+C(T))$ with $C(T)=(1000+2990.\mathrm{T})$
- maybe using the behavior VMIS_ECMI_LINE, while taking:
  D_SIGM_EPSI = $E.C(T)/(E+C(T))$ and the constant of Prager $PRAG=2/3\,C(T)$
- maybe using the behavior VMIS_CIN1_CHAB, by keeping only linear kinematic work hardening: It is enough to take then: $R_0=R_I=SIGY$, $b=0$, $C_I=C(T)$, $G_0=0$
- maybe using the behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:
  $R_0=R_I=SIGY$, $b=0$, $C1_I=C2_I=C(T)/2$, $G1_0=G2_0=0$

Temporal discretization: 1 pas de time enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ and 40 pas de time enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

4.2 Characteristics of the grid

The grid comprises a mesh QUAD4 and two meshs TRIA3

4.3 Sizes tested and results

Behavior	Moment	Displacement and effort	Reference	*Aster*	% difference
VMIS_CINE_LINE	1	NYY	210	210	0
	1	DY	$1{,}105\ 10^{-2}$	$1{,}105\ 10^{-2}$	0
	1.1	DY	$1{,}115\ 10^{-2}$	$1{,}115\ 10^{-2}$	0
	2	DY	$2.85\ \ 10^{--3}$	$2.85\ \ 10^{-3}$	0
VMIS_ECMI_LINE	1	NYY	210	210	0
	1	DY	$1{,}105\ 10^{-2}$	$1{,}105\ 10^{-2}$	0
	1.1	DY	$1{,}115\ 10^{-2}$	$1{,}115\ 10^{-2}$	0
	2	DY	$2.85\ \ 10^{-3}$	$2.85\ \ 10^{-3}$	0
VMIS_CIN1_CHAB	1	NYY	210	210	0
	1	DY	$1{,}105\ 10^{-2}$	$1{,}105\ 10^{-2}$	0
	1.1	DY	$1{,}115\ 10^{-2}$	$1{,}115\ 10^{-2}$	0
	2	DY	$2.85\ \ 10^{-3}$	$2.85\ \ 10^{-3}$	0
VMIS_CIN2_CHAB	1	NYY	210	210	0
	1	DY	$1{,}105\ 10^{-2}$	$1{,}105\ 10^{-2}$	0
	1.1	DY	$1{,}115\ 10^{-2}$	$1{,}115\ 10^{-2}$	0
	2	DY	$2.85\ \ 10^{-3}$	$2.85\ \ 10^{-3}$	0

5 Modeling C

5.1 Characteristics of modeling

Modeling COQUE_3D. Thickness unit (to find the same reference solutions as the case C_PLAN). Behaviour with linear kinematic work hardening is modelled in four ways:

- maybe using the behavior VMIS_CINE_LINE, while taking:
  D_SIGM_EPSI = $E.C(T)/(E+C(T))$ with $C(T)=(1000+2990.\mathrm{T})$
- maybe using the behavior VMIS_ECMI_LINE, while taking:
  D_SIGM_EPSI = E.C (T)/(E+C (T)) and the constant of Prager $PRAG=2/3\,C(T)$
- maybe using the behavior VMIS_CIN1_CHAB, by keeping only linear kinematic work hardening: It is enough to take then: $R_0=R_I=SIGY$, $b=0$, $C_I=C(T)$, $G_0=0$
- maybe using the behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:
  $R_0=R_I=SIGY$, $b=0$, $C1_I=C2_I=C(T)/2$, $G1_0=G2_0=0$

Temporal discretization: 1 pas de time enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ and 40 pas de time enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

5.2 Characteristics of the grid

The grid comprises a mesh QUAD8

5.3 Sizes tested and results

Behavior	Moment	Displacement and effort	Reference	*Aster*	% difference
VMIS_CINE_LINE	1	NYY	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0
VMIS_ECMI_LINE	1	NYY	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0
VMIS_CIN1_CHAB	1	NYY	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0
VMIS_CIN2_CHAB	1	NYY	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0

6 Modeling D

6.1 Characteristics of modeling

Modeling PIPE. Section unit (to find the same reference solutions as the case C_PLAN). Behaviour with linear kinematic work hardening is modelled in four ways:

- maybe using the behavior VMIS_CINE_LINE, while taking: D_SIGM_EPSI = $E.C(T)/(E+C(T))$ with $C(T)=(1000+2990.\mathrm{T})$
- maybe using the behavior VMIS_ECMI_LINE, while taking: D_SIGM_EPSI = $E.C(T)/(E+C(T))$ and the constant of Prager $PRAG=2/3\,C(T)$
- maybe using the behavior VMIS_CIN1_CHAB, by keeping only linear kinematic work hardening: It is enough to take then: $R_0=R_I=SIGY$, $b=0$, $C_I=C(T)$, $G_0=0$
- maybe using the behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then: $R_0=R_I=SIGY$, $b=0$, $C1_I=C2_I=C(T)/2$, $G1_0=G2_0=0$

Temporal discretization: 1 pas de time enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ and 40 pas de time enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

6.2 Characteristics of the grid

The grid comprises a mesh SEG3

6.3 Sizes tested and results

Behavior	Moment	Displacement and forced	Reference	Aster	% difference
VMIS_CINE_LINE	1	SIXX	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0
VMIS_ECMI_LINE	1	SIXX	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0
VMIS_CIN1_CHAB	1	SIXX	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0
VMIS_CIN2_CHAB	1	SIXX	210	210	0
	1	DY	1,105 10^{-2}	1,105 10^{-2}	0
	1.1	DY	1,115 10^{-2}	1,115 10^{-2}	0
	2	DY	2.85 10^{-3}	2.85 10^{-3}	0

Other results with the behavior VMIS_ECMI_LINE:

Moment	Mesh/Not/Under-point	Field Component	Type of reference	Value of reference	Tolerance (%)
1,975	M1/1/1	EPSI/EPXX	'NON_REGRESSION'	-	-
1,975	M1/1/1	EPSI/EPYY	'NON_REGRESSION'	-	-
1,975	M1/1/1	EPVC/EPTHER_L	'ANALYTICAL'	9.75E-4	0.1
1,975	M1/1/1	EPME/EPXX	'NON_REGRESSION'	-	-
1,975	M1/1/1	EPME/EPYY	'NON_REGRESSION'	-	-
1,975	M1/1/1	SIEF/SIXX	'NON_REGRESSION'	-	-
1,975	M1/1/1	SIEF/SIYY	'NON_REGRESSION'	-	-
1,975	M1/1/1	EPSP/EPXX	'NON_REGRESSION'	-	-
1,975	M1/1/1	EPSP/EPYY	'NON_REGRESSION'	-	-

7 Summary of the results

This test makes it possible to highlight the good taking into account of the plane constraints for the elastoplastic behaviors VMIS_ISOT_TRAC, VMIS_CINE_LINE, VMIS_CIN1_CHAB, and VMIS_CIN2_CHAB.

The results are identical to the analytical solution. Constraints $SIZZ$ who must be worthless if the assumption of the plane constraints is checked are it indeed with convergence of the iterations of Newton with a good precision ($SIZZ=2.8.\mathrm{E}-5\,MPa$ to the maximum, in comparison with $SIYY=210\,MPa$), and this for a time CPU and a nearby iteration count.