HPLA310 - Biblio_49 Fissures radial external in a circular bar subjected to a thermal shock

Summary:

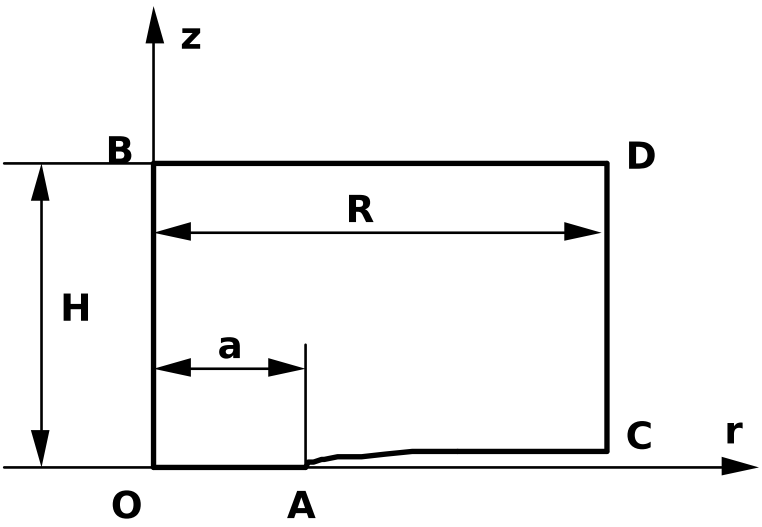
This test is resulting from the validation independent of version 3 in breaking process.

It is about a basic static test into axisymmetric under non stationary thermal loading. The behavior of the structure is thermoelastic linear isotropic.

It understands only one axisymmetric modeling.

1 Problem of reference

1.1 Geometry



External annular crack in a semi-infinite cylindrical bar

One will take $a/R=0,5$ and $H/R\geq 5$.

$a=1\,m$
$R=2\,m$
$H=10\,m$

1.2 Properties of material

The material is thermoelastic linear isotropic.

Young modulus	$E=2\text{E}11\,Pa$
Poisson's ratio	$\nu=0,3$
Linear dilation coefficient	$\alpha=1\text{E-}5\,C^{\circ -1}$
Thermal conductivity	$\lambda=50\,W/m.C^{\circ}$
Thermal diffusivity	$\kappa=\lambda/\rho\,C_p=0,5\,m^2/s$
Coefficient of heat exchange	$h=250\,W/m^2 C^{\circ}$

One will choose h such as $Bi=hR/\lambda=10.$

1.3 Boundary conditions and loading

Boundary conditions mechanical

$UX=U_r=0$ on the axis of revolution $r=0$
$UY=U_z=0$ on the ligament $0\leq r\leq a$

Worthless resulting thrust load at the higher edge; one will translate this boundary condition by a set of $(n-1)$ linear relations $UY(1)=UY(2)=...=UY(n)$ between longitudinal displacements of n nodes of the higher edge (free axial dilation, conservation of the flatness of the cross section of the bar).

Conditions of unilateral contact on the lip of the crack in order to manage the closing of this one.

Boundary conditions thermal

Heat flux no one on the axis of revolution AB (by symmetry)
Heat flux no one on the ligament OA (by symmetry) and on the crack AC .

Flow of convection $\frac{\partial T}{\partial r}=h(T_{ext}-T)$ at the edge $r=R$, T_{ext} indicating the temperature of the external medium.

Thermal loading

The temperature of the external medium undergoes an instantaneous level $T_{ext}=T0*H(t)$ where $H(t)$ is the function level-unit of Heaviside. Taking into account the boundary conditions the temperature does not vary according to z . One will take $T_0=100°C$ in order to obtain the closing of the lip, in the vicinity of the skin of the part, the beginning of the thermal shock.

1.4 Initial conditions

Mechanical initial conditions

Worthless displacements, strains and stresses in all points.

Thermal initial conditions

Worthless initial temperature in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

Field of temperature: exact analytical calculation.

Thermomechanical calculation: thermoelastic stress field in the bar not fissured given by an exact analytical expression

displacement of the lips of the crack calculated starting from functions of influence determined numerically by finite elements

factor of intensity of the constraints calculated starting from the surface stresses released along the crack, by using functions weight of the unlimited solid for a distribution of pressure on the lips constant by interval along the ray.

2.2 Results of reference

Number of Fourier: $Fo=\frac{\kappa t}{R^2}$ (adimensional time)

Number of Biot: $Bi=\frac{hR}{\lambda}$ (coefficient of exchange without dimension)

Expression of the temperature according to R and T:

$$T=T_0\left[1-2\sum_{n=1}^{\infty}\frac{BiJ_0(\mu_n r/R)}{(\mu_n^2+Bi^2)J_0(\mu_n)}\bullet\exp(-\mu_n^2 Fo)\right]$$

où

$$BiJ_0(\mu_n)=\mu_n J_1(\mu_n)$$

eigenvalues μ_n are the solutions of the equation above in which J_0 and J_1 are the functions of Bessel of first species of order 0 and 1.

The tables below summarize the values of the temperatures ($°C$) for three particular rays and three numbers of Fourier:

$F0=0{,}001$

	Ref. (10000 terms)
$r=0$	3,9968E-12
$r=1$	2,2204E-13
$r=2$	2,79689E+1

$F0=0{,}4$

	Ref. (900 terms)
$r=0$	1,6230E-1
$r=1$	6,2391E+0
$r=2$	7,7365E+1

$F0=1$

	Ref. (900 terms)
$r=0$	9,8644E+1
$r=1$	9,9018E+1
$r=2$	9,9835E+1

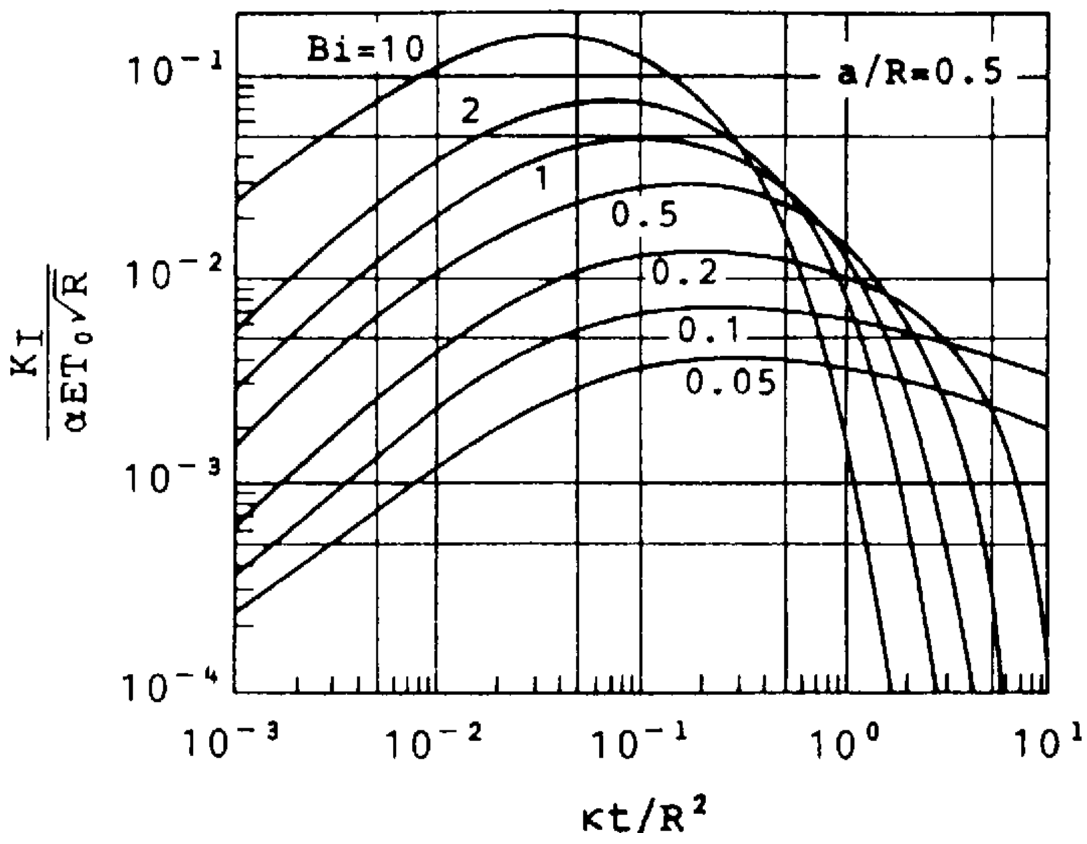
Expression of axial stress in the bar not fissured according to r and of t :

$$\sigma_{zz} = \frac{2\alpha E T_0}{(1-\nu)} \sum_{n=1}^{\infty} \left[\frac{Bi}{\left(\mu_n^2 + Bi^2\right) J_0\left(\mu_n\right)} \exp\left(-\mu_n^2 Fo\right) \right] \left[J_0\left(\mu_n \frac{r}{R}\right) - \frac{2Bi}{\mu_n^2} J_0\left(\mu_n\right) \right]$$

The table below summarizes the values of the constraints $\sigma_{zz}(Pa)$ for $r=a$ (bottom of crack) and for three numbers of Fourier:

	Ref. (900 terms)
$F0=0{,}001$	4,584029E+6
$F0=0{,}4$	6,397099E+7
$F0=1$	8,200300E+5

Factor of intensity of the constraints (adimensional) according to the number of Fourier



2.3 Uncertainty on the solution

Lower than 5% .

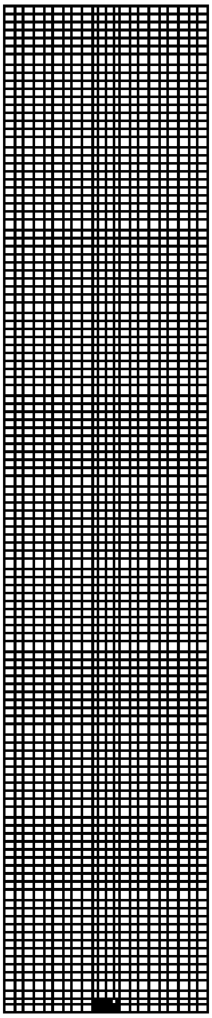
2.4 Bibliographical references

1.J.M. ZHOU, T. TAKASE and Y. IMAI: Opening and closing behavior of year external circular ace due to axisymmetrical heating. Engng.Fract.Mechs., 47, n°4, 559-568, 1994.

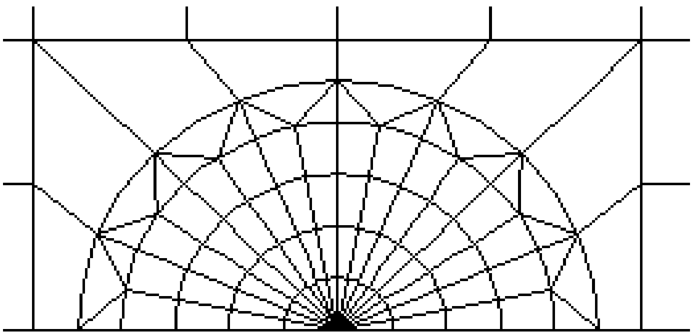
3 Modeling A

3.1 Characteristics of modeling

Non stationary thermal calculation precedes mechanical calculation. Two calculations are done using the same grid to avoid the phenomena of smoothing.



Complete grid



Zoom on the point of crack

3.2 Characteristics of the grid

The grid consists of 8651 nodes and 2772 elements, including 2732 elements `QUA8` and 40 elements `TRI6`.

The radial density of the grid is determined by successive tests in order to reduce to 1% the variation enters the theoretical solution and the digital solution, as well from the thermal point of view as thermomechanical, in the case of the bar not fissured.

The height of the half-model is fixed arbitrarily at 5 times the ray R. It is supposed a priori that the effect of the limitation of size of the grid in the direction Z on the factor of intensity of the constraints is lower than 1%.

An indeformable block, located under the lip, was with a grid in order to manage the contact without friction induced by the closing of the lip.

3.3 Values tested and results of modeling A

Identification	Reference	*Aster*	$\%$ difference
$G(Fo=0{,}001)(J.m^2)$	2,3E+2	2,9449E+2*	30
$K_I(Fo=0{,}001)(Pa.m^{0,5})$	7,0E+6	8,0438E6 **	14
$G(Fo=0{,}04)(J.m^2)$	1,0E+4	1,25016E+4*	19
$K_I(Fo=0{,}04)(Pa.m^{0,5})$	4,8E+7	5,24175E7 **	9
$G(Fo=1)(J.m^2)$	1.0	1,2104864*	15
$K_I(Fo=1)(Pa.m^{0,5})$	4,8E+5	5,1579E+5 **	7

* In the case of axisymmetric calculations, to obtain the total rate of refund, it is necessary to divide the rate of refund obtained with ASTER by $R_{fissure}=a$ (Cf Reference material [`R7.02.01`] - page18).

** Values obtained with the formula of IRWIN in plane deformations, by supposing that $K_{II}=0$, and by taking it G calculated by ASTER, which does not allow the automatic calculation of K_I into axisymmetric.

3.4 Remarks

To calculate G_{ref}, one uses the formulas of IRWIN in plane deformations:

$$G_{ref}=\frac{1-\nu^2}{E}(K_I^2+K_{II}^2)\ ,\ \ K_{II}=0$$

The raised maximum change is of 30% on G ($Fo=1$), of $14\ \%$ on K_I ($Fo=1$).

The maximum relative variation on the temperature in bottom of crack, compared to the analytical solution (summoned on 900 terms), is lower has 1%.

The maximum relative variation on σ_{zz} in the bar before cracking, compared to the analytical solution summoned on 900 terms, with the site of the later bottom of crack, is lower than $0{,}5\ \%$.

With ASTER, in axisymmetric mode, the stress field obtained is following form:

SIXX SIYY SIZZ SIXY and the associated constraints are:

SIRR SIZZ SITT SIRZ

To calculate the values of reference, we use the curve in $\log/\log$ (page 5). The precision with the reading of the values not being very good, we can estimate that the results on the rate of refund of energy G are not too far away from the reference.

It should be noted that the rate of refund of energy G is invariable on the crowns of calculation.

4 Summary of the results

With regard to the bar not fissured, the results *Aster* in temperature and constraint are very close to the reference (less 1% maximum for the temperature and less $0{,}5\%$ maximum for the constraints). On the other hand, for the rate of refund of energy, the results *Aster* are far away from the reference since we raise a maximum change of 30% for $Fo=0{,}001$, with a precision announced of 5% on the reference solution.