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## HSNV122 - Thermoplasticity and metallurgy in great deformations in simple traction

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### Summary:

One treats the determination of the mechanical evolution of a cylindrical bar subjected to known and uniform evolutions thermal and metallurgical (the metallurgical transformation is of bainitic type) and to a mechanical loading of traction.

The relation of behavior is a model of plasticity in great deformations (order `STAT_NON_LINE`, keyword `DEFORMATION: 'SIMO_MIEHE'`) with linear isotropic work hardening and plasticity of transformation.

The yield stress and the slope of the traction diagram depend on the temperature and the metallurgical composition. The dilation coefficient depends on the metallurgical composition.

The bar is modelled by axisymmetric elements.

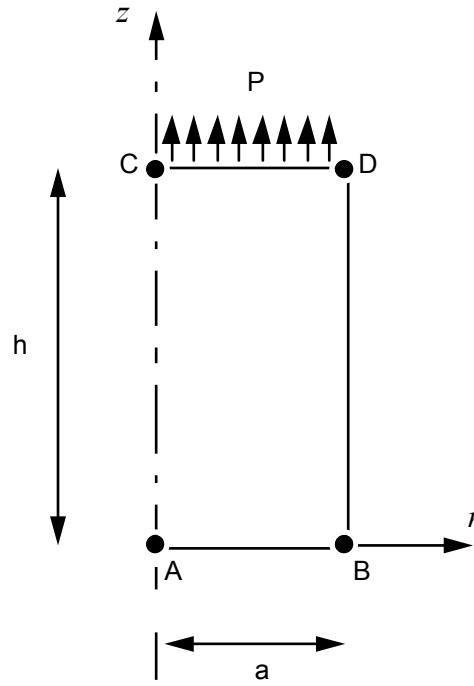
The mechanical loading applied is a following pressure.

This case test is identical to the case test HSNV101 (modeling B, [V7.22.101]) in the direction where it acts of same material, the same loading and the same thermal and metallurgical evolutions but in a version in great deformations.

## 1 Problem of reference

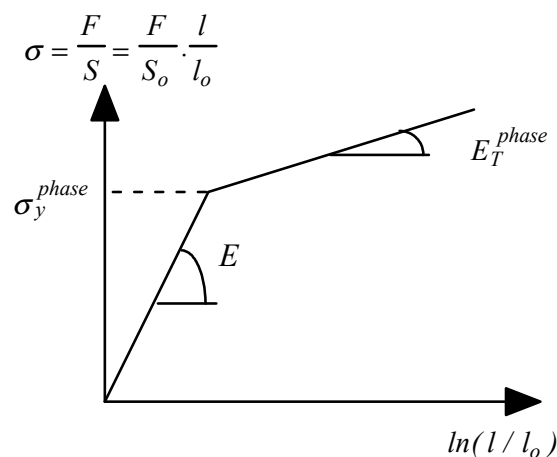
### 1.1 Geometry

Rayon :  $a = 0.05$  m  
Hauteur :  $h = 0.2$  m



### 1.2 Properties of material

The material obeys a law of behavior in great deformations with linear isotropic work hardening and plasticity of transformation. For each metallurgical phase, the slope of work hardening is given in the plan deformation logarithmic curve - rational constraint.



$l_0$  and  $l$  are, respectively, the initial length and the current length of the useful part of the test-tube.

$S_0$  and  $S$  are, respectively, surfaces initial and current.

$$\begin{aligned}
 C_p &= 2000000. \text{ J m}^{-3} \text{ } ^\circ\text{C}^{-1} & \lambda &= 9999.9 \text{ W m}^{-1} \text{ } ^\circ\text{C}^{-1} \\
 E &= 200000. \text{ } 10^6 \text{ Pa} & \sigma_y^{aust} &= 400. \text{ } 10^6 \text{ Pa} + 0.5 (T - T^o) \text{ } 10^6 \text{ Pa} \\
 \nu &= 0.3 & \sigma_y^{fbm} &= 530. \text{ } 10^6 \text{ Pa} + 0.5 (T - T^o) \text{ } 10^6 \text{ Pa} \\
 \alpha_{fbm} &= 15. \text{ } 10^{-6} \text{ } ^\circ\text{C}^{-1} & h_{aust} &= 1250. \text{ } 10^6 \text{ Pa} - 5. (T - T^o) \text{ } 10^6 \text{ Pa} \\
 \alpha_{aust} &= 23.5 \text{ } 10^{-6} \text{ } ^\circ\text{C}^{-1} & h_{fbm} &= -50. \text{ } 10^6 \text{ Pa} - 5. (T - T^o) \text{ } 10^6 \text{ Pa} \\
 \varepsilon_{fbm}^{ref} &= 2.52 \text{ } 10^{-3} & K_f &= 0. \text{ Pa}^{-1} \\
 T^{ref} &= 900 \text{ } ^\circ\text{C} & K_b = K_M &= 10^{-10} \text{ Pa}^{-1} \\
 & & F'_{fbm} &= 2. (1 - Z_{fbm})
 \end{aligned}$$

with

$C_p$	=	heat-storage capacity
$\lambda$	=	thermal conductivity
$E$	=	YOUNG modulus
$\nu$	=	Poisson's ratio
$*_{aust}$	=	characteristics relating to the austenitic phase
$*_{fbm}$	=	characteristics relating to the phases ferritic, bainitic and martensitic
$\alpha$	=	thermal dilation coefficient
$\varepsilon_{fbm}^{ref}$	=	deformation of the phases ferritic, bainitic and martensitic at the temperature of reference, austenite being regarded as not deformed at this temperature
$\sigma_y$	=	yield stress
$h$	=	$\frac{EE_T}{E - E_T}$
$K$	=	coefficient relating to the plasticity of transformation
$F$	=	function relating to the plasticity of transformation

The TRC used makes it possible to model a metallurgical evolution of bainitic type, on all the structure, of the form:

$$Z_{fbm} = \begin{cases} 0. & \text{si } t \leq \tau_1 & \tau_1 = 60\text{s} \\ \frac{t - \tau_1}{\tau_2 - \tau_1} & \text{si } \tau_1 \leq t < \tau_2 & \tau_2 = 112\text{s} \\ 1. & \text{si } t \geq \tau_2 \end{cases}$$

## 1.3 Boundary conditions and loadings

- $u_z=0$  on the face  $AB$  (condition of symmetry).
- traction imposed (following pressure) on the face  $CD$  :

$$p(t) = \begin{cases} p_o t & \text{pour } t \leq \tau_1 & p_o = 6 \text{ } 10^6 \text{ Pa} \\ 360 \text{ } 10^6 \text{ Pa} & \text{pour } t \geq \tau_1 & \tau_1 = 60\text{s} \end{cases}$$

**Note:**

*In great deformations, it is essential to use the following pressure to take account of current surface and not of initial surface (before deformation).*

- $T = T^o + \mu t$ ,  $\mu = -5 \text{ } ^\circ\text{C.s}^{-1}$  on all the structure.

## 1.4 Initial conditions

$$T^o = 900^\circ\text{C} = T^{ref}$$

## 2 Reference solution

### 2.1 Calculation of the reference solution (cf feeding-bottle [1] and [3])

For a tensile test according to the direction  $x$ , tensors of Kirchhoff  $\tau$  and of Cauchy  $\sigma$  are form:

$$\tau = \begin{bmatrix} \tau & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \sigma = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ with } \tau = J\sigma$$

Variation of volume  $J$  is given by the resolution of

$$J^3 - (3\varepsilon^{th} + \frac{2\sigma}{3K})J^2 - J - 3\varepsilon^{th} = 0$$

where  $\varepsilon^{th}$  is the thermal deformation. This one applies to an austenitic transformation – bainitic:

$$\varepsilon^{th} = Z_{aust} \alpha_{aust} (T - T^{ref}) + Z_b \left[ \alpha_{fbm} (T - T^{ref}) + \varepsilon_{fbm}^{ref} \right]$$

#### Note:

The coefficient  $K$  is the module of compression (not to be confused with the coefficients  $K_{phase}$  relating to the law of plasticity of transformation)

In plastic load, for an isotropic work hardening  $R$  linear, such as:

$$R = (Z_{aust} h_{aust} + Z_b h_{fbm}) p$$

cumulated plastic deformation  $p$  is worth

$$p = \frac{J\sigma - \sigma_y}{Z_{aust} h_{aust} + Z_b h_{fbm}}$$

with

$$\sigma_y = Z_{aust} \sigma_y^{aust} + Z_b \sigma_y^{fbm}$$

Tensor gradients of the transformation  $\mathbf{F}$  and  $\bar{\mathbf{F}}$  and the tensor of plastic deformations  $\mathbf{G}^P$  are form:

$$\mathbf{F} = \begin{pmatrix} F & 0 & 0 \\ 0 & F_{yy} & 0 \\ 0 & 0 & F_{yy} \end{pmatrix} \quad \text{et } J = \det \mathbf{F} = FF_{yy}^2 \Rightarrow F_{yy} = \sqrt{J/F}$$

$$\bar{\mathbf{F}} = J^{-1/3} \mathbf{F} = \begin{pmatrix} \bar{F} & 0 & 0 \\ 0 & \bar{F}_{yy} & 0 \\ 0 & 0 & \bar{F}_{yy} \end{pmatrix} \quad \text{et } \det \bar{\mathbf{F}} = 1 \Rightarrow \begin{cases} \bar{F} = J^{-1/3} F \\ \bar{F}_{yy} = \bar{F}^{-1/2} \end{cases}$$

$$\mathbf{G}^P = \begin{pmatrix} G^P & 0 & 0 \\ 0 & G_{yy}^P & 0 \\ 0 & 0 & G_{yy}^P \end{pmatrix} \quad \text{et } \det \mathbf{G}^P = 1 \Rightarrow G_{yy}^P = (G^P)^{-1/2}$$

The law of evolution of the plastic deformation  $G^P$  is written:

$$\dot{G}^P / G^P = -2\dot{p} - 4\tau K_b (1 - Z_b) \langle \dot{Z}_b \rangle$$

- For  $0s \leq t \leq 60s$ , one has  $\dot{Z}_b = 0$ . There is no plasticity of transformation. One obtains then:

$$G^P = e^{-2p}$$

- For  $60s \leq t \leq 176s$ , one has  $\sigma = \text{constante}$ .

To integrate the law of evolution of the plastic deformation, it should be supposed that the constraint of Kirchhoff  $\tau$  vary very little, i.e. variation of volume  $J$  is very small. Under this assumption, one obtains

$$G^P = e^{-2p} e^{-4\tau K_b (Z_b - Z_b^2/2)}$$

The component  $\bar{F}$  gradient of the transformation is given by the resolution of:

$$\bar{F}^3 - \frac{\tau}{\mu G^P} \bar{F} - \frac{1}{(G^P)^{3/2}} = 0$$

Lastly, the field of displacement  $u$  (in the initial configuration) is form  $\mathbf{u} = u_x \mathbf{X} + u_y \mathbf{Y} + u_z \mathbf{Z}$ . The components are given by:

$$\begin{aligned} u_x &= (F - 1) X \\ u_y &= (\sqrt{J/F} - 1) Y \\ u_z &= (\sqrt{J/F} - 1) Z \end{aligned}$$

## 2.2 Notice

In the case test HSNV101 (modeling B), the coefficients of material were selected in such manner not to have classical plasticity  $\dot{p} = 0$  during the metallurgical transformation which takes place between the moments 60 and 122s . Indeed if one writes the criterion of load-discharge in this time interval, one obtains

$$f = \sigma - 2750 p - 250 \text{ with } \sigma = 360 \text{ MPa}$$

who cancels himself only for only one value of the cumulated plastic deformation  $p$  .

For the law of behavior written in great deformations, the criterion of load-discharge is written between these two moments

$$f = J(t)\sigma - 2750 p - 250 \text{ with } \sigma = 360 \text{ MPa}$$

In this case, as long as the variable  $J$  remain lower than the value obtained at time  $t = 60\text{s}$  , one will have  $\dot{p} = 0$  . However the value of  $J$  is function only of the value of the thermal deformation (the constraint  $\sigma$  is constant and the coefficient  $K$  is independent of the metallurgical phases and temperature).

In this time interval, thermal deformation  $\varepsilon^{th}$  is given by the following equation:

$$\varepsilon^{th} = 8.173 \cdot 10^{-7} t^2 - 1.1807 \cdot 10^{-4} t - 2.90763 \cdot 10^{-3}$$

One traces below the thermal deformation as well as the variation of volume  $J$  , solution of the equation of the 3<sup>ème</sup> degree, according to time.

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### Thermal deformation according to time

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### Variation of volume $J$ according to time

It is noted that the variable  $J$  decrease and increases same manner as the thermal deformation. In this case, to know the moment from which the variable  $J$  is higher than the value obtained at time 60s , it is enough to know the moment for which the thermal deformation is identical to that obtained at time  $t=60s$  . One finds by the resolution of the equation above  $t=84.46s$  .

## 2.3 Uncertainty on the solution

The solution is analytical. Two mistakes are made on this solution. The first door on the calculation of the bainitic proportion of phase created. Preliminary metallurgical calculation does not restore exactly the equation of [§1.2] giving  $Z_{fbm}$  according to time, this is why the results of reference presented below are calculated with the bainitic proportion of phase calculated by *Code\_Aster*.

The second error is the assumption made on the constraint of Kirchhoff  $\tau$  who is not constant on the time interval understood enters 60 and 176s . This will impact the calculation of displacement  $u_x$  and of the plastic deformation  $G^P$  .

## 2.4 Results of reference

One will adopt like results of reference displacement in the direction of the loading of traction, the constraint of Cauchy  $\sigma$  , the Boolean indicator of plasticity  $\chi$  and cumulated plastic deformation  $P$  . The various moments of calculations are  $t=47, 48, 60, 83, 84, 85$  and 176s . For the calculation of displacement, the initial length of the bar in the direction of loading is of 0.2m .

In all the cases, one has

- $3K = 500\,000\text{ MPa}$  (module of compression)       $\mu = 76923.077\text{ MPa}$

**At time**  $t=47\text{s}$ , one has  $Z_b = 0$ ,  $T = 665^\circ\text{C}$ ,  $\sigma = 282\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -5.5225 \cdot 10^{-3} & J &= 0.983855 & \tau &= 277.45\text{ MPa} \\ \sigma_y &= 282.5\text{ MPa} & p &= 0 & \chi &= 0 \\ G^p &= 1 & \bar{F} &= 1.0012 & u &= -8.4347 \cdot 10^{-4}\text{ m}\end{aligned}$$

**At time**  $t=48\text{s}$ , one has  $Z_b = 0$ ,  $T = 660^\circ\text{C}$ ,  $\sigma = 288\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -5.64 \cdot 10^{-3} & J &= 0.983508 & \tau &= 283.25\text{ MPa} \\ \sigma_y &= 280.\text{ MPa} & p &= 1.327 \cdot 10^{-3} & \chi &= 1 \\ G^p &= 0.997 & \bar{F} &= 1.00256 & u &= -5.9639 \cdot 10^{-4}\text{ m}\end{aligned}$$

**At time**  $t=60\text{s}$ , one has  $Z_b = 0$ ,  $T = 600^\circ\text{C}$ ,  $\sigma = 360\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -7.05 \cdot 10^{-3} & J &= 0.979337 & \tau &= 352.56\text{ MPa} \\ \sigma_y &= 250.\text{ MPa} & p &= 3.7295 \cdot 10^{-2} & \chi &= 1 \\ G^p &= 0.9281 & \bar{F} &= 1.03959 & u &= 6.47595 \cdot 10^{-3}\text{ m}\end{aligned}$$

**At time**  $t=83\text{s}$ , one has  $Z_b = 0.442138$ ,  $T = 485^\circ\text{C}$ ,  $\sigma = 360\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -7.07867 \cdot 10^{-3} & J &= 0.979249 & \tau &= 352.53\text{ MPa} \\ \sigma_y &= 249.978\text{ MPa} & p &= 3.7295 \cdot 10^{-2} & \chi &= 0 \\ G^p &= 0.8841277 & \bar{F} &= 1.06514 & u &= 1.15441 \cdot 10^{-2}\text{ m}\end{aligned}$$

**At time**  $t=84\text{ S}$ , one has  $Z_b = 0.461361$ ,  $T = 480^\circ\text{C}$ ,  $\sigma = 360\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -7.06031 \cdot 10^{-3} & J &= 0.979305 & \tau &= 352.55\text{ MPa} \\ \sigma_y &= 249.977\text{ MPa} & p &= 3.7296 \cdot 10^{-2} & \chi &= 1 \\ G^p &= 0.8828104 & \bar{F} &= 1.06593 & u &= 1.17051 \cdot 10^{-2}\end{aligned}$$

**At time**  $t=85\text{s}$ , one has  $Z_b = 0.480584$ ,  $T = 475^\circ\text{C}$ ,  $\sigma = 360\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -7.04032 \cdot 10^{-3} & J &= 0.979367 & \tau &= 352.57\text{ MPa} \\ \sigma_y &= 249.976\text{ MPa} & p &= 3.73044 \cdot 10^{-2} & \chi &= 1 \\ G^p &= 0.8815276 & \bar{F} &= 1.06671 & u &= 1.18644 \cdot 10^{-2}\end{aligned}$$

**At time**  $t=176\text{s}$ , one has  $Z_b = 1$ ,  $T = 20^\circ\text{C}$ ,  $\sigma = 360\text{ MPa}$

$$\begin{aligned}\varepsilon^{th} &= -1.068 \cdot 10^{-2} & J &= 0.968132 & \tau &= 348.527\text{ MPa} \\ \sigma_y &= 90.\text{ MPa} & p &= 5.9432 \cdot 10^{-2} & \chi &= 1 \\ G^p &= 0.82814 & \bar{F} &= 1.10053 & u &= 1.7743 \cdot 10^{-2}\text{ m}\end{aligned}$$



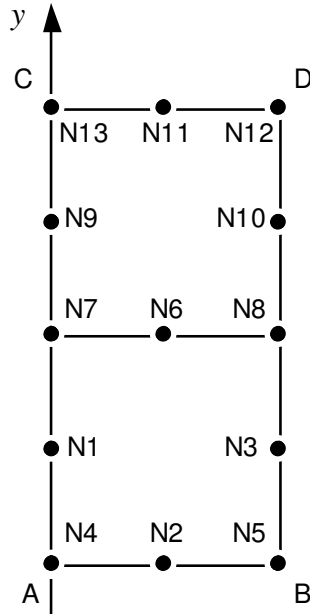
## 2.5 Bibliographical references

One will be able to refer to:

- 1) V. CANO, E. LORENTZ: Introduction into *Code\_Aster* of a model of behavior in great deformations elastoplastic with isotropic work hardening – Note interns EDF DER HI - 74/98/006/0
- 2) A.M. DONORE, F. WAECKEL: Influence of structure transformations in the elastoplastic laws of behavior Notes HI-74/93/024
- 3) F. WAECKEL, V. CANO: Law of behavior great deformations élasto (visco) plastic with metallurgical transformations [R4.04.03]

## 3 Modeling A

### 3.1 Characteristics of modeling



$A = N4$ ,  $B = N5$ ,  $C = N13$ ,  $D = N12$ .

Load: the full number of increments is of 102 (4 increments of 0 with 46s, 2 increments of 46 with 48s, 6 increments of 48 with 60s, 26 of 60 with 112s, 4 of 112 with 116s and 60 increments until 176s). Convergence is carried out if the residue (RESI\_GLOB\_RELA) is lower or equal to  $10^{-6}$ .

### 3.2 Characteristics of the grid

Many nodes: 13  
Many meshes and types: 2 meshes QUAD8, 6 meshes SEG3

### 3.3 Values tested

Identification	Reference
$t=47$ Displacement $DY$ (N13)	$- 8.4347 \cdot 10^{-4}$ m
$t=47$ Variable $p$ VARI (MI, PG1)	0.
$t=47$ $\chi$ VARI (MI, PG1)	0
$t=47$ Constraint SIGYY (MI, PG1)	$282. \cdot 10^{-6}$ Pa
$t=48$ Displacement $DY$ (N13)	$- 5.9639 \cdot 10^{-4}$ m
$t=48$ Variable $p$ VARI (MI, PG1)	$1.3260 \cdot 10^{-3}$
$t=48$ $\chi$ VARI (MI, PG1)	1
$t=48$ Constraint SIGYY (MI, PG1)	$288. \cdot 10^6$ Pa
$t=60$ Displacement $DY$ (N13)	$6,476 \cdot 10^{-3}$ m
$t=60$ Variable $p$ VARI (MI, PG1)	$3.7295 \cdot 10^{-2}$
$t=60$ $\chi$ VARI (MI, PG1)	1
$t=60$ Constraint SIGYY (MI, PG1)	$360. \cdot 10^6$ Pa

$t=83$	Displacement $DY$ (NI3)	$1.1544 \cdot 10^{-2}$ m
$t=83$	Variable $p$ $VARI$ (MI, PGI)	$3.7295 \cdot 10^{-2}$
$t=83$	$\chi$ $VARI$ (MI, PGI)	0
$t=83$	Constraint $SIGYY$ (M1, PG1)	$360. \cdot 10^6$ Pa
$t=84$	Displacement $DY$ (NI3)	$1.1705 \cdot 10^{-2}$ m
$t=84$	Variable $p$ $VARI$ (MI, PGI)	$3.7296 \cdot 10^{-2}$
$t=84$	$\chi$ $VARI$ (MI, PGI)	1
$t=84$	Constraint $SIGYY$ (MI, PGI)	$360. \cdot 10^6$ Pa
$t=85$	Displacement $DY$ (NI3)	$1.1864 \cdot 10^{-2}$ m
$t=85$	Variable $p$ $VARI$ (MI, PGI)	$3.7304 \cdot 10^{-2}$
$t=85$	$\chi$ $VARI$ (MI, PGI)	1
$t=85$	Constraint $SIGYY$ (M1, PG1)	$360. \cdot 10^6$ Pa
$t=176$	Displacement $DY$ (NI3)	$1.7743 \cdot 10^{-2}$ m
$t=176$	Variable $p$ $VARI$ (MI, PGI)	$5,943 \cdot 10^{-2}$
$t=176$	$\chi$ $VARI$ (MI, PGI)	1
$t=176$	Constraint $SIGYY$ (MI, PGI)	$360. \cdot 10^6$ Pa

## 4 Summary of the results

Results found with *Code\_Aster* are very satisfactory with percentages of error lower than 0.9%, knowing that the analytical solution of reference makes the dead end on certain aspects which into account precisely the solution takes of *Code\_Aster*. This can explain the differences observed.