SSNV182 – Block with interface in contact rubbing with X-FEM

Summary

The purpose of this test is to validate the taking into account of the contact on the lips of the crack, while being limited if the crack crosses the structure completely. The contact is taken into account by the method continues [bib1] adapted to the framework of method X-FEM [bib2]. Two algorithms of contact are tested: method of Lagrangian increased and penalized method [bib2].

This test brings into play a parallelepipedic block in compression modelled in $2D$ and $3D$. The interface the beam is represented by a level-set within the framework of method X-FEM. One takes into account several angular positions of the interface: the interface follows the faces of the elements ($\theta=0°$) and interfaces it cut the elements ($\theta=26.56°$ in $3D$ and $\theta=30°$ in $2D$). By taking a coefficient of friction of sufficiently high Coulomb so that there is adherence, one finds the solution of the same problem without interface. Modelings D, E and F make it possible to also validate the method in $2D$.

1 Problem of reference

1.1 Geometry

The structure is a right at square base and healthy parallelepiped. Dimensions of the block (see [Figure -1.1-a]) are: $LX=5\,m$, $LY=20\,m$ and $LZ=20\,m$. It does not comprise any crack.

The interface is introduced by functions of levels (level sets) directly into the command file using the operator DEFI_FISS_XFEM [U4.82.08]. The interface is present in the middle of the structure by the means of its representation by the level sets. The level set normal (LSN) allows to define a plane interface forming an angle θ with the plan Oxy by the following equation:

$$LSN=Z-(aY+b) \qquad \text{éq 1.1-1}$$

where a is the slope of the interface, that is to say $a=-\tan(\theta)$ and $b=\frac{LZ}{2}-a\frac{LY}{2}$.

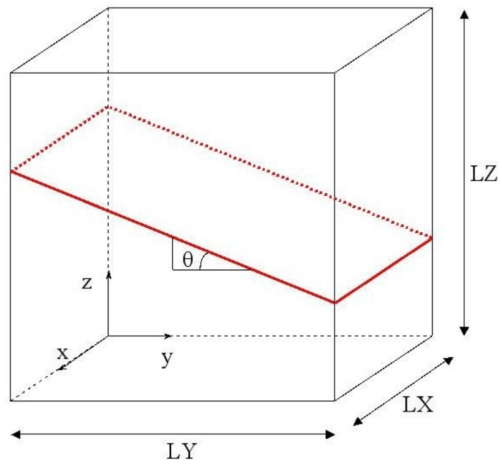

Figure -1.1-a : Geometry of the bar and positioning of the interface

In $2D$, there is an equivalent structure.

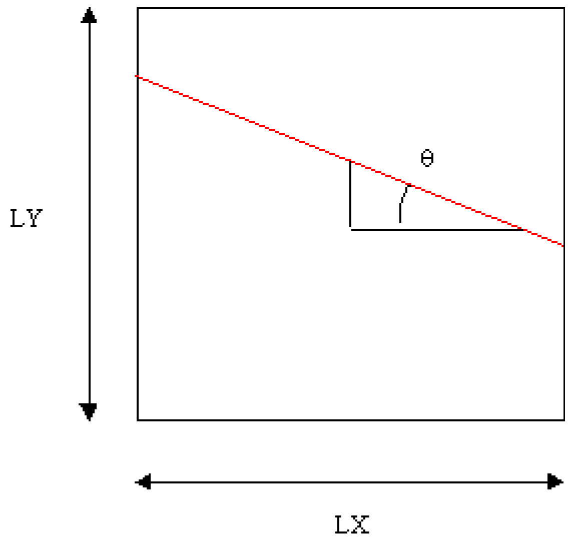

Figure 1.1-b **: Geometry and positioning of the interface in** $2D$

1.2 Properties of material

Young modulus: $E=100\,MPa$
Poisson's ratio: $\nu=0$.

1.3 Boundary conditions and loadings

The nodes of the lower face of the bar are embedded and a displacement $UZ=-10^{-6}m$ is imposed on those of the higher face which corresponds to a loading in pressure along the axis z. Displacements along the axes x and y are blocked for the nodes of the upper surface. In $2D$, displacement is carried out according to the axis y.

1.4 Characteristics of the grid

The structure is modelled in $3D$ by a regular grid composed of $5\times20\times20$ HEXA8 [Figure 1.4-a].

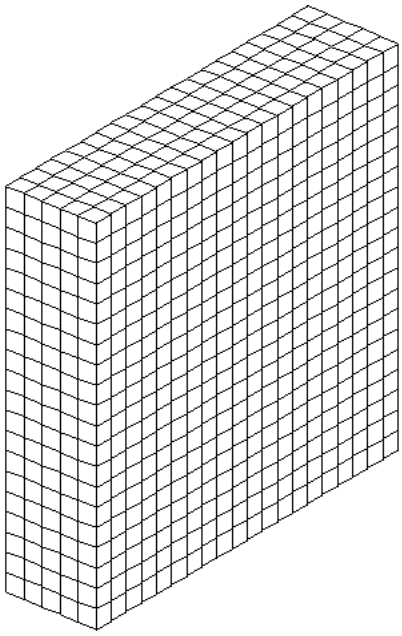

Figure -1.4-a : Grid $3D$

This grid is composed of linear finite elements. However, within the framework of the continuous method [bib1] with X-FEM [bib2], it is necessary to pass to a little special linear elements. These elements have linear functions of form and a quadratic mesh support. On these elements, the nodes top carry the unknown factors of displacement, and the nodes medium carry the unknown factors related to the contact. Moreover, when the interface follows the edge of an element, the nodes top of the element carry also the unknown factors of contact.

For the CAS-test $2D$, the structure is modelled by a regular grid composed of 20×20 `QUAD4`.

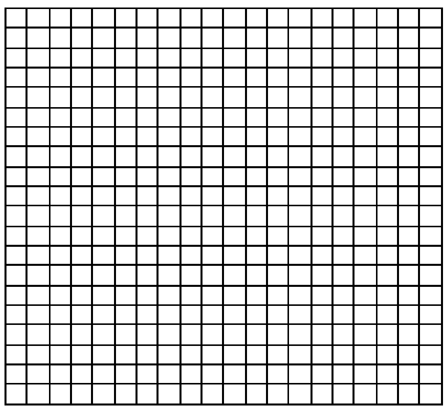

Figure -1.4-b : Grid $2D$

1.5 Bibliography

1. MASSIN P., BEN DHIA H., ZARROUG MR.: Elements of contacts derived from a continuous hybrid formulation, Handbook of reference of *Code_Aster*, [R5.03.52]
2. MASSIN P., GENIAUT S.: Method X-FEM, Handbook of reference of *Code_Aster*, [R7.02.12]

3. TARDIEU NR., VAUTIER I., LORENTZ E.: Quasi-static nonlinear algorithm, Manuel de Référence of *Code_Aster*, [R5.03.01]

4. DHATT G., TOUZOT G.: A presentation of the finite element method, Maloine ED., PARIS

2 Modeling a: interfaces right, method of Lagrangian increased

In this modeling, one represents a right interface, the angle θ 0 are worth then. The interface coincides with the faces of certain finite elements. The method of Lagrangian increased is used for the treatment of the contact/friction.

2.1 Analytical resolution

When the adhering contact is taken into account on the interface, the problem is equivalent to that of a healthy bar in pure compression. The solution of the problem is that of the same problem without interface.
The constraint in the structure is:

$$\sigma_{zz}=E\frac{UZ}{LZ} \quad \textbf{éq 2.1-1}$$

and the value of the contact pressure on the interface is:

$$\lambda=\sigma_{zz} \quad \textbf{éq 2.1-2}$$

With the digital values previously introduced, $\lambda=-5.0\,Pa$.

2.2 Features tested

As it is announced to the preceding paragraph, it is necessary to activate friction. In the operator `DEFI_CONTACT`, one stipulates then `FRICTION`= '`COULOMB`', and the coefficient of friction is selected equal to 1.

Moreover, as of the first iteration of the active constraints, one makes the assumption that the points of contact have a contacting statute. This is possible while specifying `CONTACT_INIT`= '`YES`'.
If not, at the end of the first iteration, the contact not being activated, the higher block returns in the lower block but the two blocks did not become deformed. Their state of stresses is thus null, and should then be chosen a total criterion (`RESI_GLOB_MAXI`) for the convergence of the algorithm of Newton - Raphson [bib3], criterion which is likely to be unsuited in the continuation of calculations when the contact is activated.

To avoid that, and to have a relative criterion, one needs a state of stresses not no one as of the first iteration, and thus to activate the contact as of the beginning.

The algorithm of the active constraints thus converges in an iteration.

2.3 Sizes tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of the operator `STAT_NON_LINE` and of the loop on the active constraints. All the points of contact are tested, which correspond to the nodes of the grid on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% Tolerance
`LAGS_C` for all the nodes of the interface	-5.00	1.00e-10

To test all the nodes in only once, one tests the minimum and the maximum of column.

2.4 Comments

This modeling shows the possibilities of the continuous method of contact applied to framework X - FEM. The advantage is that the procedure of pairing is intrinsic with method X-FEM since here, there are not really surface Master and slave considering whom one remains in small displacements.

3 Modeling b: interfaces leaning, method of Lagrangian increased

In this modeling, a leaning interface is represented. The angle θ is worth $\arctan(1/2)$, that is to say a slope α being worth $-1/2$. The interface does not coincide any more with the faces of the finite elements, and cuts the elements now. The normal with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}0\\1/\sqrt{5}\\2/\sqrt{5}\end{pmatrix},\qquad \tau=\begin{pmatrix}0\\2/\sqrt{5}\\-1/\sqrt{5}\end{pmatrix}$$ **éq 4-1**

The method of Lagrangian increased is used for the treatment of the contact/friction.

3.1 Analytical resolution

The interface being leaning, there is likely to be slip. To avoid that, one forces adherence by choosing a coefficient of friction of sufficiently high Coulomb. Theoretically, it is enough to take:

$$\mu>\tan(\theta)$$ **éq 4.1-1**

Thus, the solution of the problem remains identical to that of the same problem without interface.
The constraint in the structure is always that of [éq 2.1-1], and the value of the contact pressure on the interface is function of the normal n with the interface:

$$\lambda=n.\sigma.n=n_z\sigma_{zz}n_z$$ **Q 4.1-2**

where n_z is the component according to z of n .

The semi-multiplier of friction Λ is defined by:

$$r_\tau=\lambda\mu\Lambda$$ **éq 4.1-3**

With the density of tangential stress being written as follows:

$$r_\tau=(\tau.\sigma.n)\tau$$ **éq 4.1-4**

From where:

$$\Lambda=\left(\frac{1}{\mu}\frac{\tau.\sigma.n}{n.\sigma.n}\right)\tau=\left(\frac{1}{\mu}\frac{\tau_z}{n_z}\right)\tau$$ **éq 4.1-5**

One takes $\mu=1$.
With the digital values previously introduced, $\lambda=-4.0\,MPa$ and $\Lambda.\tau=-0.5$.

3.2 Sizes tested and results

The values of the pressio are testedN normal of contact and the semi-multiplier of friction after convergence of the iterations of the operator STAT_NON_LINE, loop on the active constraints and of the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the grid of visualization: GROUP_NO points of contact is extracted by the keyword PREF_GROUP_CO order POST_CHAM_XFEM.
It is checked that one finds well the values determined with [§3.1]. LAGS_F1 corresponds to the semi - multiplying of friction in the direction Ox (it is thus null), whereas LAGS_F2 corresponds to the semi-multiplier of following friction τ .

Identification	Reference	% Tolerance
LAGS_C for all the points of contact	-4.00	1.00e-8
LAGS_F1 for all the points of contact	0.00	1.00e-8
LAGS_F2 for all the points of contact	-0.50	1.00e-8

To test all the points of contact in only once, the value is tested MIN and the value MAX column.

3.3 Comments

Let us specify that in this study, the keyword CONTACT_INIT = 'YES' allows to begin the loop on the active constraints with an assumption of statute contacting for all the points of contact. That authorizes to take a relative criterion ('RESI_RELA_MAX') for the convergence of the iterations of Newton. Indeed, if one chooses CONTACT_INIT = 'NOT', at the time of the phase of prediction of Newton, the contact not being activated, the higher structure moves without becoming deformed, and that lower remains motionless. The constraints are then worthless and a relative criterion is not usable, only a total criterion is, whose value is left with the choice of the user. The problem is that this value can appear calculation (active contact thereafter….) inadequate with the loadings and the constraints then concerned. Thus, it is to better provide to take a single relative criterion as of the beginning.

Moreover, the initial value of the threshold of friction was taken with -10^9 in order to be sure that one has adherence as of the 1era iteration on the thresholds of friction.

4 Modeling C: right interface and under-integration, method of Lagrangian increased

This modeling is identical to modeling A, except that the digital diagram of integration of the terms of contact changed.

In modeling A, one uses a diagram of Gauss at 12 points by triangular facets of contact. In modeling C, one only uses a diagram reduced to 4 points.

Indeed, the diagram must allow the exact integration of a constant field of pressure. The intégrande on the facet is then a students' rag procession in $x^i y^i$ with $i+j \leq 3$.

According to [bib4], a diagram at 4 points of Gauss is enough.

4.1 Sizes tested and results

One tests the same values as for modeling A.

Identification	Reference	% difference
LAGS_C for all the points of contact	-5.00	0.20
LAGS_F1 for all the points of contact	0.00	1.00e-10
LAGS_F2 for all the points of contact	0.00	1.00e-10

To test all the nodes of the interface in only once, one tests the values minimum and maximum.

4.2 Comments

This modeling shows that a diagram of integration reduced to 4 points makes it possible to pass the patch test where the solution in pressure is constant.

5 Modeling D: right interface in plane constraints, method of Lagrangian increased

In this modeling, one represents a right interface, the angle θ 0 are worth then. The interface coincides with the faces of certain finite elements.

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there a difference between plane constraints and deformations).

The method of Lagrangian increased is used for the treatment of the contact/friction.

5.1 Analytical resolution

The interface being right, and the compactness is uniaxial and normal with the interface, there is no possible slip. The solution of the problem is that of the same problem without interface.
The constraint in the structure is:

$$\sigma_{zz}=E\frac{UY}{LY} \quad \textbf{éq 8.1-1}$$

and the value of the contact pressure on the interface is:

$$\lambda=\sigma_{zz} \quad \textbf{éq 8.1-2}$$

With the digital values previously introduced, $\lambda=-5.0\,Pa$.

5.2 Features tested

This case does not require the activation of friction. In the operator DEFI_CONTACT, one stipulates then FRICTION= 'WITHOUT'.

Moreover, as of the first iteration of the active constraints, one makes the assumption that the points of contact have a contacting statute. This is possible while specifying CONTACT_INIT= 'YES'.
If not, at the end of the first iteration, the contact not being activated, the higher block returns in the lower block but the two blocks did not become deformed. Their state of stresses is thus null, and should then be chosen a total criterion (RESI_GLOB_MAXI) for the convergence of the algorithm of Newton - Raphson [bib3], criterion which is likely to be unsuited in the continuation of calculations when the contact is activated.
To avoid that, and to have a relative criterion, one needs a state of stresses not no one as of the first iteration, and thus to activate the contact as of the beginning.

The algorithm of the active constraints thus converges in an iteration.

5.3 Sizes tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of the operator STAT_NON_LINE and of the loop on the active constraints. All the points of contact are tested, which correspond to the nodes of the grid on the interface. It is checked that one finds well the values determined with [§5.1].

Identification	Reference	% difference
LAGS_C for all the nodes of the interface	-5.00	1.00e-10
LAGS_F1 for all the nodes of the interface	0.00	1.00e-5

To test all the nodes in only once, one tests the minimum and the maximum of column.

5.4 Comments

This modeling shows the possibilities of the continuous method of contact applied to framework X - FEM in $2D$. The advantage is that the procedure of pairing is intrinsic with method X-FEM since here, there are not really surface Master and slave for the treatment of the contact since one remains in small displacements.

6 Modeling E: leaning interface in plane constraints, method of Lagrangian increased

In this modeling, a leaning interface is represented. The angle θ is worth $30°$. The interface does not coincide any more with the faces of the finite elements, and cuts the elements now. The normal with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}1/2\\ \sqrt{3}/2\end{pmatrix}, \qquad \tau=\begin{pmatrix}-\sqrt{3}/2\\ 1/2\end{pmatrix}$$ **éq 10-1**

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there a difference between plane constraints and deformations).

The method of Lagrangian increased is used for the treatment of the contact/friction.

6.1 Analytical resolution

To avoid the slip one takes $\mu=1$.

One must have:

$$\lambda=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}=n_y\sigma_{yy}n_y$$ **éq 10.1-2**

where n_y is the component according to y of n .

The semi-multiplier of friction Λ is defined by:

$$r_\tau=\lambda\mu\Lambda$$ **éq 10.1-3**

With the density of tangential stress being written as follows:

$$r_\tau=(\tau.\sigma.n)\tau$$ **éq 10.1-4**

From where:

$$\Lambda=\left(\frac{1}{\mu}\frac{\tau.\sigma.n}{n.\sigma.n}\right)\tau=\left(\frac{1}{\mu}\frac{\tau_y}{n_y}\right)\tau$$ **éq 10.1-5**

One takes $\mu=1$.
With the digital values previously introduced, $\lambda=-3.75\,Pa$ and $\Lambda.\tau=1/\sqrt{3}$.

6.2 Sizes tested and results

The value is testedS of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of the operator STAT_NON_LINE, loop on the active constraints and of the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the grid of visualization: GROUP_NO points of contact is extracted by the keyword PREF_GROUP_CO order POST_CHAM_XFEM.

Identification	Reference	% difference
LAGS_C for all the points of contact	-3.75	0.01

LAGS_F1 for all the points of contact	5.77350E-01	0.01

To test all the points of contact in only once, one tests the minimum and the maximum of column.

7 Modeling F: leaning interface in plane deformations, method of Lagrangian increased

This modeling is the same one as modeling E, except the fact that one considers the assumption of plane deformations. As the Poisson's ratio is null, that does not affect the results.

7.1 Sizes tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of the opérateur STAT_NON_LINE, loop on the active constraints and of the loop on the thresholds of friction. All the points of contact are tested. One tests the value of the multipliers of contact and friction at the points of contact of the grid of visualization: GROUP_NO points of contact is extracted by the keyword PREF_GROUP_CO order POST_CHAM_XFEM.

Identification	Reference	% difference
LAGS_C for all the points of contact	-3.75	0.01
LAGS_F1 for all the points of contact	5.77350E-01	0.01

To test all the points of contact in only once, one tests the minimum and the maximum of column.

8 Modeling G: right interface, penalized method

In this modeling, one represents a right interface, the angle θ 0 are worth then. The interface coincides with the faces of certain finite elements. The penalized method is used for the treatment of the contact/friction.

8.1 Analytical resolution

The analytical solution is the same one as that of modeling A.

8.2 Features tested

In the operator DEFI_CONTACT, one stipulates then FRICTION= 'COULOMB', and the coefficient of friction is selected equal to 1.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the matrix of rigidity.

8.3 Sizes tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of the operator STAT_NON_LINE and of the loop on the active constraints. All the points of contact are tested, which correspond to the nodes of the grid on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% difference
LAGS_C for all the nodes of the interface	-5.00	1.00e-10
LAGS_F1 for all the nodes of the interface	0.	1.00e-10
LAGS_F2 for all the nodes of the interface	0.	1.00e-10

To test all the nodes in only once, the value is tested MIN and the value MAX column.

8.4 Comments

With the help of important coefficients of penalization, the penalized method makes it possible to get the same results as the method of Lagrangian increased.

9 Modeling H: leaning interface, penalized method

In this modeling, a leaning interface is represented. The angle θ is worth $\arctan(1/2)$, that is to say a slope α being worth $-1/2$. The interface does not coincide any more with the faces of the finite elements, and cuts the elements now. The normal with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}0\\1/\sqrt{5}\\2/\sqrt{5}\end{pmatrix},\qquad \tau=\begin{pmatrix}0\\2/\sqrt{5}\\-1/\sqrt{5}\end{pmatrix} \qquad \text{éq 16-1}$$

The penalized method is used for the treatment of the contact/friction.

9.1 Analytical resolution

The analytical solution is the same one as that of modeling B.

9.2 Features tested

This case requires the activation of friction. In the operator DEFI_CONTACT, one stipulates then FRICTION= 'COULOMB'.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the matrix of rigidity.

9.3 Sizes tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of the operator STAT_NON_LINE, loop on the active constraints and of the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the grid of visualization: GROUP_NO points of contact is extracted by the keyword PREF_GROUP_CO order POST_CHAM_XFEM.
It is checked that one finds well the values determined with [§3.1]. LAGS_F1 corresponds to the semi - multiplying of friction in the direction Ox (it is thus no one), whereas LAGS_F2 corresponds to the semi-multiplier of following friction τ .

Identification	Reference	% difference
LAGS_C for all the points of contact	-4.00	5.00e-3
LAGS_F1 for all the points of contact	0.00	5.00e-3
LAGS_F2 for all the points of contact	-0.50	5.00e-3

To test all the points of contact in only once, the value is tested MIN and the value MAX column.

9.4 Comments

With the help of important coefficients of penalization, the penalized method makes it possible to get the same results as the method of Lagrangian increased.

10 Modeling I: right interface in plane constraints, penalized method

In this modeling, one represents a right interface, the angle θ 0 are worth then. The interface coincides with the faces of certain finite elements.

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there a difference between plane constraints and deformations).

The penalized method is used for the treatment of the contact/friction.

10.1 Analytical resolution

The analytical solution is the same one as that of modeling D.

10.2 Features tested

This case does not require the activation of friction. In the operator DEFI_CONTACT, one stipulates then FRICTION= 'WITHOUT'.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the matrix of rigidity.

10.3 Sizes tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of the operator STAT_NON_LINE and of the loop on the active constraints. All the points of contact are tested, which correspond to the nodes of the grid on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% Tolerance
LAGS_C for all the nodes of the interface	-5.00	0,001
LAGS_F1 for all the nodes of the interface	0.00	0,001

To test all the nodes in only once, one tests the minimum and the maximum of column.

10.4 Comments

With the help of important coefficients of penalization, the penalized method makes it possible to get the same results as the method of Lagrangian increased.

11 Modeling J: leaning interface in plane constraints, penalized method

In this modeling, a leaning interface is represented. The angle θ is worth $30°$. The interface does not coincide any more with the faces of the finite elements, and cuts the elements now. The normal with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}1/2\\ \sqrt{3}/2\end{pmatrix}, \qquad \tau=\begin{pmatrix}-\sqrt{3}/2\\ 1/2\end{pmatrix} \quad \textbf{éq 20-1}$$

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there a difference between plane constraints and deformations).

The penalized method is used for the treatment of the contact/friction.

11.1 Analytical resolution

The analytical solution is the same one as that of modeling E.

11.2 Features tested

In the operator DEFI_CONTACT, one stipulates then FRICTION= 'COULOMB', and the coefficient of friction is selected equal to 1.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the matrix of rigidity.

11.3 Sizes tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of the operator STAT_NON_LINE, loop on the active constraints and of the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the grid of visualization: GROUP_NO points of contact is extracted by the keyword PREF_GROUP_CO order POST_CHAM_XFEM.

Identification	Reference	% Tolerance
LAGS_C for all the points of contact	-3.75	0.01
LAGS_F1 for all the points of contact	5.77350E-01	0.01

To test all the points of contact in only once, the MIN and the MAX of the column are tested.

11.4 Comments

With the help of important coefficients of penalization, the penalized method makes it possible to get the same results as the method of Lagrangian increased.

12 Modeling K: right interface in plane constraints, method of Lagrangian increased on a quadratic grid

In this modeling, one represents a right interface, the angle θ 0 are worth then. The interface coincides with the faces of certain finite elements.

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there a difference between plane constraints and deformations).

The contact friction is treated with quadratic elements X-FEM $P2P1$, i.e. carrying the degrees of freedom of displacement on all the nodes and the lagranges of contact friction on the nodes tops.

12.1 Analytical resolution

The analytical solution is the same one as that of modeling D.

12.2 Features tested

This case does not require the activation of friction. In the operator `DEFI_CONTACT`, one stipulates then `FRICTION`= '`WITHOUT`'.

12.3 Sizes tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of the operator `STAT_NON_LINE` and of the loop on the active constraints. All the points of contact are tested, which correspond to the nodes of the grid on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% Tolerance
`LAGS_C` for all the nodes of the interface	-5.00	1.00e-10

To test all the nodes in only once, one tests the minimum and the maximum of column.

12.4 Comments

This valid test:

- the calculation of the matrix of rigidity (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom)
- the calculation of the matrices of contact (integration on one `SEG3` at the points of Gauss)

It does not make it possible to validate under cutting since the interface coincides with the faces of the elements.

13 Modeling L: leaning interface in plane constraints, method of Lagrangian increased on a quadratic grid

In this modeling, a leaning interface is represented. The angle θ is worth $30°$. The interface does not coincide any more with the faces of the finite elements, and cuts the elements now. The normal with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}1/2\\ \sqrt{3}/2\end{pmatrix}, \qquad \tau=\begin{pmatrix}-\sqrt{3}/2\\ 1/2\end{pmatrix} \qquad \textbf{éq 20-1}$$

In this modeling, the assumption of plane constraints is considered (although here, the Poisson's ratio being null, it does not have there a difference between plane constraints and deformations).

The contact friction is treated with quadratic elements X-FEM $P2P1$, i.e. carrying the degrees of freedom of displacement on all the nodes and the lagranges of contact friction on the nodes tops.

13.1 Analytical resolution

The analytical solution is the same one as that of modeling E.

13.2 Features tested

In the operator DEFI_CONTACT, one stipulates then FRICTION= 'COULOMB', and the coefficient of friction is selected equal to 1.

13.3 Sizes tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of the operator STAT_NON_LINE, loop on the active constraints and of the loop on the thresholds of friction. One tests all the points of contact, knowing that the points of contact are nodes tops of the elements cut by the interface.

Identification	Reference	% Tolerance
LAGS_C for all the points of contact	-3.75	0.01
LAGS_F1 for all the points of contact	5.77350E-01	0.01

To test all the points of contact in only once, the value is tested MIN and the value MAX column.

13.4 Comments

This valid test:

- the calculation of the matrix of rigidity (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom)
- the calculation of the matrices of contact (integration on one SEG3 at the points of Gauss)
- under cutting (configurations in right interface and elements on right board)

Linear solvor MUMPS detects a singularity in the matrix for the node $N252$ on the component DY . This problem already arose on many other cases tests X-FEM in contact. The readjustment of the level set normal makes it possible to improve conditioning of the matrix but to the detriment of a too great error of discretization of the level set. One thus keeps the option of desactivation of the detection of singularity of the solvor before suggesting a more satisfactory solution.

14 Modeling M: leaning interface in 3D, method of Lagrangian increased on a quadratic grid

We take again characteristics identical to modeling B, put except for the grid which is quadratic. For recall, the interface is thus tilted of an angle θ being worth $\arctan(1/2)$, that is to say a slope α being worth $-1/2$. Modeling is three-dimensional, the interface not coinciding with the faces of the finite elements. The normal $\boldsymbol{n}$ and the vector tangent τ to the interface are given by (éq 4-1).

The method of Lagrangian increased is used for the treatment of the rubbing contact. The contact rubbing is treated with elements XFEM quadratic P2P1, i.e. carrying the degrees of freedom of displacement on all Nœuds and "lagranges" of contact/friction on Nœuds tops.

14.1 Analytical resolution

The analytical solution is the same one as that of modeling B.

14.2 Features tested

In the operator DEFI_CONTACT, one stipulates then FROTTEMENT=' COULOMB', and the coefficient of friction is selected equal to 1. This test is used to test the contact XFEM on the quadratic grids (attribute CONTACT=' P2P1' in the order MODI_MODELE_XFEM) in the three-dimensional case. Friction is activated, therefore it is also tested (FROTTEMENT=' COULOMB').

14.3 Sizes tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence. All the points of contact are tested. Let us recall that in this discretization 'P2P1' the degrees of freedom of contact are carried by Nœuds tops of the elements cut by the interface: it is thus in these points that one tests the values.

Identification	Reference	% Tolerance
LAGS_C for all the points of contact	-3.75	0.01
LAGS_F1 for all the points of contact	5.77350E-01	0.01

To test all the points of contact in only once, the value is tested MIN and the value MAX column.

14.4 Comments

This test validates modeling P2P1 in 3D namely:

- the calculation of the matrix of rigidity (the good shift during the filling of the matrix because Nœuds does not increase the same number of degrees of freedom),
- the calculation of the matrices of contact.

At present, one under-cutting quadratic elements 3D with linear elements, quadratic under-cutting, possibly with curved faces, not being yet available. In the same way, the facets of contact are TRI3. This test thus does not constitute a validation of subelements or facets of contact individuals.

15 Summaries of the results

The goals of this test are achieved:

- It is a question of showing the feasibility of the taking into account of the contact rubbing on the lips of the crack with the method continues adapted to framework X-FEM. Only the case of a crack crossing the structure completely was considered (interface).
- Cases where the interface follows the border of the elements ($\theta=0°$) and where the interface cuts the elements ($\theta=26.56°$ in $3D$ and $\theta=30°$ in $2D$) were validated.
- The method was validated in $2D$ for $P1P1A$ and $P2P1$ and in $3D$ for $P1P1A$.
- The method was validated at the same time with the method of Lagrangian increased and the method penalized for the treatment of the contact/friction.