

## SSNV247 - Application of a pressure distributed on the lips of an interface XFEM curves crossing a segment of a sphere

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### Summary:

It is about a test of validation of the facets of contact resulting from under elements from integration XFEM.

The facets of contact used by default do not result from under elements from integration and do not give the possibility of having quadratic facets of contact in 3D. One can activate the recovery of the facets of contact resulting from under elements from integration in the operator `MODI_MODELE_XFEM` with the keyword `DECOUPE_FACETTE=' SOUS_ELEMENTS'`. One then has linear and quadratic facets in 2D as in 3D which have the advantage of being in conformity with under elements of integration since they are selected among the sides of under elements of integration in 2D and the faces of under elements of integration in 3D. This mode of recovery also has the following advantage: the coordinates of the nodes of these facets are not recomputed as it is the case by default, because these nodes are recovered during the cutting which is carried out in `TOPOSE`.

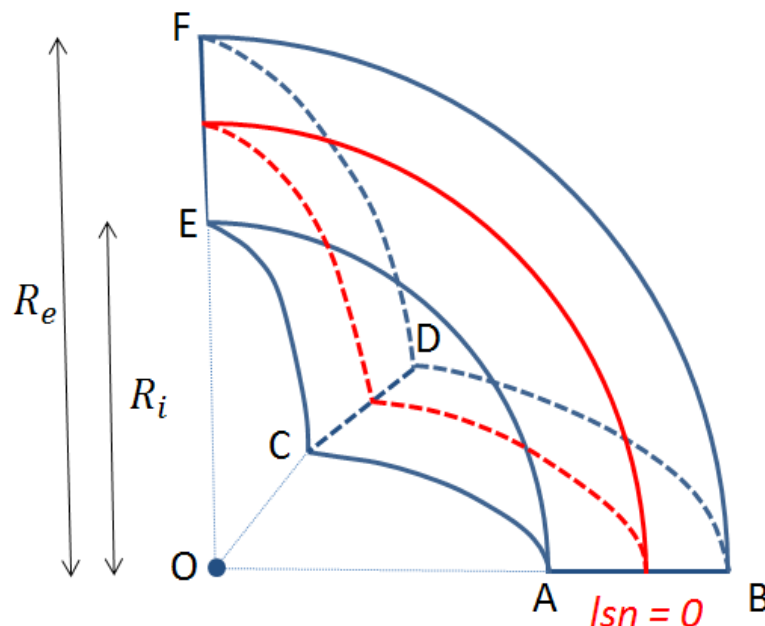
In this test, one makes sure of the good performance of the recovery of these facets when a mechanical pressure is applied to the lips of an interface for a three-dimensional geometry. The got results are compared with an analytical solution. One voluntarily chooses an Iso-zero "lsn" curve representing the interface to appreciate the performance profit brought by the quadratic "facetisation".

## 1 Problem of reference

### 1.1 Geometry of the problem

It is about a eighth of sphere of interior ray  $R_i=1m$ , of external ray  $R_e=2m$  and of centre  $O(0,0,0)$ . This portion of sphere is crossed by a discontinuity of the type interfaces (interface nonwith a grid which is introduced into the model via the level-sets thanks to the operator `DEFI_FISS_XFEM`), concentric of ray  $R=1.5m$ .

One represents on the Figure 3.3-a geometry of the column.



### 1.2 Properties materials

Parameters given in the Table 1.2-1, correspond to the parameters used for 4 modelings. The behavior is elastic ( 'ELAS' ).

Elastic parameters	Young modulus $E(enMPa)$	5800
	Poisson's ratio $\nu$	0
	Thermal dilation coefficient $\alpha(enK^{-1})$	0

Table 1.2-1 : Properties of material

## 1.3 Boundary conditions and loading

The following conditions of Dirichlet are applied:

- on the lower face [ABDC], displacements according to  $z$  are blocked  $u_z=0$ ,
- on face [ABFE], displacements according to  $y$  are blocked  $u_y=0$ ,
- on face [CDFE], displacements according to  $x$  are blocked  $u_x=0$ ,
- on the external cap BDF and the interior cap ACE, displacements are blocked in all the directions ( $u_x=0$ ,  $u_y=0$  and  $u_z=0$ ).

The loading is the following:

- on each lip of the interface with  $r=R$  a pressure distributed uniform is imposed  $p=10\text{ MPa}$  via `AFPE_CHAR_MECA` and of the keyword `CRACK` of `PRES_REP`.

## 2 Reference solution

### 2.1 Method of calculating

It is about an analytical solution. Taking into account the boundary conditions, displacements can be obtained starting from the analytical resolution of the conservation equation of the momentum.

By neglecting gravity, the equation is written (in total constraints):

$$\text{Div}(\boldsymbol{\sigma})=0$$

The Fish module  $\nu$  being null, and being in the elastic case, one has  $\boldsymbol{\sigma}=E\boldsymbol{\epsilon}$ .

The volume studied with spherical symmetry, consists of a homogeneous and isotropic material; the boundary conditions have also spherical symmetry. One is thus brought to seek a solution of the problem in a spherical frame of reference  $(r, \theta, \varphi)$  such as the fields of displacement, of constraint and deformation are respectively of the form:

$$\begin{cases} u_r=h(r) \\ u_\theta=u_\varphi=0 \end{cases} \quad \begin{cases} \sigma_{rr}=f_1(r) \\ \sigma_{\theta\theta}=\sigma_{\varphi\varphi}=g_1(r) \\ \sigma_{r\theta}=\sigma_{r\varphi}=\sigma_{\theta\varphi}=0 \end{cases} \quad \begin{cases} \epsilon_{rr}=f_2(r) \\ \epsilon_{\theta\theta}=\epsilon_{\varphi\varphi}=g_2(r) \\ \epsilon_{r\theta}=\epsilon_{r\varphi}=\epsilon_{\theta\varphi}=0 \end{cases}$$

The equilibrium equation  $\text{Div}(\boldsymbol{\sigma})=0$  is reduced then to:  $\frac{d\sigma_{rr}}{dr} + \frac{2}{r} * (\sigma_{rr} - \sigma_{\theta\theta}) = 0$

The boundary conditions static are form:  $\sigma_{rr}(R) = -p$

The equations kinematics have the form:  $\begin{cases} \epsilon_{rr} = \frac{du_r}{dr} \\ \epsilon_{\theta\theta} = \frac{u_r}{r} \end{cases}$  that is to say finally:  $\begin{cases} \sigma_{rr} = E * \frac{du_r}{dr} \\ \sigma_{\theta\theta} = E * \frac{u_r}{r} \end{cases}$

In substituent these two relations in the equilibrium equation one obtains:

$$\frac{d^2 u_r}{dr^2} + \frac{2}{r} * \left( \frac{du_r}{dr} - \frac{u_r}{r} \right) = 0 \quad \text{that is to say} \quad \frac{d}{dr} \left( \frac{1}{r^2} \frac{d(r^2 u_r)}{dr} \right) = 0$$

The solution of this differential equation is:  $u_r(r) = C_1 r + \frac{C_2}{r^2}$ . The solution sought being discontinuous in  $R$ , one separately solves this equation on the two fields  $[R_i, R]$  and  $[R, R_e]$ .

$$\text{Finally: } \begin{cases} u_r(r) = C_1 r + \frac{C_2}{r^2} & \text{sur } [R_i, R] \\ u_r(r) = C_3 r + \frac{C_4}{r^2} & \text{sur } [R, R_e] \end{cases}$$

$$\text{According to the boundary conditions kinematics, } u_r(R_i) = u_r(R_e) = 0 \text{ thus } \begin{cases} C_1 R_i + \frac{C_2}{R_i^2} = 0 \\ C_3 R_e + \frac{C_4}{R_e^2} = 0 \end{cases}$$

$$\text{In addition } \sigma_{rr} = E * \frac{du_r}{dr} = \begin{cases} E * C_1 - 2 E * \frac{C_2}{r^3} & \text{sur } [R_i, R] \\ E * C_3 - 2 E * \frac{C_4}{r^3} & \text{sur } [R, R_e] \end{cases} \quad \text{thus according to the boundary}$$

$$\text{conditions static: } \begin{cases} C_1 - 2 * \frac{C_2}{R^3} = -\frac{p}{E} \\ C_3 - 2 * \frac{C_4}{R^3} = -\frac{p}{E} \end{cases}$$

$$\text{The resolution of the system gives } \begin{cases} C_1 = \frac{-p}{E * \left( 2 * \frac{R_i^3}{R^3} + 1 \right)} \\ C_2 = \frac{p}{E * \left( \frac{2}{R^3} + \frac{1}{R_i^3} \right)} \\ C_3 = \frac{-p}{E * \left( 2 * \frac{R_e^3}{R^3} + 1 \right)} \\ C_4 = \frac{p}{E * \left( \frac{2}{R^3} + \frac{1}{R_e^3} \right)} \end{cases}$$

## 2.2 Sizes and results of reference

One tests the value of radial displacements of share and others of the interface.

Sizes tested	Type of reference	Value of reference
UR (in lower part)	'ANALYTICAL'	-0.0001142742582
UR (in top)	'ANALYTICAL'	6.173526141E-05

## 2.3 Uncertainties on the solution

No, the solution is analytical.

## 3 Modeling A

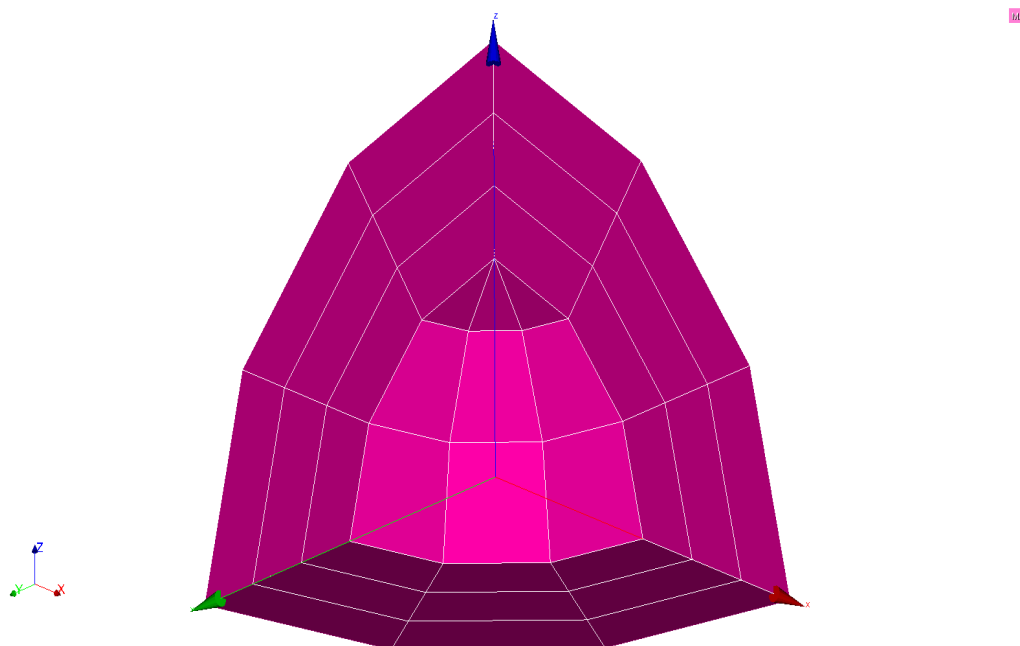
### 3.1 Characteristics of modeling

It is about a modeling 3D using linear elements XFEM.

### 3.2 Characteristics of the grid

The segment of a sphere on which one carries out modeling is divided into 18 HEXA8 and 9 PENTA6. The interface is nonwith a grid and cuts the cap in its thickness. The grid is represented Figure Error: Reference source not found.

Figure 3.2-a: Linear grid 3D



### 3.3 Sizes tested and results

The results are got with *Code\_Aster* (resolution with `STAT_NON_LINE`). Radial displacements are tested  $u_r$  on the lips of the crack. For each crack, it is tested MIN and it MAX of these two sizes for all the nodes of the crack. The got results are synthesized in the table below.

Sizes tested	Type of reference	Analytical values	Tolerance (%)
DR. MIN (int)	'ANALYTICAL'	-0.0001142742582	10.
DR. MAX (int)	'ANALYTICAL'	-0.0001142742582	10.
DR. MIN (ext.)	'ANALYTICAL'	6.173526141E-05	10.
DR. MAX (ext.)	'ANALYTICAL'	6.173526141E-05	10.

Radial displacement  $u_r$  and the deformation are represented on the Figure Error: Reference source not found. One observes a clear discontinuity of displacements and the spherical symmetry of the fields.

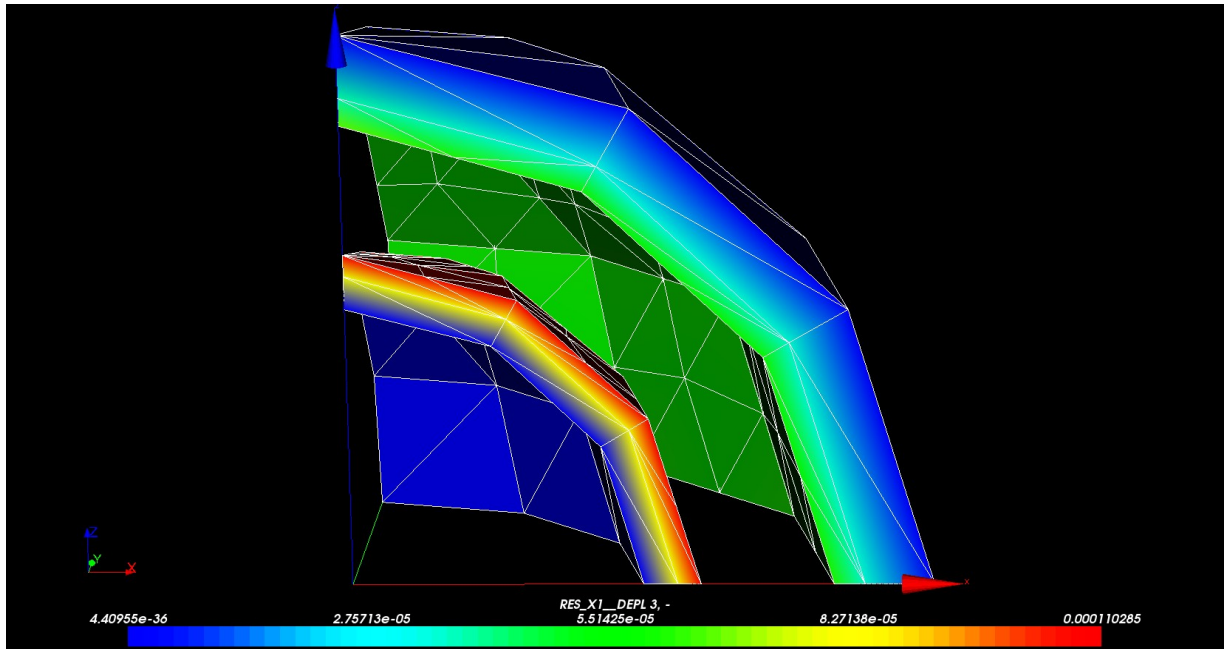


Figure 3.3-a: Field of radial displacement

The variations noted with the analytical solution are to be put in prospect with poverty for the grid used. One uses only 3 meshes in the thickness of the cap and the longitudinal and southernmost directions.

## 4 Modeling B

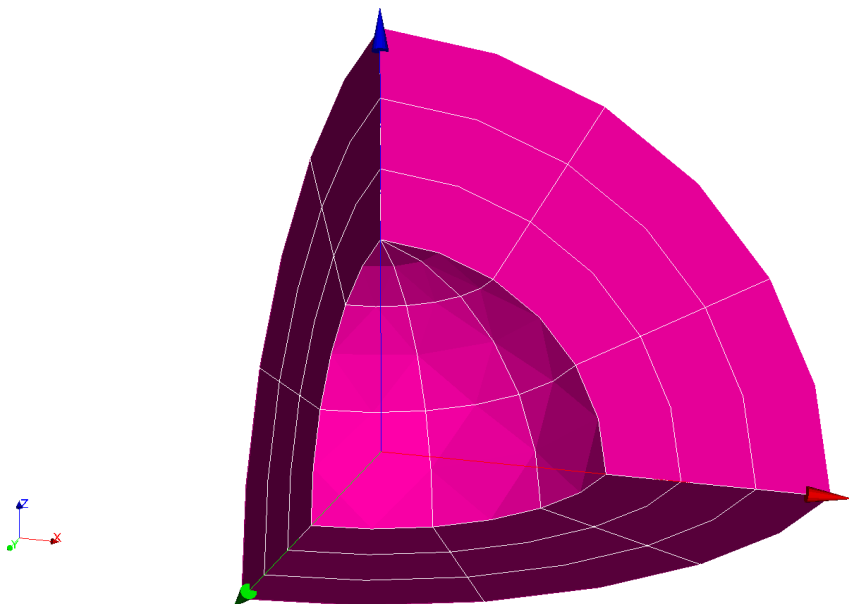
### 4.1 Characteristics of modeling

This modeling is strictly identical to the preceding one, except for the elements used which are quadratic.

### 4.2 Characteristics of the grid

The cap on which one carries out modeling is divided into 18 HEXA20 and 9 PENTA15. The interface is nonwith a grid and cuts the cap in its thickness. The grid is represented Figure Error: Reference source not found.

Figure 4.2-a: Quadratic grid 3D



### 4.3 Sizes tested and results

The results are got with *Code\_Aster* (resolution with `STAT_NON_LINE`). Radial displacements are tested  $u_r$  on the lips of the crack. For each crack, it is tested MIN and it MAX of these two sizes for all the nodes of the crack. The got results are synthesized in the table below.

Sizes tested	Type of reference	Analytical values	Tolerance (%)
DR. MIN (int)	'ANALYTICAL'	-0.0001142742582	1.
DR. MAX (int)	'ANALYTICAL'	-0.0001142742582	1.
DR. MIN (ext.)	'ANALYTICAL'	6.173526141E-05	1.
DR. (ext.)	'ANALYTICAL'	6.173526141E-05	1.



MAX			
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Radial displacement  $u_r$  and the deformation are represented on the Figure Error: Reference source not found. One observes a clear discontinuity of displacements and the spherical symmetry of fields.

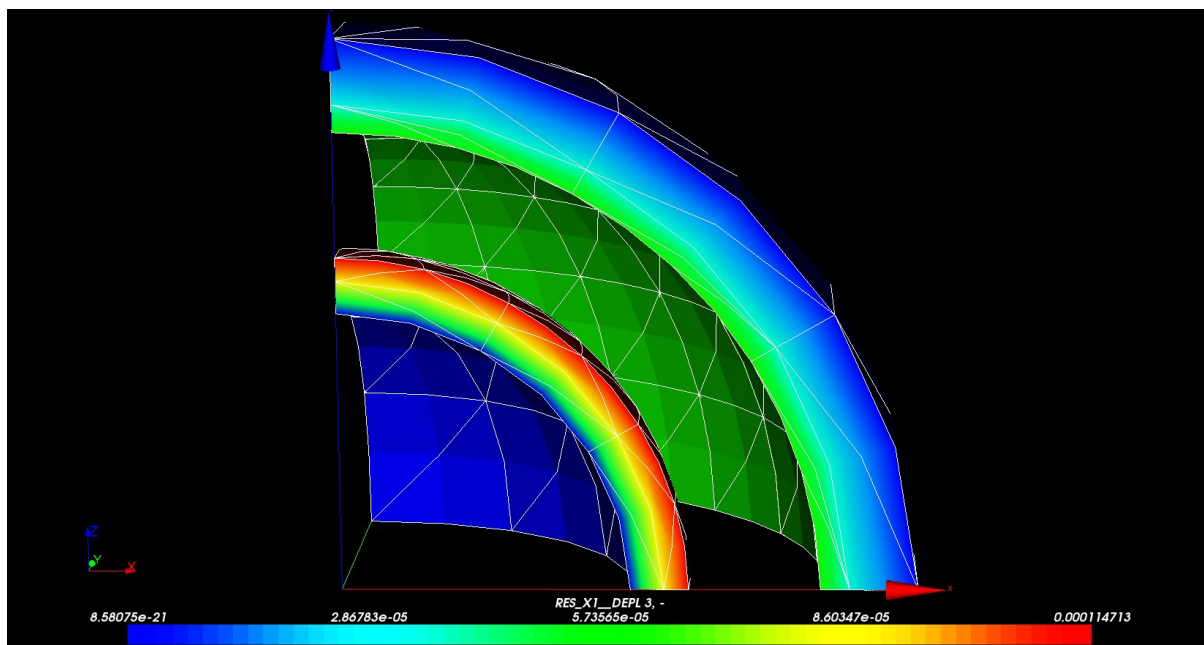


Figure 4.3-a : Field of radial displacement

With the same number of meshes that in preceding modeling, one gets results definitely more precise (one gains a factor 10 in precision on displacements). These two modelings once again illustrate the considerable contribution of the quadratic elements compared to the linear elements. The results got in this last modeling are completely satisfactory, especially taking into consideration the low number of meshes used and curve of the Isn.

## 5 Conclusion

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2 modelings validate the use of the facets of contact resulting from under elements from integration XFEM for the application from mechanical efforts on the lips from an interface nonwith a grid in 3D. They also illustrate the profit of precision obtained with quadratic elements compared to linear elements.