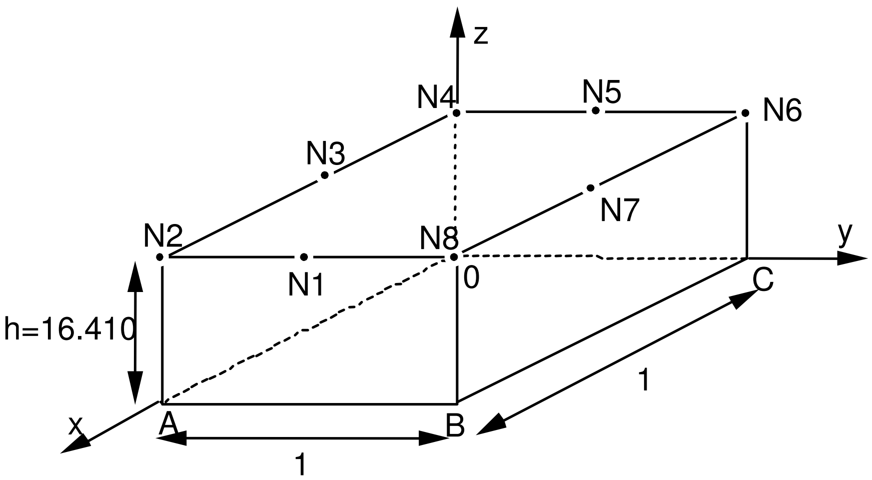
HPLV101 - Homogenisation of a material homogeneous

Summary:

This test tests, in a commonplace situation where the material is homogeneous, the resolution of the thermal and mechanical problems stationary, with loadings corresponding to a variation in temperature and an imposed deformation, close to those corresponding to the elementary problems of the method of periodic homogenisation.

1 Problem of reference

1.1 Geometry



	x	y	z
N4	0	0	16.410
N8	1.	1.	16.410
N3	0.5	0	16.410

1.2 Material properties

Modeling A

$E=1.0\,MPa$
$\nu=0.3$
$k=1.0\,W/(m.°C)$
$C_p=0\,J/(°C.m^3)$

One tests the plasticity and the compatibility of MFront with Deborst (forced plane)

Modeling B

$E_L=1.0\,MPa$
$E_T=0.9\,MPa$
$E_N=0.8\,MPa$
$\nu_{LT}=0.1$
$\nu_{LN}=0.25$
$\nu_{TN}=0.3333333$
$k=1.0\,W/(m.°C)$
$C_p=0\,J/(°C.m^3)$

1.3 Boundary conditions and loadings

- Mechanics 3D:

Plan $z=0$:	$dz=0$	for the membrane loading;
	$dx=0$, $dy=0$	for the loading of inflection
Plans $y=0$, $y=1$:	$dy=0$	
Plans $x=0$, $x=1$:	$dx=0$	
Node: O	$dz=0$	(for the only loading of inflection)

Loading:

déformation membranaire: $\mathsf{E} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

flexion uniforme imposée: $\mathsf{E} = \begin{bmatrix} \frac{z}{z_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$z_0 = 1\text{m}$

- Mechanics 2D, forced plane:
  Axis: $x=0$ $dx=0$
  Node: O $dy=0$
  (these conditions do not correspond to the application of the method of homogenisation).

  Loading: deformation $\mathsf{E} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ uniforme imposée

- Thermics 3D and 2D:
  Plan $x=0$ $temp=0$ (this condition does not correspond to the application of the method of homogenisation).

  Loading: gradient $\mathbf{G} = (-1,0,0)$ imposed uniform.

2 Reference solution

2.1 Method of calculating used for the reference solution

- In thermics: the stationary thermal problem is solved:

$$\int_\Omega \nabla T.\mathbf{K}.\nabla\theta = \int_\Omega \mathbf{G}.\mathbf{K}.\nabla\theta \quad , \quad \forall\theta\in V \quad , \quad \text{avec } \mathbf{G} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Note:

The boundary conditions chosen here are not those necessary to the method of homogenisation: one would find indeed $T=0$ everywhere.

The solution is then (checking the conditions defined in [§1.3]): $T(x,y,z) = -x$

The potential energy is then with balance: $W^{th} = -\frac{1}{2}\int_\Omega \nabla T.\mathbf{K}.\nabla T = -\frac{1}{2}|\Omega|$ ici

- In mechanics: one solves the problem of elastostatic:

$$\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{v}) = \int_\Omega E.\mathbf{A}.\varepsilon(\mathbf{v}) \quad , \quad \forall \mathbf{v}\ \varepsilon\ W \quad ,$$

for the cases:

loading 3D membrane	loading 3D of inflection	loading 2D plane constraints
$E = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$E = \begin{pmatrix} \frac{z}{z_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$E = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

The solutions are:

- in 3D, loading **membrane** and isotropic elasticity: $\mathbf{u}(x,y,z) = \left(0,0,-\frac{\nu z}{(1-\nu)}\right)$

  the potential energy with balance is:

$$W^{pot} = -\frac{1}{2}\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{u}) = -\left(\frac{\nu}{1-\nu}\right)^2.\frac{|\Omega|}{2}(\lambda+2\mu)$$

- in 3D, loading **membrane** and orthotropic elasticity: $\mathbf{u}(x,y,z) = (0,0,-\beta z)$ with $\beta = \frac{C_{13}}{C_{33}}$ that is to say $\beta = \frac{\frac{E_N}{E_L}(\nu_{LN} + \nu_{LT}\nu_{TN})}{1-\frac{E_N}{E_T}{\nu_{TN}}^2}$ because the local reference mark is not confused with the total reference mark (nautical angles being worth all 90°).

$$W^{pot} = -\frac{1}{2}\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{u}) = -\beta^2.\frac{|\Omega|}{2}C_{33}$$

where $C_{33}=\frac{1-\nu_{NT}\nu_{TN}}{E_T E_N \Delta}$ and $\Delta=\frac{1-\nu_{LT}\nu_{TL}-\nu_{LN}\nu_{NL}-\nu_{TN}\nu_{NT}-2\nu_{LT}\nu_{TN}\nu_{NL}}{E_L E_T E_N}$

- in 3D, loading of **inflection** : $\mathbf{u}(x,y,z)=\left(0,0,\frac{+\nu z^2}{2z_0(1-\nu)}\right)$;

$$W^{pot}=-\left(\frac{\nu}{1-\nu}\right)^2.\frac{|\Omega|}{2}(\lambda+2\mu).\frac{h^2}{3z_0^2}$$

- in 3D, loading **of inflection** and orthotropic elasticity: $\mathbf{u}(x,y,z)=\left(0,0,-\beta\frac{z}{L}\right)$ with $\beta=\frac{C_{13}}{C_{33}}$ that is to say $\beta=\frac{\frac{E_N}{E_L}(\nu_{LN}+\nu_{LT}\nu_{TN})}{1-\frac{E_N}{E_T}\nu_{TN}^{\;2}}$ because the local reference mark is not confused with the total reference mark (nautical angles being worth all 90°).

$$W^{pot}=-\frac{1}{2}\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{u})=-\beta^2.\frac{|\Omega|}{2}C_{33}\frac{h^2}{3z_0^2}$$

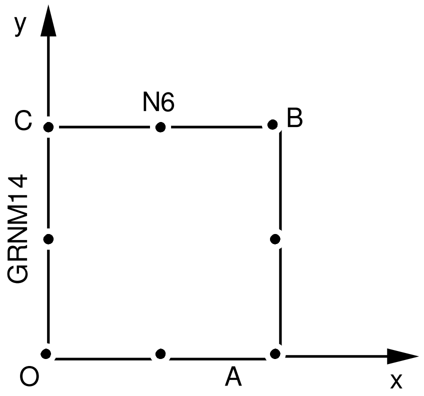
where $C_{33}=\frac{1-\nu_{NT}\nu_{TN}}{E_T E_N \Delta}$ and $\Delta=\frac{1-\nu_{LT}\nu_{TL}-\nu_{LN}\nu_{NL}-\nu_{TN}\nu_{NT}-2\nu_{LT}\nu_{TN}\nu_{NL}}{E_L E_T E_N}$

- in 2D, loading plan: $\mathbf{u}(x,y)=(-x,\ 0)$;

$$W^{pot}=\frac{-|\Omega|}{2(1-\nu^2)}$$

3 Modeling A

3.1 Characteristics of modeling



Boundary conditions and loading:

Thermics:	GROUP_NO: GRNM14: TEMP: 0.0 PRE_GRAD_TEMP: FLUX_X: – 1.0
Mechanics: (plane constraints)	GROUP_NO: GRNM14: DX: 0.0 NODE: O DY: 0.0 PRE_EPSI: EPXX: – 1.0

3.2 Characteristics of the grid

Many nodes: 8

Many meshs and types: 1 QUAD8

3.3 Values tested

Not	Size CMP	Reference
A	$TEMP$	– 1.0000
A	DX	– 1.0000
$N6$	DX	– 0.5000

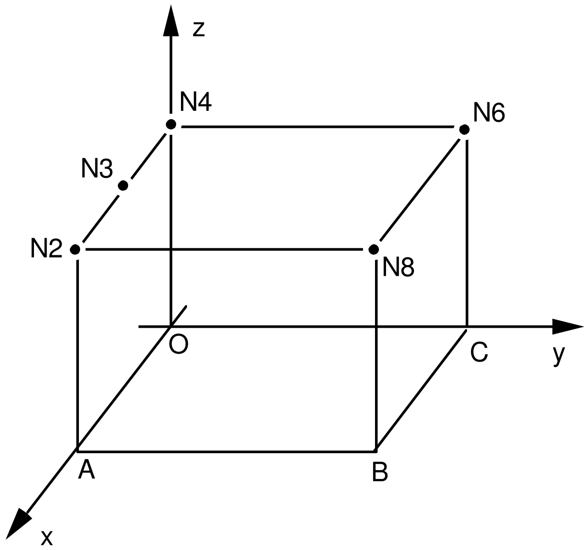
Mesh	Energy potential with balance	Reference
$M1$	Thermics	– 0.500000000
$M1$	Mechanics	– 0.549450550

3.4 Remarks

Code_Aster provides the value of the deformation energy, equal contrary to the potential energy to balance (elastic case).

4 Modeling B

4.1 Characteristics of modeling



Name of the meshs of the faces:	*ZEGAL0*	*YEGAL0*	*YEGAL1*	*XEGAL0*	*XEGAL1*
Summits:	*B C O A*	*O A N2 N4*	*B C N6 N8*	*C O N4 N6*	*A B N8 N2*

Boundary conditions:

Thermics:	`ZERO: DEFI_CONSTANTE (VALE: 0.0) ;`
	`FCT1: DEFI_FONCTION (Nom_para: 'Z', VALE: (0.0 0.0.1.0.1.0));`
	`GROUP_NO: XEGAL0: TEMP: 0.0`

Mechanics:	`PRE_GRAD_TEMP: FLUX_X: -1.0`
	`GROUP_NO: YEGALO: DY = 0.0`
	`XEGAL1: DX = 0.0`
	`YEGAL1: DY = 0.0`
	`XEGALO: DZ = 0.0`

Membrane case:	`GROUP_NO: ZEGALO: DZ = 0.0` `PRE_EPSI: EPXX: -1.0`

Case inflection:	`GROUP_NO: ZEGALO: DX = ZERO, DY = ZERO`
	`NODE: 0 DZ = ZERO`
	`PRE_EPSI: EPXX: FCT1`

4.2 Characteristics of the grid

Many nodes: 20
Many meshs and types: 1 HEXA20

4.3 Values tested

In isotropic elasticity

Case	Size	Not	Reference
Thermics	*temp*	*N8*	– 1.000000
	temp	*N3*	– 0.500000
Mechanics	*dz*	*N4*	– 7.03285714

membrane	dz	$N8$	– 7.03285714
Mechanics inflection	dz	$N4$	57.70459285
	dz	$N8$	57.70459285

Mesh	**Energy potential with balance**	**Reference**
$M1$	Thermics	– 8.20500
$M1$	Mechanics Membrane Inflection	 – 2.0287088 – 1.8210238 10^2

In orthotropic elasticity

Case	**Size**	**Not**	**Reference**
Thermics	$temp$	$N8$	– 1.000000
	$temp$	$N3$	– 0.500000
Mechanics membrane	dz	$N4$	– 6.63044894
	dz	$N8$	– 6.63044894
Mechanics inflection	dz	$N4$	54.40283358
	dz	$N8$	54.40283358

Mesh	**Energy potential with balance**	**Reference**
$M1$	Thermics	– 8.20500

5 Summary of the results

The results are exact with errors rounding close, since the sought solutions are part of the space of the finite elements selected for modeling.