HSNV102 - Thermo-metal-worker-plasticity coupled in simple traction

Summary:

One treats the determination of the mechanical evolution of a cylindrical bar subjected to evolutions thermics $T(t)$ and metallurgical $Z(t)$ known and uniform (the metallurgical transformation is of martensitic type).

The elements used are axisymmetric elements.
For modelings A and B, the relation of behavior is plasticity of Von Mises with isotropic work hardening. For modeling B, one also takes account of the plasticity of transformation.
In modeling C, a viscoplastic behaviour with linear isotropic work hardening is considered and the plasticity of transformation is not taken into account.

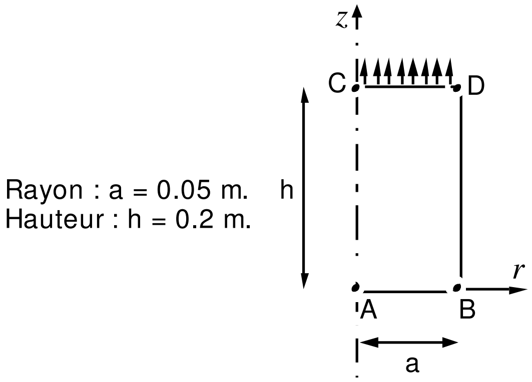
The yield stress and the slope of the traction diagram depend on the temperature and the metallurgical composition.

The dilation coefficient α depends on the metallurgical composition.

The metallurgical transformations take place with $\dot{\varepsilon}^p \neq 0$ (it is in the sense that the test **couples** the plasticity of transformation of classical plasticity).

1 Problem of reference

1.1 Geometry



1.2 Properties of materials

Following convention is adopted in order to distinguish the parameters from the hot phase (austenitic) parameters of the cold phases (ferrito-perlitic, bainitic and martensitic):

$*^{aust}$ = characteristics relating to the austenitic phase
$*^{fbm}$ = characteristics relating to the phases ferrito-perlitic, bainitic and martensitic

Metallurgical parameters:
TRC to model a metallurgical evolution of martensitic type of the form:

$$Z_{fbm}=\begin{cases} 0. & si \quad t\le\tau_1 & \tau_1=25\,s \\ 1-e^{\varphi\lambda(t-\tau_1)} & si \quad \tau_1\le t<\tau_2 & \tau_2=40\,s \\ 1. & si \quad t\ge\tau_2 & \end{cases}$$

with $\varphi=0.03$ and $\lambda=-10\,°C.s^{-1}$

Thermal parameters:
Heat-storage capacity: $\rho C_p=2.\,10^6\,J.m^{-3}.\,°C^{-1}$
Conductivity: $\lambda=9999.9\,W.m^{-1}.\,°C^{-1}$

Thermomechanical parameters:
- Thermoelastic parameters:

Young modulus $E=200000\,10^6\,Pa$
Poisson's ratio $\nu=0.3$
Dilation coefficients thermal
$\alpha_{fbm}=15.\,10^{-6}\,°C^{-1}$
$\alpha_{aust}=23.5\,10^{-6}\,°C^{-1}$
Temperature of definition of the dilation coefficient: $T_{ref}=900\,°C$
Thermal state of deformation of reference: $\Delta\varepsilon_{f\gamma}^{T_{ref}}=2.52\,10^{-3}$
Elastic limit:
$\sigma_y^{fbm}=\sigma_0^{fbm}+s^{fbm}(T-T^0)$ with $\sigma_0^{fbm}=50.\,10^6\,Pa$ and $s^{fbm}=1.\,10^6\,Pa.\,°C^{-1}$
$\sigma_y^{aust}=\sigma_0^{aust}+s^{aust}(T-T^0)$ with $\sigma_0^{aust}=50.\,10^6\,Pa$ and $s^{aust}=1.3\,10^6\,Pa.\,°C^{-1}$

- Thermoplastic parameters (law with linear work hardening)
  Tangent modules: E_T^{fbm} and E_T^{aust} are selected such as:
  $H^{fbm}=H_0^{fbm}+\lambda^{fbm}(T-T^0)$ with $H_0^{fbm}=0.Pa$ and $\lambda^{fbm}=-6.10^6 Pa.°C^{-1}$
  $H^{aust}=H_0^{aust}+\lambda^{aust}(T-T^0)$ with $H_0^{aust}=0.Pa$ and $\lambda^{aust}=-1.10^6 Pa.°C^{-1}$
  It is pointed out that $H=\dfrac{EE_T}{E-E_T}$

- Parameters for the plasticity of transformation:

  Recall:

In the case of one metallurgical evolution of martensitic type, the model of the plasticity of transformation is the following: $\dot{\varepsilon}^{pt}=\frac{3}{2}\tilde{\sigma}\, k^{fbm} F'(Z_{fbm})\dot{Z}_{fbm}$

  Parameters of the model: $k^{fbm}=1.10^{-10} Pa^{-1}$ and $F'(Z_{fbm})=2(1-Z_{fbm})$

Viscous parameters complementary to modeling C:
Two parameters η and C are taken worthless so that the viscoplastic model tends towards a plastic model independent of time.

1.3 Boundary conditions and loadings

- $u_Z=0$ on the side AB (condition of symmetry).
- traction imposed on the side CD, $p(t)=p_0 t$, $p_0=15.10^6\ Pa$.
- $T(t)=T^0+\mu t$, $\mu=-10°C.s^{-1}$ on all the structure.

1.4 Initial conditions

$T^0=900°C$.

2 Reference solution

2.1 Method of calculating used for the reference solution

Before transformation, thermoelastic solution for $t<\tau_1$ (not of metallurgical transformation $\dot{Z}=0$).

$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t) \\ \sigma_{zz}(t)=p_0 t \\ \varepsilon_{zz}^{e}(t)=\dfrac{\sigma_{zz}(t)}{E} \\ \varepsilon_{zz}^{th}(t)=\alpha_{aust}(T-T^0) \end{cases}$$

The yield stress is reached for:

$$\tau_1=\frac{\sigma_0^{aust}}{p_0-s^{aust}\mu}$$

During the transformation, solution thermo-metal-worker-élasto-plastic, for $\tau_1\le t\le\tau_2$, $\tau_2=40\ s$.

$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t)+\varepsilon_{zz}^{pt}(t) \\ \sigma_{zz}(t)=p_0 t \\ \varepsilon_{zz}^{e}(t)=\dfrac{\sigma_{zz}(t)}{E} \\ \varepsilon_{zz}^{th}(t)=Z_{aust}\alpha_{aust}(T-T^0)+Z_{fbm}\left(\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}}\right) \\ \varepsilon_{zz}^{p}=\dfrac{\sigma_{zz}(t)-(Z_{aust}\sigma_y^{aust}(T)+Z_{fbm}\sigma_y^{fbm}(T))}{Z_{aust}H^{aust}(T)+Z_{fbm}H^{fbm}(T)} \\ \varepsilon_{zz}^{pt}(t)=k^{fbm}\left(\sigma_{zz}(\tau_1)-\dfrac{p_0}{2\lambda\varphi}\right)-k^{fbm}\left(\sigma_{zz}(t)-\dfrac{p_0}{2\lambda\varphi}\right)(1-Z_{fbm})^2 \end{cases}$$

After transformation, thermoelastoplastic solution, for $t\ge\tau_2$

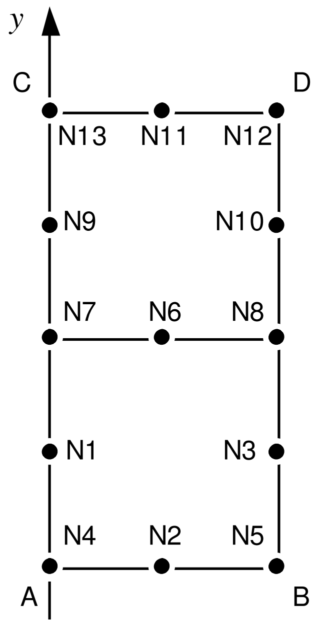
$$\begin{cases} \varepsilon_{zz}(t)=\varepsilon_{zz}^{e}(t)+\varepsilon_{zz}^{th}(t)+\varepsilon_{zz}^{p}(t)+\varepsilon_{zz}^{pt}(\tau_2) \\ \sigma_{zz}(t)=p_0 t \\ \varepsilon_{zz}^{e}(t)=\dfrac{\sigma_{zz}(t)}{E} \\ \varepsilon_{zz}^{th}(t)=\alpha_{fbm}(T-T^0)+\Delta\varepsilon_{f\gamma}^{T_{ref}} \\ \varepsilon_{zz}^{p}(t)=\dfrac{\sigma_{zz}(t)-(\sigma_0^{fbm}+s^{fbm}\mu t)}{H_0^{fbm}+\lambda^{fbm}\mu t} \end{cases}$$

2.2 Results of reference

σ_{zz} , ε_{zz} , ε_{zz}^{p} , ε_{zz}^{th} , ε_{zz}^{meca} , ε_{zz}^{plas} and χ with $24\ s$, $26\ s$, $40\ s$ and $90\ s$.
with:
ε^{meca} : mechanical deformations
ε^{plas} : plastic deformations (including the plasticity of transformation)

3 Modeling A

3.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

3.2 Characteristics of the grid

Many nodes: 13

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3

4 Results of modeling A

4.1 Values tested

One tests the structural parameters of data results:

Identification	Reference
`INST` for `NUME_ORDRE= 27`	90
`ITER_GLOB for NUME_ORDRE=27`	2

Identification	Reference	Test	Tolerance
ε_{zz}^{p} $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
χ $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
σ_{zz} $t=24\,s$	360. 10^{6}	ANALYTICAL	0.1 %
ε_{zz} $t=24\,s$	-0.00384	ANALYTICAL	0.1 %
ε_{zz}^{th} $t=24\,s$	-0.005640	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=24\,s$	0,018	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
ε_{zz}^{p} $t=26\,s$	0.0372	ANALYTICAL	0.4 %
χ $t=26\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=26\,s$	390. 10^{6}	ANALYTICAL	0.1 %
ε_{zz} $t=26\,s$	0.03428	ANALYTICAL	0.4 %
ε_{zz}^{th} $t=26\,s$	-0.004884	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=26\,s$	0.039164	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=26\,s$	0.0372	ANALYTICAL	0.1 %
ε_{zz}^{p} $t=40\,s$	0.0625	ANALYTICAL	0.04 %
χ $t=40\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=40\,s$	600. 10^{6}	ANALYTICAL	0.1 %
ε_{zz} $t=40\,s$	0.06198	ANALYTICAL	0.07 %
ε_{zz}^{th} $t=40\,s$	-0.003546	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=40\,s$	0.065526	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=40\,s$	0.0625	ANALYTICAL	0.1 %
ε_{zz}^{p} $t=90\,s$	0.0741	ANALYTICAL	0.08 %
χ $t=90\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=90\,s$	1350. 10^{6}	ANALYTICAL	0.1 %
ε_{zz} $t=90\,s$	0.069844	ANALYTICAL	0.03 %
ε_{zz}^{th} $t=90\,s$	-0,011	ANALYTICAL	0.4 %
ε_{zz}^{meca} $t=90\,s$	0.08085	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=90\,s$	0.0741	ANALYTICAL	0.1 %

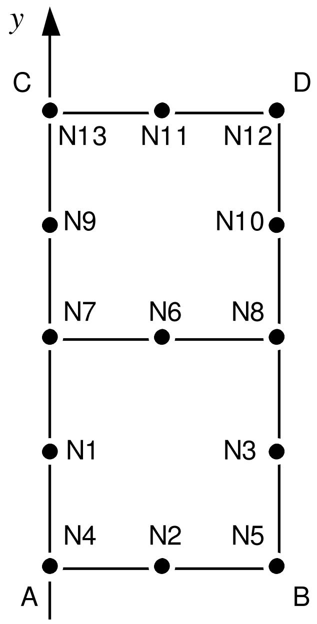
4.2 Remarks

In this modeling:

$$\varepsilon^{pt}(T, Z)=0$$

5 Modeling B

5.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

5.2 Characteristics of the grid

Many nodes: 13

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3

6 Results of modeling B

6.1 Values tested

Identification	Reference	Test	Tolerance
ε_{zz}^{p} $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
χ $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
σ_{zz} $t=24\,s$	$360.\ 10^{6}$	ANALYTICAL	0.1 %
ε_{zz} $t=24\,s$	$-3.84\ 10^{-3}$	ANALYTICAL	0.1 %
ε_{zz}^{th} $t=24\,s$	-0.005640	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=24\,s$	0,018	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
ε_{zz}^{p} $t=26\,s$	0.037217	ANALYTICAL	0.1 %
χ $t=26\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=26\,s$	$390.\ 10^{6}$	ANALYTICAL	0.1 %
ε_{zz} $t=26\,s$	0.051507	ANALYTICAL	1.1 %
ε_{zz}^{th} $t=26\,s$	-0.004884	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=26\,s$	0.05639	ANALYTICAL	1.0 %
ε_{zz}^{plas} $t=26\,s$	0.05444	ANALYTICAL	1.0 %
ε_{zz}^{p} $t=40\,s$	0.062523	ANALYTICAL	0.1 %
χ $t=40\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=40\,s$	$600.\ 10^{6}$	ANALYTICAL	0.1 %
ε_{zz} $t=40\,s$	0.10197	ANALYTICAL	1.1 %
ε_{zz}^{th} $t=40\,s$	-0.003546	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=40\,s$	0.01055	ANALYTICAL	1.1 %
ε_{zz}^{plas} $t=40\,s$	0.01025	ANALYTICAL	1.1 %
ε_{zz}^{p} $t=90\,s$	0.0741	ANALYTICAL	0.1 %
χ $t=90\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=90\,s$	$1350.\ 10^{6}$	ANALYTICAL	0.1 %
ε_{zz} $t=90\,s$	0.10984	ANALYTICAL	1.0 %
ε_{zz}^{th} $t=90\,s$	-0.01098	ANALYTICAL	0.6 %
ε_{zz}^{meca} $t=90\,s$	0.012082	ANALYTICAL	1.1 %
ε_{zz}^{plas} $t=90\,s$	0.011407	ANALYTICAL	1.1 %

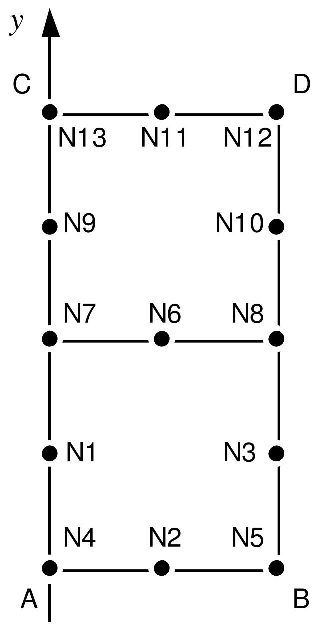
6.2 Remarks

In this modeling, one takes into account the term due to the plasticity of transformation:

$$\dot{\varepsilon}^{pt}(T,Z)\neq 0 \text{ when } \dot{Z}\neq 0$$

7 Modeling C

7.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

7.2 Characteristics of the grid

Many nodes: 13

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3

8 Results of modeling C

8.1 Values tested

One tests the structural parameters of data results:

Identification	Reference
INST for NUME_ORDRE= 27	90
ITER_GLOB for NUME_ORDRE=27	2

Identification	Reference	Test	Tolerance
ε_{zz}^{p} $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
χ $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
σ_{zz} $t=24\,s$	360. 10^6	ANALYTICAL	0.1 %
ε_{zz} $t=24\,s$	-0.00384	ANALYTICAL	0.1 %
ε_{zz}^{th} $t=24\,s$	-0.005640	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=24\,s$	0,018	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=24\,s$	0	ANALYTICAL	1.0E-6 (absolute)
ε_{zz}^{p} $t=26\,s$	0.0372	ANALYTICAL	0.1 %
χ $t=26\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=26\,s$	390. 10^6	ANALYTICAL	0.1 %
ε_{zz} $t=26\,s$	0.03428	ANALYTICAL	0.1 %
ε_{zz}^{th} $t=26\,s$	-0.004884	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=26\,s$	0.039164	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=26\,s$	0.0372	ANALYTICAL	0.1 %
ε_{zz}^{p} $t=40\,s$	0.0625	ANALYTICAL	0.1 %
χ $t=40\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=40\,s$	600. 10^6	ANALYTICAL	0.1 %
ε_{zz} $t=40\,s$	0.06198	ANALYTICAL	0.1 %
ε_{zz}^{th} $t=40\,s$	-0.003546	ANALYTICAL	0.1 %
ε_{zz}^{meca} $t=40\,s$	0.065526	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=40\,s$	0.0625	ANALYTICAL	0.1 %
ε_{zz}^{p} $t=90\,s$	0.0741	ANALYTICAL	0.1 %
χ $t=90\,s$	1	ANALYTICAL	0.1 %
σ_{zz} $t=90\,s$	1350. 10^6	ANALYTICAL	0.1 %
ε_{zz} $t=90\,s$	0.069844	ANALYTICAL	0.1 %
ε_{zz}^{th} $t=90\,s$	-0,011	ANALYTICAL	0.4 %
ε_{zz}^{meca} $t=90\,s$	0.08085	ANALYTICAL	0.1 %
ε_{zz}^{plas} $t=90\,s$	0.0741	ANALYTICAL	0.1 %

9 Summary of the results

Three modelings give very good approximations of the reference solution.