HSNV103 - Thermoplasticity and metallurgy in plane deformations

Summary:

One treats the determination of the mechanical evolution of a right-angled parallépipède in plane deformations subjected to evolutions thermics $T(t)$ and metallurgical $Z(t)$ known and uniform (the metallurgical transformation is of bainitic type).

The elements used are two-dimensional elements in plane deformations and the relation of behavior is the plasticity of von Mises with linear isotropic work hardening (for modeling B, one also takes account of the plasticity of transformation).

The yield stress and the slope of the traction diagram depend on the temperature and the metallurgical composition.

The dilation coefficient α depends on the metallurgical composition.

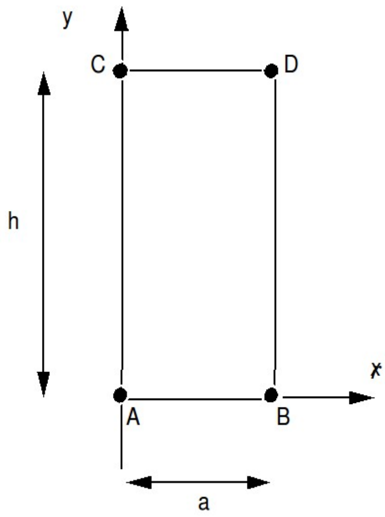
For modeling A (without plasticity of transformation), the reference solution is obtained by the analytical resolution of the problem. For modeling B (with plasticity of transformation), the reference solution is obtained by the digital resolution of the problem by using axisymmetric elements for which one imposes the condition of plane deformations.

Results provided by *Code_Aster* are very satisfactory with errors lower than 0,5%.

1 Problem of reference

1.1 Geometry

Width: $a=0.05\,m$
Height: $h=0.2\,m$



1.2 Properties of materials

Following convention is adopted in order to distinguish the parameters from the hot phase (austenitic) parameters of the cold phases (ferrito-perlitic, bainitic and martensitic):

$*^{aust}$ = characteristics relating to the austenitic phase
$*^{fbm}$ = characteristics relating to the phases ferrito-perlitic, bainitic and martensitic

Metallurgical parameters:
TRC to model a metallurgical evolution of bainitic type, on all the structure, of the form:

$$Z_{fbm}=\begin{cases} 0. & si\ \ t\leq\tau_1 & \tau_1=60\,s \\ \dfrac{t-\tau_1}{\tau_2-\tau_1} & si\ \ \tau_1\leq t<\tau_2 & \tau_2=112\,s \\ 1. & si\ \ t\geq\tau_2 & \end{cases}$$

Thermal parameters:
Heat-storage capacity: $\rho C_p=2.10^6\,J.m^{-3}.°C^{-1}$
Conductivity: $\lambda=9999.9\,W.m^{-1}.°C^{-1}$

Thermomechanical parameters:

- Thermoelastic parameters:

  Young modulus $E=200000\,10^6\,Pa$
  Poisson's ratio $\nu=0.3$
  Dilation coefficients thermal

$$\alpha_{fbm}=15.10^{-6}\,°C^{-1}$$

$$\alpha_{aust}=23.5\,10^{-6}\,°C^{-1}$$

Temperature of definition of the dilation coefficient: $T_{ref}=900\,°C$

Thermal state of deformation of reference: $\Delta\varepsilon_{f\gamma}^{T_{ref}}=2.52\,10^{-3}$

Elastic limit:

$\sigma_y^{fbm}=\sigma_0^{fbm}+s^{fbm}(T-T^0)$ with $\sigma_0^{fbm}=530.10^6\,Pa$ and $s^{fbm}=0.5\,10^6\,Pa.\,°C^{-1}$

$\sigma_y^{aust}=\sigma_0^{aust}+s^{aust}(T-T^0)$ with $\sigma_0^{aust}=400.10^6\,Pa$ and $s^{aust}=0.5\,10^6\,Pa.\,°C^{-1}$

- Thermoplastic parameters (law with linear work hardening)
  Tangent modules: E_T^{fbm} and E_T^{aust} are selected such as:
  $H^{fbm}=H_0^{fbm}+\lambda^{fbm}(T-T^0)$ with $H_0^{fbm}=-50.10^6\,Pa$ and $\lambda^{fbm}=-5.10^6\,Pa.\,°C^{-1}$
  $H^{aust}=H_0^{aust}+\lambda^{aust}(T-T^0)$ with $H_0^{aust}=1250.10^6\,Pa$ and $\lambda^{aust}=-5.10^6\,Pa.\,°C^{-1}$
  It is pointed out that $H=\frac{EE_T}{E-E_T}$

- Parameters for the plasticity of transformation:

  Recall:

In the case of one metallurgical evolution of bainitic type, the model of the plasticity of transformation is the following: $\dot{\boldsymbol{\varepsilon}}^{pt}=\frac{3}{2}\tilde{\boldsymbol{\sigma}}\,k^{fbm}F'(Z_{fbm})\dot{Z}_{fbm}$

Parameters of the model: $k^{fbm}=1.10^{-10}\,Pa^{-1}$ and $F'(Z_{fbm})=2(1-Z_{fbm})$

1.3 Boundary conditions and loadings

- $u_Y=0$ on the side AB ; $u_X=0$ in A .
- $T=T^0+\mu t$, $\mu=-5\,°C.s^{-1}$ on all the structure.

  Notations: $T(\tau_1)=T_1$
  $T(\tau_2)=T_2$

- The loading on the structure is due with the phenomena of thermal and metallurgical dilation constrained in the direction z by the condition of plane deformations.

1.4 Initial conditions

$$T^0=900\,°C=T^{ref}$$

2 Reference solution (for modeling A)

2.1 The shape of the field solution

The stress field solution $\boldsymbol{\sigma}(t)$ is form:

$$\boldsymbol{\sigma}(t)=\sigma_o(t)\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

One from of deduced the following form from the tensor of the elastic strain:

$$\boldsymbol{\varepsilon}^{e}(t)=\frac{\sigma_o(t)}{E}\begin{pmatrix}-\nu & 0 & 0\\ 0 & -\nu & 0\\ 0 & 0 & 1\end{pmatrix}$$

Moreover, since $\boldsymbol{\sigma}(t)$ keep a constant direction, one a:

$$\boldsymbol{\varepsilon}^{p}(t)=\varepsilon_o^{p}(t)\begin{pmatrix}\frac{-1}{2} & 0 & 0\\ 0 & \frac{-1}{2} & 0\\ 0 & 0 & 1\end{pmatrix}\quad ;\quad \boldsymbol{\varepsilon}^{pt}(t)=\varepsilon_o^{pt}(t)\begin{pmatrix}\frac{-1}{2} & 0 & 0\\ 0 & \frac{-1}{2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

where $\boldsymbol{\varepsilon}^{p}$ and $\boldsymbol{\varepsilon}^{pt}$ are respectively the tensor of the plastic deformations and the tensor of deformations due to the metallurgical plasticity of transformation.

2.2 Method of calculating used for the reference solution

Before transformation, thermoelastic solution for $t<t_1$.

$$\begin{cases}\varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)=0\\ \varepsilon_{zz}^{th}(t)=\alpha_{aust}(T-T^{0})\\ \sigma_{zz}(t)=-E\,\varepsilon_{zz}^{th}(t)\end{cases}$$

The yield stress is reached for $t=t_1$ such as:

$$\sigma_{zz}(t_1)=-E\,\varepsilon_{zz}^{th}(t_1)=\sigma_y^{aust}(t_1)\quad\Leftrightarrow\quad T(t_1)-T^{0}=\frac{-\sigma_0^{aust}}{E\,\alpha_{aust}+s^{aust}}\simeq-76.92\,°C$$

$$\Rightarrow\quad t_1=\frac{T(t_1)-T^{0}}{\mu}\simeq 15.38\,s$$

Before transformation, thermoelastoplastic solution, $t_1\le t\le\tau_1$.

$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t)=0 \\ \varepsilon_{zz}^{th}(t)=\alpha_{aust}(T-T^0) \\ \varepsilon_{zz}^{p}(t)=\dfrac{-\sigma_y^{aust}(T)-E\,\alpha_{aust}(T-T^0)}{E+H^{aust}(T)} \\ \sigma_{zz}(t)=-E(\varepsilon_{zz}^{p}(t)+\varepsilon_{zz}^{th}(t)) \end{cases}$$

During the transformation, thermoelastoplastic solution with phase shift, $\tau_1 \le t < t_2$.

$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t)=0 \\ \varepsilon_{zz}^{th}(t)=Z_{aust}\alpha_{aust}(T-T^0)+Z_{fbm}\left(\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}}\right) \\ \varepsilon_{zz}^{p}(t)=\dfrac{-\sigma_y^{moy}(T)-E\,\varepsilon_{zz}^{th}(t)}{E+H^{moy}(T)} \\ \sigma_{zz}(t)=-E(\varepsilon_{zz}^{p}(t)+\varepsilon_{zz}^{th}(t)) \end{cases}$$

with $\sigma_y^{moy}=Z_{aust}\sigma_y^{aust}+Z_{fbm}\sigma_y^{fbm}$ and $H^{moy}=Z_{aust}H^{aust}+Z_{fbm}H^{fbm}$

There is a thermoelastoplastic solution as long as one remains in load, i.e. as long as $\dot{\varepsilon}^{th}<0$:

$$\dot{\varepsilon}^{th}<0 \quad\Leftrightarrow\quad T=\frac{(\alpha_{fbm}-\alpha_{aust})T^0-\Delta\varepsilon_{f\gamma}^{T_{ref}}+\alpha_{fbm}T_1-\alpha_{aust}T_2}{2(\alpha_{fbm}-\alpha_{aust})}>538.82\,°C$$
$$\Leftrightarrow\quad t<t_2=72.23\,s$$

For $t\ge t_2$, the solution is thermoelastic.

During the transformation, thermoelastic solution with phase shift, $t_2 \le t < \tau_2$.

$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t_2)=0 \\ \varepsilon_{zz}^{th}(t)=Z_{aust}\alpha_{aust}(T-T^0)+Z_{fbm}\left(\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}}\right) \\ \sigma_{zz}(t)=-E(\varepsilon_{zz}^{p}(t_2)+\varepsilon_{zz}^{th}(t)) \end{cases}$$

After the transformation, thermoelastic solution for $\tau_2 < t < t_3$.

$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t_2)=0 \\ \varepsilon_{zz}^{th}(t)=\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}} \\ \sigma_{zz}(t)=-E(\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t_2)) \end{cases}$$

The yield stress is reached for $t=t_3$ such as:

$$\sigma_{zz}(t_3)=-E(\varepsilon_{zz}^{th}(t_3)+\varepsilon_{zz}^{p}(t_2))=H^{fbm}\varepsilon_{zz}^{p}(t_2)+\sigma_y^{fbm}(t_3)$$

$$\Leftrightarrow \quad T(t_3)-T^0=-\frac{E(\Delta \varepsilon_{f\gamma}^{T_{ref}}+\varepsilon_{zz}^p(t_2))+H_0^{fbm}\varepsilon_{zz}^p(t_2)+\sigma_0^{fbm}}{E\alpha_{fbm}+s^{fbm}+\lambda^{fbm}\varepsilon_{zz}^p(t_2)}=-634{,}68°C$$

$$\Leftrightarrow \quad t_3=\frac{T(t_3)-T^0}{\mu}=126.94\,s$$

After the transformation, thermoelastoplastic solution for $t \geq t_3$.

$$\begin{cases}\varepsilon_{zz}(t)=\varepsilon_{zz}^e(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^p(t)=0\\ \varepsilon_{zz}^{th}(t)=\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}}\\ \varepsilon_{zz}^p(t)=\dfrac{-\sigma_y^{fbm}(T)-E(\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}})}{E+H^{fbm}(T)}\\ \sigma_{zz}(t)=-E(\varepsilon_{zz}^p(t)+\varepsilon_{zz}^{th}(t))\end{cases}$$

2.3 Results of reference

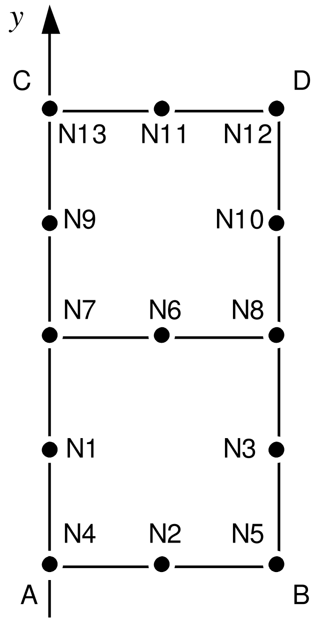
With $t=16\,s$:	χ	p	ε_{xx}	σ_{zz}	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}
With $t=60\,s$:	χ	p	ε_{xx}	σ_{zz}	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}
With $t=72\,s$:	χ	p					
With $t=112\,s$:	χ	p		σ_{zz}	ε_{xx}^{th}	ε_{xx}^{meca}	ε_{xx}^{plas}
With $t=176\,s$:	χ		ε_{xx}	σ_{zz}			

2.4 Bibliography

1) DONORE A.M. - WAECKEL F. - Influence of structure transformations in the elastoplastic laws of behavior Notes HI-74/93/024.

3 Modeling A

3.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

3.2 Characteristics of the grid

Many nodes: 13.

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3.

4 Results of modeling A

4.1 Values tested

One tests the structural parameters of data results:

Identification	Reference
INST for NUME_ORDRE= 176	176.0
ITER_GLOB for NUME_ORDRE=176	2

Identification	Reference	Test	Tolerance
ε_{xx} $t=16\,s$	-2.4599 10^{-3}	ANALYTICAL	0.1 %
χ $t=16\,s$	1	ANALYTICAL	0.1 %
σ $t=16\,s$	360.13 10^{6}	ANALYTICAL	0.1 %
p $t=16\,s$	7.9345 10^{-5}	ANALYTICAL	0.1 %
ε_{xx}^{th} $t=16\,s$	-1.88 10^{-3}	ANALYTICAL	0.1 %
ε_{xx}^{meca} $t=16\,s$	-5,799 10^{-4}	ANALYTICAL	0.1 %
ε_{xx}^{plas} $t=16\,s$	3.9672 10^{-5}	ANALYTICAL	0.1 %
ε_{xx} $t=60\,s$	-1.0309 10^{-2}	ANALYTICAL	0.1 %
p $t=60\,s$	5.7213 10^{-3}	ANALYTICAL	0.1 %
σ $t=60\,s$	265.73 10^{6}	ANALYTICAL	0.1 %
ε_{xx}^{th} $t=60\,s$	-7.05 10^{-3}	ANALYTICAL	0.1 %
ε_{xx}^{meca} $t=60\,s$	-3,259 10^{-3}	ANALYTICAL	0.1 %
ε_{xx}^{plas} $t=60\,s$	2.86065 10^{-3}	ANALYTICAL	0.1 %
p $t=72\,s$	5.8420 10^{-3}	ANALYTICAL	0.5 %
χ $t=112\,s$	0	ANALYTICAL	0.1 %
σ $t=112\,s$	12.82 10^{6}	ANALYTICAL	0.5 %
p $t=112\,s$	5.8421 10^{-3}	ANALYTICAL	0.5 %
ε_{xx}^{th} $t=112\,s$	-5.88 10^{-3}	ANALYTICAL	0.1 %
ε_{xx}^{meca} $t=112\,s$	-2.90182 10^{-3}	ANALYTICAL	1.0%
ε_{xx}^{plas} $t=112\,s$	-2.92105 10^{-3}	ANALYTICAL	0.5 %
ε_{xx} $t=176\,s$	– 1.5886 10^{-2}	ANALYTICAL	0.1 %
χ $t=176\,s$	1	ANALYTICAL	0.1 %
σ $t=176\,s$	133.55 10^{6}	ANALYTICAL	0.1 %

4.2 Remarks

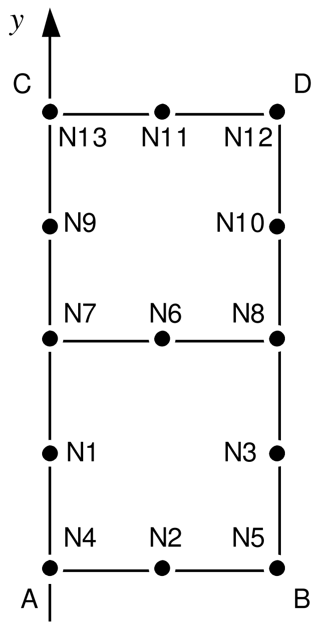
In this modeling:

$$\varepsilon_{zz}^{pt}(T\,,Z)=0$$

The error on the plastic deformation cumulated at 72 seconds comes by way of the mistake made on the digital description of the metallurgical transformation which is, at this moment, from approximately 0.5%.

5 Modeling B

5.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

5.2 Characteristics of the grid

Many nodes: 13.

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3.

6 Results of modeling B

6.1 Values tested

Identification	Reference	Test	Tolerance
ε_{yy} $t=60\,s$	– 1.0309 10^{-2}	AUTRE_ASTER	0.02 %
σ_{zz} $t=60\,s$	265.73 10^{6}	AUTRE_ASTER	0,002 %
p $t=60\,s$	5.7213 10^{-3}	AUTRE_ASTER	0.1 %
ε_{yy}^{th} $t=60\,s$	-7.05 10^{-3}	AUTRE_ASTER	0.1 %
ε_{yy}^{meca} $t=60\,s$	-3.2593 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy}^{plas} $t=60\,s$	-2.8607 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy} $t=89\,s$	– 1.0325 10^{-2}	AUTRE_ASTER	0.02 %
p $t=89\,s$	5.7213 10^{-3}	AUTRE_ASTER	0,001 %
σ_{zz} $t=89\,s$	– 13,545 10^{6}	AUTRE_ASTER	0.04 %
ε_{yy}^{th} $t=89\,s$	-6.8751 10^{-3}	AUTRE_ASTER	0.1 %
ε_{yy}^{meca} $t=89\,s$	-3.4511 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy}^{plas} $t=89\,s$	-3.4714 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy} $t=112\,s$	– 8.9197 10^{-3}	AUTRE_ASTER	0.02 %
σ_{zz} $t=112\,s$	101.39 10^{6}	AUTRE_ASTER	0.05 %
p $t=112\,s$	5.7213 10^{-3}	AUTRE_ASTER	0,001 %
ε_{yy}^{th} $t=112\,s$	-5.8800 10^{-3}	AUTRE_ASTER	0.1 %
ε_{yy}^{meca} $t=112\,s$	-3.0413 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy}^{plas} $t=112\,s$	-3.1934 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy} $t=176\,s$	– 1.5884 10^{-2}	AUTRE_ASTER	0.04%
p $t=176\,s$	9.3610 10^{-2}	AUTRE_ASTER	0,001 %
σ_{zz} $t=176\,s$	130.72 10^{6}	AUTRE_ASTER	0,001 %
ε_{yy}^{th} $t=176\,s$	-1,068 10^{-2}	AUTRE_ASTER	0.1 %
ε_{yy}^{meca} $t=176\,s$	-5.2093 10^{-3}	AUTRE_ASTER	0,002 %
ε_{yy}^{plas} $t=176\,s$	-5.0132 10^{-3}	AUTRE_ASTER	0,002 %

6.2 Remarks

In this modeling, one takes into account the term due to the plasticity of transformation:

$$\dot{\varepsilon}^{pt}(Z,T)\neq 0 \text{ when } \dot{Z}\neq 0$$

The reference solution is obtained by the digital resolution of the problem with axisymmetric elements for which one imposes the condition of plane deformations.

7 Summary of the results

Results found with *Code_Aster* are very satisfactory, with percentages of error lower than 0.5%.