

## SSNV104 - Contact of two Sphères

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### Summary:

Calculation consists in crushing two hemispheres one on the other to test the algorithm of unilateral contact in statics. The solution is compared with the analytical result of Hertz.

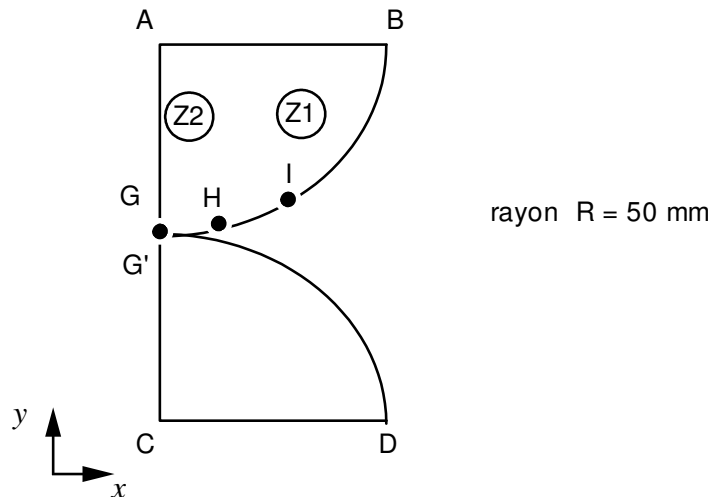
Modelings implement calculations in axisymmetric as in 3D with linear/quadratic elements and various methods of resolutions of the contact (dualisation, penalization, continuous formulation, Mortar pairing).

This classical test for which one has an analytical solution is also used to validate several options of the algorithm of pairing.

## 1 Problem of reference

### 1.1 Geometry

The geometry consists of two half-spheres of identical ray which one represents a cut in the figure below.



Taking into account the differences in grid, positions of the nodes  $H$  and  $I$  indicated here in an approximate way are different in 3 modelings.

### 1.2 Properties of material

$$E = 20\,000. \text{ MPa}$$

$$\nu = 0.3$$

### 1.3 Boundary conditions and loadings

Crushing imposed on the spheres:

- $DY$  imposed  $-2\text{ mm}$  for the nodes of  $AB$
- $DY$  imposed  $+2\text{ mm}$  for the nodes of  $CD$

For axisymmetric calculations:

- $DX$  blocked on the axis  $AC$

For calculations 3D (2 modelled quarters of sphere):

- conditions of symmetry on each quarter of sphere

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

In 1881, Hertz established under certain assumptions a solution with the problem which bears its name[1]. Thus by supposing a contact without friction and for which the half width of contact  $a$  is very small in front of the ray of the spheres  $R$  ( $a \ll R$ ), the contact pressure at the points  $C1$  and  $C2$  is worth [2] :

$$P_0 = -\frac{E}{\pi(1-\nu^2)} \sqrt{\frac{2h}{R}} \quad \text{éq 2.1-1}$$

where  $h$  corresponds to the imposed crushing, which is worth here  $4 \text{ mm}$ . That is to say  $P_0 = -2798.3 \text{ Mpa}$ .

The half-width of contact  $a$  express yourself according to imposed crushing and of the ray of the spheres:

$$a = \sqrt{\frac{Rh}{2}} \quad \text{éq 2.1-2}$$

In this test, for a crushing of  $4 \text{ mm}$ ,  $a = 10 \text{ mm}$ .

The surface of contact is a disc of ray  $a$ , the distribution of pressure in this zone is the following one:

$$\text{If } x \leq a \text{ then } P(x) = P_0 \sqrt{1 - \left(\frac{x}{a}\right)^2} \quad \text{éq 2.1-3}$$

### 2.2 Sizes and results of reference

$\sigma_{yy}$  at the point  $G$  (analytical solution).

Displacements in three points of the edge (not-regression except for  $G$  analytical).

$\sigma_{yy}$  in meshes resting on  $G$  (not-regression).

Statute and game in several points of the surface of contact (analytical).

### 2.3 Uncertainties on the solution

Analytical calculation although valid for  $a \ll R$  in general give a very good approximation of the solution.

### 2.4 Bibliographical reference

## Bibliography

1: HERTZ H, Über die Berührung fester elastischer Körper, 1881

2: DUMONT G, La méthode des contraintes actives appliquée au contact unilatéral, 1993

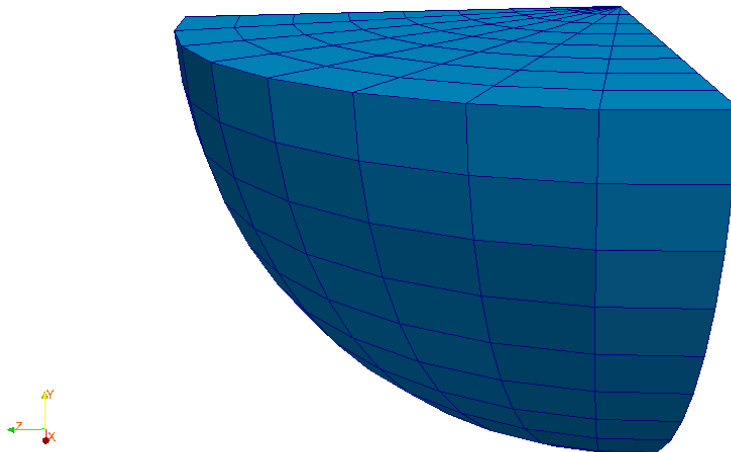
## 3 Modeling A

### 3.1 Characteristics of modeling

A modeling is used 3D. Due to symmetry, only one quarter of hemisphere is represented, a condition of type unilateral connection makes it possible to supplement modeling.

In this modeling, points  $H$  and  $H_p$  (respectively  $I$  and  $I_p$ ) are the same points (compared to those described in the geometry) but located on the two faces supporting the conditions of symmetry. The results got with this modeling are used as reference (AUTRE\_ASTER) for three-dimensional modelings I and J.

### 3.2 Characteristics of the grid



Nodes: 1849.

Meshes: 96 TETRA4, 2936 PENTA6, 112 PYRAM5.

### 3.3 Sizes tested and results

The first calculation (formulation 'LIAISON\_UNIL')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M2948 not G	'ANALYTICAL'	-2798.3 N	14.0%
$\sigma_{yy}$ mesh M2948 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M2960 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M2972 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M2984 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M2996 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M3008 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M3020 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
$\sigma_{yy}$ mesh M3032 not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%

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$DX$ not $G$	'ANALYTICAL'	0 mm	1.0E-10
$DX$ not $H$	'NON_REGRESSION'	-0.11343 mm	1,0E-4%
$DZ$ not $H_p$	'NON_REGRESSION'	-0.11343 mm	1,0E-4%
$DY$ not $H$	'NON_REGRESSION'	-0.16291 mm	1,0E-4%
$DY$ not $H_p$	'NON_REGRESSION'	-0.16291 mm	1,0E-4%
$DX$ not $I$	'NON_REGRESSION'	-0.17845 mm	1,0E-4%
$DZ$ not $I_p$	'NON_REGRESSION'	-0.17845 mm	1,0E-4%
$DY$ not $I$	'NON_REGRESSION'	-0.62966 mm	1,0E-4%
$DY$ not $I_p$	'NON_REGRESSION'	-0.62966 mm	1,0E-4%

## 3.4 Remarks

The results got in this three-dimensional modeling are in concord with the analytical solution. One notes moreover that the radiant grid makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point  $G$  are similar.

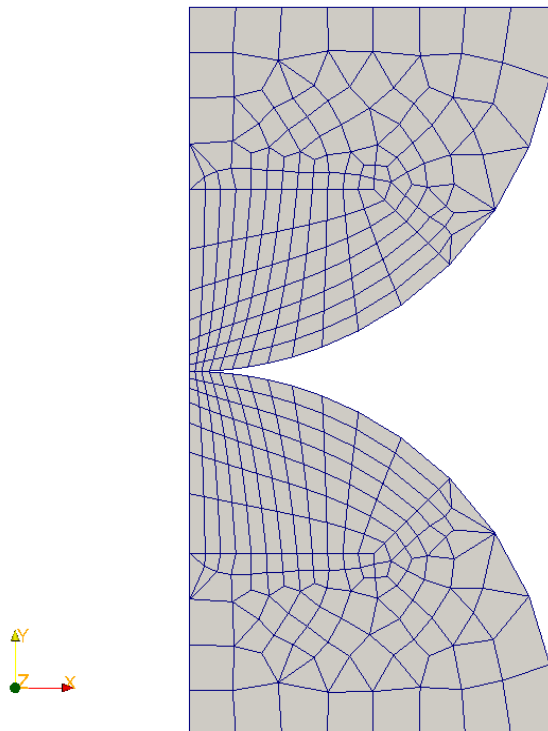
This modeling validates the use of a unilateral condition function of the space, i.e. whose "game" is not the same one for all the constrained nodes. The got results are identical to a modeling of the contact.

## 4 Modeling B

### 4.1 Characteristics of modeling

A modeling is used `AXIS`. Three calculations are carried out with options of pairing or different algorithms of contact.

### 4.2 Characteristics of the grid



Nodes: 376 nodes.

Meshes: 30 `TRIA3` and 324 `QUAD4`.

### 4.3 Sizes tested and results

The first calculation (algorithm `'PENALIZATION'`, nodal pairing, normal slave)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291(G)</i>	'ANALYTICAL'	-2798.3 <i>N</i>	7,0%
<i>DX</i> node <i>N291(G)</i>	'ANALYTICAL'	0 <i>mm</i>	1.0E-10
<i>DX</i> node <i>N287(H)</i>	'NON_REGRESSION'	-0.111338 <i>mm</i>	-
<i>DY</i> node <i>N287(H)</i>	'NON_REGRESSION'	-0.161811 <i>mm</i>	-
<i>DX</i> node <i>N285(I)</i>	'NON_REGRESSION'	-0.168635 <i>mm</i>	-
<i>DY</i> node <i>N285(I)</i>	'NON_REGRESSION'	-0.628585 <i>mm</i>	-

The second calculation (algorithm 'PENALIZATION')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G)	'ANALYTICAL'	-2798.3 N	7,0%
DX node N291(G)	'ANALYTICAL'	0 mm	1.0E-10
DX node N287(H)	'NON_REGRESSION'	-0.108104 mm	-
DY node N287(H)	'NON_REGRESSION'	-0.164375 mm	-
DX node N285(I)	'NON_REGRESSION'	-0.160912 mm	-
DY node N285(I)	'NON_REGRESSION'	-0.631411 mm	-

The third calculation (algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G)	'ANALYTICAL'	-2798.3 N	7,0%
DX node N291(G)	'ANALYTICAL'	0 mm	1.0E-10
DX node N287(H)	'NON_REGRESSION'	-0.108104 mm	-
DY node N287(H)	'NON_REGRESSION'	-0.164375 mm	-
DX node N285(I)	'NON_REGRESSION'	-0.160912 mm	-
DY node N285(I)	'NON_REGRESSION'	-0.631409 mm	-

## 4.4 Remarks

The variation enters the contact pressure at the point  $G$  and the analytical solution is explained on the one hand by extrapolation (passage points of Gauss towards nodes) and on the other hand by the weak refinement of the grid.

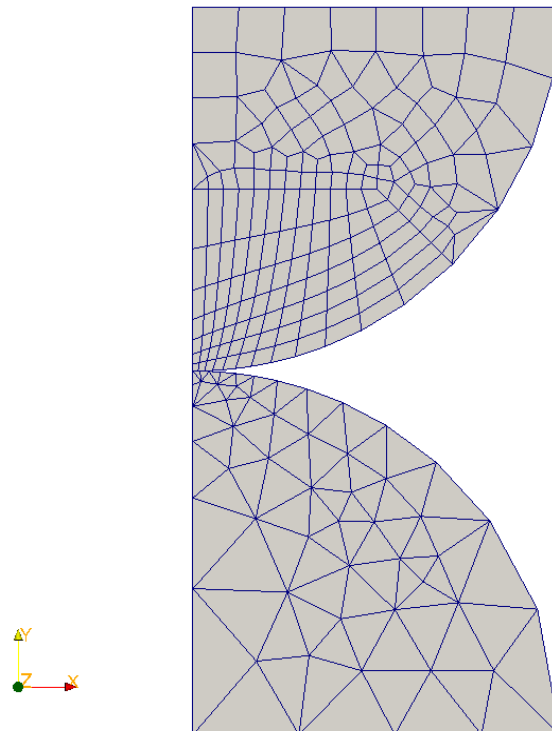
It is noted that the algorithms of contact 'PENALIZATION' and 'FORCED' the same results give exactly and that nodal pairing makes it possible to preserve the symmetry of the problem while giving results very close to pairing node-facet.

## 5 Modeling D

### 5.1 Characteristics of modeling

A modeling is used `AXIS`. Two calculations are carried out with different options of pairing. In this modeling, imposed crushing goes until  $10\text{ mm}$ .

### 5.2 Characteristics of the grid



Nodes: 243.

Meshes: 100 TRIA3, 162 QUAD4.

### 5.3 Sizes tested and results

The first calculation (method `'PENALIZATION'`, smoothing)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291</i> ( <i>G</i> ) moment 1.0	'ANALYTICAL'	-1978.7 <i>N</i>	7.0%
$\sigma_{yy}$ mesh <i>M286</i> node <i>N55</i> ( <i>G'</i> ) moment 1.0	'ANALYTICAL'	-1978.7 <i>N</i>	1.0%
$\sigma_{yy}$ mesh <i>M293</i> node <i>N55</i> ( <i>G'</i> ) moment 1.0	'ANALYTICAL'	-1978.7 <i>N</i>	15.0%
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291</i> ( <i>G</i> ) moment 2.0	'ANALYTICAL'	-2798.3 <i>N</i>	5.0%
$\sigma_{yy}$ mesh <i>M286</i> node <i>N55</i> ( <i>G'</i> ) moment 2.0	'ANALYTICAL'	-2798.3 <i>N</i>	3.0%
$\sigma_{yy}$ mesh <i>M293</i> node <i>N55</i> ( <i>G'</i> ) moment 2.0	'ANALYTICAL'	-2798.3 <i>N</i>	15.0%

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$\sigma_{yy}$ mesh M31 node N291(G) moment 3.0	'ANALYTICAL'	-3427.2 N	2.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	15.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 5.0	'ANALYTICAL'	-4424.5 N	6.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	4.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

The second calculation (method 'FORCED', nodal pairing and normal slave)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G) moment 1.0	'ANALYTICAL'	-1978.7 N	2.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 1.0	'ANALYTICAL'	-1978.7 N	4.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 2.0	'ANALYTICAL'	-2798.3 N	7.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 2.0	'ANALYTICAL'	-2798.3 N	7.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 2.0	'ANALYTICAL'	-2798.3 N	4.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 3.0	'ANALYTICAL'	-3427.2 N	18.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	13.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 4.0	'ANALYTICAL'	-3957.4 N	28.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	20.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	2.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 5.0	'ANALYTICAL'	-4424.5 N	38.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	27.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	1.0%

Third calculation (method 'CONTINUES', smoothing)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G) moment 1.0	'ANALYTICAL'	-1978.7 N	7,5%
$\sigma_{yy}$ mesh M286 node N55(G') moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 2.0	'ANALYTICAL'	-2798.3 N	5.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 2.0	'ANALYTICAL'	-2798.3 N	3.0%

$\sigma_{yy}$ mesh M293 node N55(G')	moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%
$\sigma_{yy}$ mesh M31 node N291(G)	moment 3.0	'ANALYTICAL'	-3427.2 N	2.0%
$\sigma_{yy}$ mesh M286 node N55(G')	moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G')	moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
$\sigma_{yy}$ mesh M31 node N291(G)	moment 4.0	'ANALYTICAL'	-3957.4 N	4, 0%
$\sigma_{yy}$ mesh M286 node N55(G')	moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
$\sigma_{yy}$ mesh M293 node N55(G')	moment 4.0	'ANALYTICAL'	-3957.4 N	15.0%
$\sigma_{yy}$ mesh M31 node N291(G)	moment 5.0	'ANALYTICAL'	-4424.5 N	9, 0%
$\sigma_{yy}$ mesh M286 node N55(G')	moment 5.0	'ANALYTICAL'	-4424.5 N	4.0%
$\sigma_{yy}$ mesh M293 node N55(G')	moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

## 5.4 Remarks

The grid consists of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).

On lower surface, one notes contrary to the considerable variations because there exist 2 meshes which contribute to the point G' (grid of TRIA3), however if one makes the average of the results got on these 2 meshes, one finds a pressure in concord with the analytical solution.

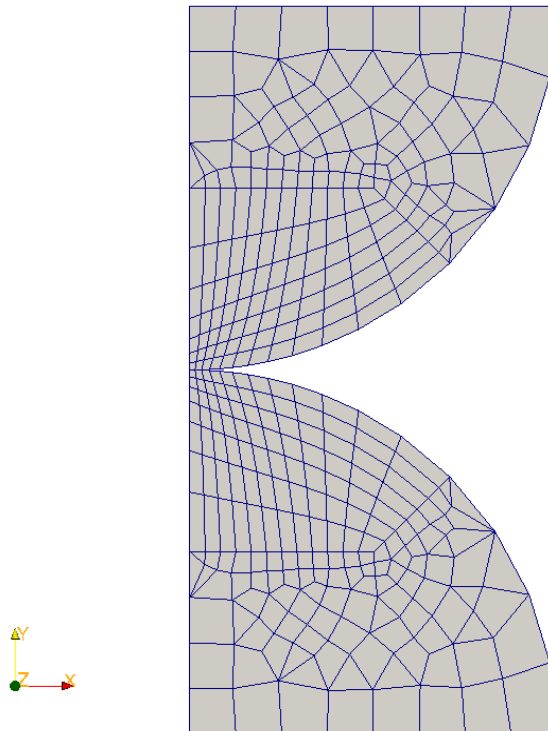
As regards the second calculation the important errors are due to the use of a nodal pairing whereas the grid is not compatible.

## 6 Modeling E

### 6.1 Characteristics of modeling

A modeling is used `AXIS` with the formulation `CONTINUOUS` contact. This modeling validates the functionality `SLIDE` by slackening imposed crushing.

### 6.2 Characteristics of the grid



Nodes: 376 nodes.

Meshes: 30 TRIA3 and 324 QUAD4.

### 6.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G)	'ANALYTICAL'	-2798.3 N	7,0%
$\sigma_{yy}$ mesh M31 node N291(G)	'NON_REGRESSION'	-2971.37 N	1.0%
DX node N291(G)	'ANALYTICAL'	0 mm	1.0E-10
DX node N287(H)	'NON_REGRESSION'	-0.110211 mm	3.0%
DY node N287(H)	'NON_REGRESSION'	-0.162911 mm	1.0%
DX node N285(I)	'NON_REGRESSION'	-0.165946 mm	4.0%
DY node N285(I)	'NON_REGRESSION'	-0.629666 mm	0.5%

Identification	Type of reference	Value of reference	Tolerance
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Statute	CONT	node	N72	moment	'ANALYTICAL'	0	0.1
		0.2					
Statute	CONT	node	N80	moment	'ANALYTICAL'	0	0.1
		0.2					
Statute	CONT	node	N81	moment	'ANALYTICAL'	0	0.1
		0.2					
Statute	CONT	node	N72	moment	'ANALYTICAL'	0	0.1
		0.5					
Statute	CONT	node	N80	moment	'ANALYTICAL'	0	0.1
		0.5					
Statute	CONT	node	N81	moment	'ANALYTICAL'	2	0.1
		0.5					
Statute	CONT	node	N72	moment	'ANALYTICAL'	2	0.1
		1.0					
Statute	CONT	node	N80	moment	'ANALYTICAL'	2	0.1
		1.0					
Statute	CONT	node	N81	moment	'ANALYTICAL'	2	0.1
		1.0					
Statute	CONT	node	N72	moment	'ANALYTICAL'	2	0.1
		1.5					
Statute	CONT	node	N80	moment	'ANALYTICAL'	2	0.1
		1.5					
Statute	CONT	node	N81	moment	'ANALYTICAL'	2	0.1
		1.5					
Game	JEU	node	N72	moment	'ANALYTICAL'	0	1.0E-8
		1.5					
Game	JEU	node	N80	moment	'ANALYTICAL'	0	1.0E-8
		1.5					
Game	JEU	node	N81	moment	'ANALYTICAL'	0	1.0E-8
		1.5					
Statute	CONT	node	N72	moment	'ANALYTICAL'	2	0.1
		2.0					
Statute	CONT	node	N80	moment	'ANALYTICAL'	2	0.1
		2.0					
Statute	CONT	node	N81	moment	'ANALYTICAL'	2	0.1
		2.0					
Game	JEU	node	N72	moment	'ANALYTICAL'	0	1.0E-8
		2.0					
Game	JEU	node	N80	moment	'ANALYTICAL'	0	1.0E-8
		2.0					
Game	JEU	node	N81	moment	'ANALYTICAL'	0	1.0E-8
		2.0					

## 6.4 Remarks

Results got with the formulation CONTINUOUS are very close to those obtained in formulation DISCRETE and in concord with the analytical solution.

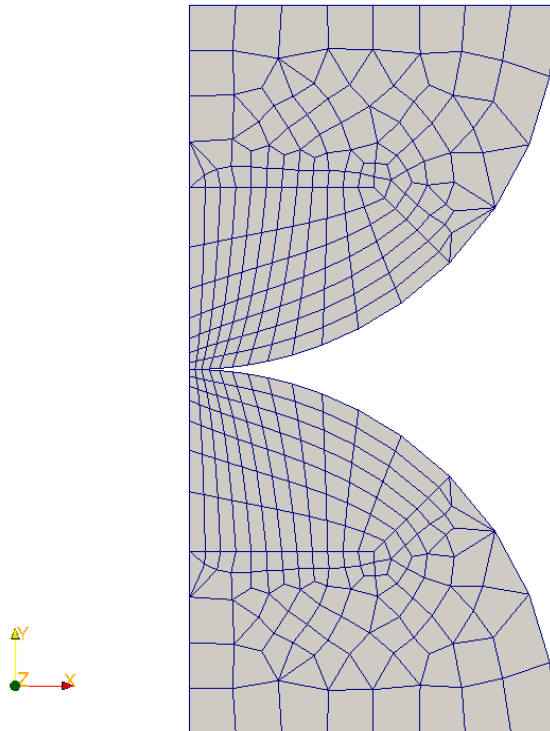
The differences noted on displacements are explained by the diagram of integration used in formulation CONTINUOUS : it is about a diagram to the nodes which does not make it possible correctly to treat the nullity of Jacobien on the axis of revolution.

## 7 Modeling F

### 7.1 Characteristics of modeling

A modeling is used `AXIS`. Four calculations are carried out with options of pairing or different algorithms of contact.

### 7.2 Characteristics of the grid



Nodes: 376 nodes.

Meshes: 30 TRIA3 and 324 QUAD4.

### 7.3 Sizes tested and results

The first calculation (nodal pairing, master-slave normal and algorithm `'PENALIZATION'`)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291(G)</i>	<code>'ANALYTICAL'</code>	-2798.3 <i>N</i>	7,0%
<i>DX</i> node <i>N291(G)</i>	<code>'ANALYTICAL'</code>	0 <i>mm</i>	1.0E-10
<i>DX</i> node <i>N287(H)</i>	<code>'NON_REGRESSION'</code>	-0.110211 <i>mm</i>	-
<i>DY</i> node <i>N287(H)</i>	<code>'NON_REGRESSION'</code>	-0.162912 <i>mm</i>	-
<i>DX</i> node <i>N285(I)</i>	<code>'NON_REGRESSION'</code>	-0.165946 <i>mm</i>	-
<i>DY</i> node <i>N285(I)</i>	<code>'NON_REGRESSION'</code>	-0.629667 <i>mm</i>	-

The second calculation (master-slave normal, algorithm `'PENALIZATION'`)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G)	'ANALYTICAL'	-2798.3 N	7,0%
DX node N291(G)	'ANALYTICAL'	0 mm	1.0E-10
DX node N287(H)	'NON_REGRESSION'	-0.110678 mm	-
DY node N287(H)	'NON_REGRESSION'	-0.162865 mm	-
DX node N285(I)	'NON_REGRESSION'	-0.167194 mm	-
DY node N285(I)	'NON_REGRESSION'	-0.628961 mm	-

The third calculation (master-slave normal, algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G)	'ANALYTICAL'	-2798.3 N	7,0%
DX node N291(G)	'ANALYTICAL'	0 mm	1.0E-10
DX node N287(H)	'NON_REGRESSION'	-0.110678 mm	-
DY node N287(H)	'NON_REGRESSION'	-0.162901 mm	-
DX node N285(I)	'NON_REGRESSION'	-0.167194 mm	-
DY node N285(I)	'NON_REGRESSION'	-0.628947 mm	-

The fourth calculation (smoothing, master-slave normal and algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G)	'ANALYTICAL'	-2798.3 N	7,0%
DX node N291(G)	'ANALYTICAL'	0 mm	1.0E-10
DX node N287(H)	'NON_REGRESSION'	-0.110211 mm	-
DY node N287(H)	'NON_REGRESSION'	-0.162911 mm	-
DX node N285(I)	'NON_REGRESSION'	-0.165946 mm	-
DY node N285(I)	'NON_REGRESSION'	-0.629666 mm	-

## 7.4 Remarks

On this modeling, it is noted that smoothing makes it possible to find the symmetry of the problem (nodes perfectly in opposite once the established contact), last calculation indeed which gets the same results as in nodal pairing.

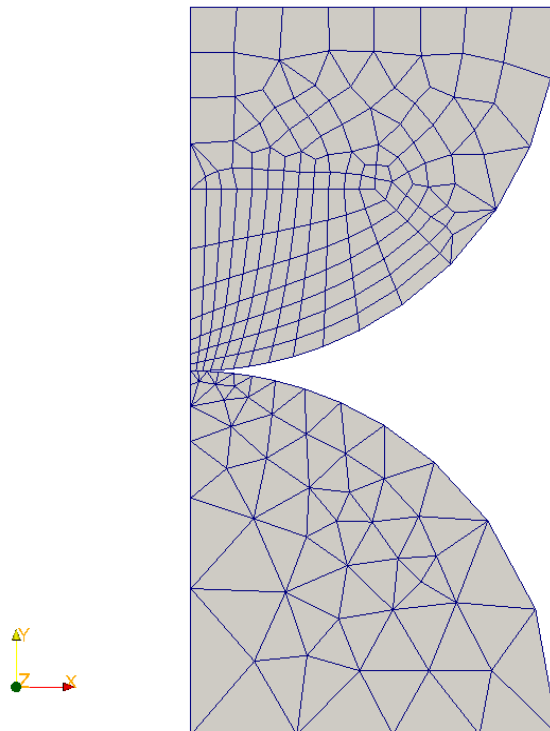
The values obtained are in concord general with the analytical solutions.

## 8 Modeling G

### 8.1 Characteristics of modeling

A modeling is used `AXIS`. Two calculations are carried out with different options of pairing. In this modeling, imposed crushing goes until  $10\text{ mm}$ .

### 8.2 Characteristics of the grid



Nodes: 243.

Meshes: 100 `TRIA3`, 162 `QUAD4`.

### 8.3 Sizes tested and results

The first calculation (smoothing, master-slave normal and algorithm `'PENALIZATION'`)

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291</i> ( <i>G</i> ) moment 1.0	'ANALYTICAL'	-1978.7 <i>N</i>	8.0%
$\sigma_{yy}$ mesh <i>M286</i> node <i>N55</i> ( <i>G'</i> ) moment 1.0	'ANALYTICAL'	-1978.7 <i>N</i>	1.0%
$\sigma_{yy}$ mesh <i>M293</i> node <i>N55</i> ( <i>G'</i> ) moment 1.0	'ANALYTICAL'	-1978.7 <i>N</i>	15.0%
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291</i> ( <i>G</i> ) moment 2.0	'ANALYTICAL'	-2798.3 <i>N</i>	6.0%
$\sigma_{yy}$ mesh <i>M286</i> node <i>N55</i> ( <i>G'</i> ) moment 2.0	'ANALYTICAL'	-2798.3 <i>N</i>	4.0%
$\sigma_{yy}$ mesh <i>M293</i> node <i>N55</i> ( <i>G'</i> ) moment 2.0	'ANALYTICAL'	-2798.3 <i>N</i>	15.0%

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$\sigma_{yy}$ mesh M31 node N291(G) moment 3.0	'ANALYTICAL'	-3427.2 N	1.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	14.5%
$\sigma_{yy}$ mesh M31 node N291(G) moment 5.0	'ANALYTICAL'	-4424.5 N	8.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

The second calculation (smoothing, master-slave normal and algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M31 node N291(G) moment 1.0	'ANALYTICAL'	-1978.7 N	8.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 2.0	'ANALYTICAL'	-2798.3 N	6.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 2.0	'ANALYTICAL'	-2798.3 N	4.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 3.0	'ANALYTICAL'	-3427.2 N	1.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 4.0	'ANALYTICAL'	-3957.4 N	14.0%
$\sigma_{yy}$ mesh M31 node N291(G) moment 5.0	'ANALYTICAL'	-4424.5 N	8.0%
$\sigma_{yy}$ mesh M286 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	3.0%
$\sigma_{yy}$ mesh M293 node N55(G') moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

## 8.4 Remarks

The grid consists of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).

On lower surface, one notes contrary to the considerable variations because there exist 2 meshes which contribute to the point G' (grid of TRIA3), however if one makes the average of the results got on these 2 meshes, one finds a pressure in concord with the analytical solution.

The two algorithms of contact used give the same results appreciably.

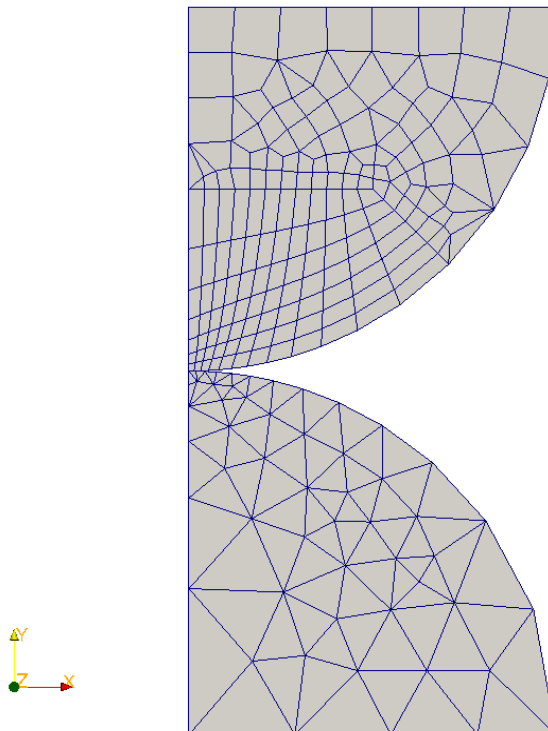


## 9 Modeling H

### 9.1 Characteristics of modeling

A modeling is used `AXIS`. Only one calculation with penalization of the contact is carried out. In this modeling, imposed crushing goes until  $10\text{ mm}$ .

### 9.2 Characteristics of the grid



Nodes: 243.

Meshes: 100 `TRIA3`, 162 `QUAD4`.

### 9.3 Sizes tested and results

The first calculation (smoothing, algorithm '`PENALIZATION`')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291</i> ( <i>G</i> ) moment 1.0	'ANALYTICAL'	-1978.7 N	7.0%
$\sigma_{yy}$ mesh <i>M286</i> node <i>N55</i> ( <i>G'</i> ) moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
$\sigma_{yy}$ mesh <i>M293</i> node <i>N55</i> ( <i>G'</i> ) moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
$\sigma_{yy}$ mesh <i>M31</i> node <i>N291</i> ( <i>G</i> ) moment 2.0	'ANALYTICAL'	-2798.3 N	5.0%
$\sigma_{yy}$ mesh <i>M286</i> node <i>N55</i> ( <i>G'</i> ) moment 2.0	'ANALYTICAL'	-2798.3 N	3.0%
$\sigma_{yy}$ mesh <i>M293</i> node <i>N55</i> ( <i>G'</i> ) moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%

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$\sigma_{yy}$ mesh M31 node N291( $G$ ) moment 3.0	'ANALYTICAL'	-3427.2	$N$	2.0%
$\sigma_{yy}$ mesh M286 node N55( $G'$ ) moment 3.0	'ANALYTICAL'	-3427.2	$N$	3.0%
$\sigma_{yy}$ mesh M293 node N55( $G'$ ) moment 3.0	'ANALYTICAL'	-3427.2	$N$	14.0%
$\sigma_{yy}$ mesh M31 node N291( $G$ ) moment 4.0	'ANALYTICAL'	-3957.4	$N$	3.0%
$\sigma_{yy}$ mesh M286 node N55( $G'$ ) moment 4.0	'ANALYTICAL'	-3957.4	$N$	4.0%
$\sigma_{yy}$ mesh M293 node N55( $G'$ ) moment 4.0	'ANALYTICAL'	-3957.4	$N$	15.0%
$\sigma_{yy}$ mesh M31 node N291( $G$ ) moment 5.0	'ANALYTICAL'	-4424.5	$N$	6.0%
$\sigma_{yy}$ mesh M286 node N55( $G'$ ) moment 5.0	'ANALYTICAL'	-4424.5	$N$	4.0%
$\sigma_{yy}$ mesh M293 node N55( $G'$ ) moment 5.0	'ANALYTICAL'	-4424.5	$N$	15.0%

## 9.4 Remarks

The grid consists of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).

On lower surface, one notes contrary to the considerable variations because there exist 2 meshes which contribute to the point  $G'$  (grid of TRIA3), however if one makes the average of the results got on these 2 meshes, one finds a pressure in concord with the analytical solution.

The coefficient of penalization used here (large in front  $R_{sphere} * E$ ) allows to get results comparable to those of the exact algorithms of dualisation (modeling G).

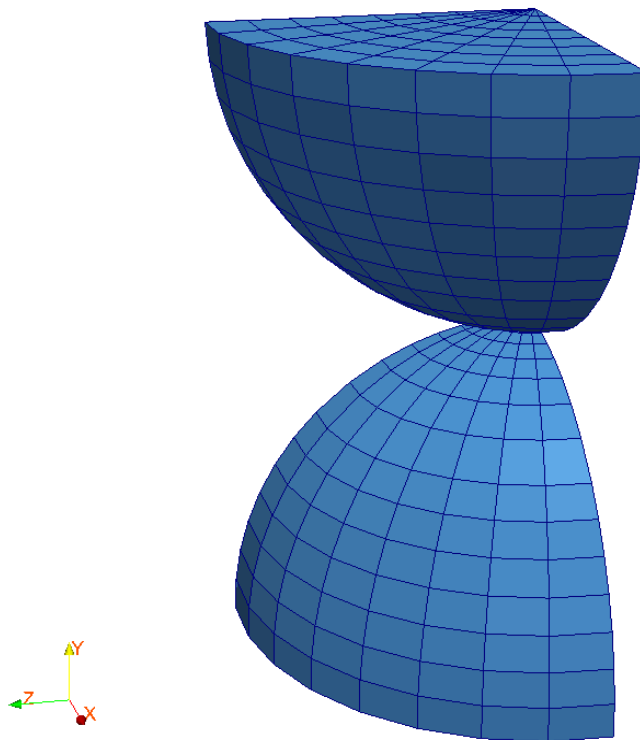
## 10 Modeling I

### 10.1 Characteristics of modeling

A modeling is used 3D. Only one calculation with the parameters by default of the contact is carried out.

In this modeling, points  $H$  and  $H_p$  (resp.  $I$  and  $I_p$ ) are the same points (compared to those described in the geometry) but located on the two faces supporting the conditions of symmetry.

### 10.2 Characteristics of the grid



Nodes: 3698.

Meshes: 192 TETRA4, 5872 PENTA6, 224 PYRAM5.

### 10.3 Sizes tested and results

The first calculation (algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M5884 not G	'ANALYTICAL'	-2798.3 N	14.0%
$\sigma_{yy}$ mesh M5884 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$\sigma_{yy}$ mesh M5896 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$\sigma_{yy}$ mesh M5908 not G	'AUTRE_ASTER'	-3176.3 N	2.0%

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$\sigma_{yy}$ mesh M5920 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$\sigma_{yy}$ mesh M5932 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$\sigma_{yy}$ mesh M5944 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$\sigma_{yy}$ mesh M5956 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$\sigma_{yy}$ mesh M5968 not G	'AUTRE_ASTER'	-3176.3 N	2.0%
$DX$ not G	'ANALYTICAL'	0 mm	1.0E-10
$DX$ not H	'AUTRE_ASTER'	-0.11343 mm	4.0%
$DZ$ not $H_p$	'AUTRE_ASTER'	-0.11343 mm	4.0%
$DY$ not H	'AUTRE_ASTER'	-0.16291 mm	2.0%
$DY$ not $H_p$	'AUTRE_ASTER'	-0.16291 mm	2.0%
$DX$ not I	'AUTRE_ASTER'	-0.17845 mm	3.0%
$DZ$ not $I_p$	'AUTRE_ASTER'	-0.17845 mm	3.0%
$DY$ not I	'AUTRE_ASTER'	-0.62966 mm	0.1%
$DY$ not $I_p$	'AUTRE_ASTER'	-0.62966 mm	0.1%

## 10.4 Remarks

The results got in this modeling differ up to 4 % from the reference (modeling A): it is because the geometrical non-linearity of the contact is voluntarily solved coarsely (criterion fixed at 5 %). One notes that the radiant grid makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point  $G$  are similar.

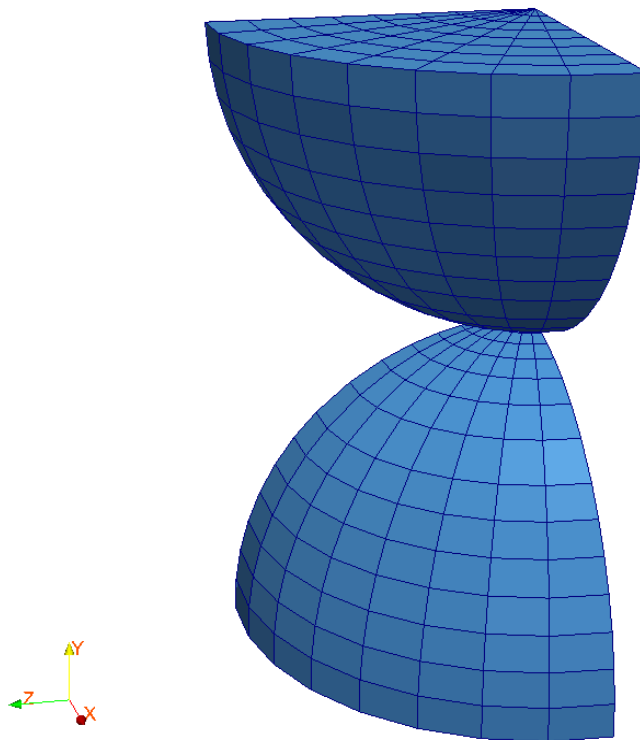
## 11 Modeling J

### 11.1 Characteristics of modeling

A modeling is used 3D. Only one calculation with an alternative of the algorithm of contact by default is carried out.

In this modeling, points  $H$  and  $H_p$  (resp.  $I$  and  $I_p$ ) are the same points (compared to those described in the geometry) but located on the two faces supporting the conditions of symmetry.

### 11.2 Characteristics of the grid



Nodes: 3698.

Meshes: 192 TETRA4, 5872 PENTA6, 224 PYRAM5.

### 11.3 Sizes tested and results

The first calculation (algorithm 'GCP')

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ mesh M5884 not G	'ANALYTICAL'	-2798.3 N	14.0%
$\sigma_{yy}$ mesh M5884 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
$\sigma_{yy}$ mesh M5896 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
$\sigma_{yy}$ mesh M5908 not G	'AUTRE_ASTER'	-3176.3 N	0.1%

$\sigma_{yy}$ mesh M5920 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
$\sigma_{yy}$ mesh M5932 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
$\sigma_{yy}$ mesh M5944 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
$\sigma_{yy}$ mesh M5956 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
$\sigma_{yy}$ mesh M5968 not G	'AUTRE_ASTER'	-3176.3 N	0.1%
DX not G	'ANALYTICAL'	0 mm	1.0E-10
DX not H	'AUTRE_ASTER'	-0.11343 mm	0.1%
DZ not $H_p$	'AUTRE_ASTER'	-0.11343 mm	0.1%
DY not H	'AUTRE_ASTER'	-0.16291 mm	0.1%
DY not $H_p$	'AUTRE_ASTER'	-0.16291 mm	0.1%
DX not I	'AUTRE_ASTER'	-0.17845 mm	0.1%
DZ not $I_p$	'AUTRE_ASTER'	-0.17845 mm	0.1%
DY not I	'AUTRE_ASTER'	-0.62966 mm	0.1%
DY not $I_p$	'AUTRE_ASTER'	-0.62966 mm	0.1%

## 11.4 Remarks

The results got in this three-dimensional modeling are in very good agreement with the analytical solution as well as the reference (modeling A). One notes moreover that the radiant grid makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.

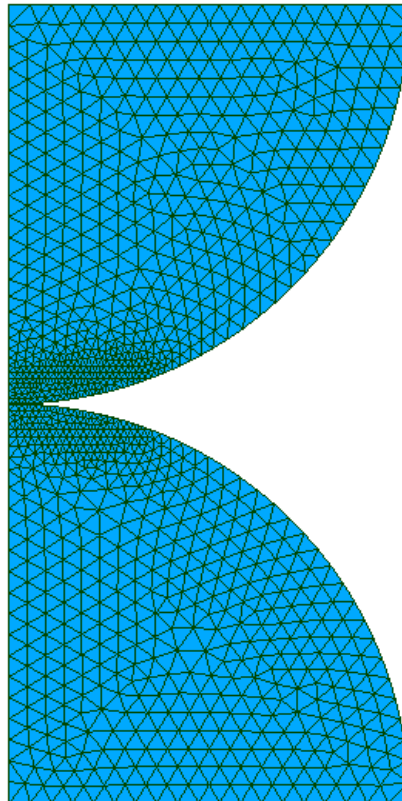
The results differ from those obtained by modeling I not because of the algorithm of resolution of the contact but because the geometrical convergence criteria are not the same one (5 % in modeling I, 1 % in modeling J).

## 12 Modeling K

### 12.1 Characteristics of modeling

A modeling is used `AXIS`. Only one calculation is carried out with Mortar pairing and cutting `LAKE`.

### 12.2 Characteristics of the grid



Nodes: 1099.

Meshes: 2008 `TRIA3`.

The grids are incompatible on the level of the zone of contact.

### 12.3 Sizes tested and results

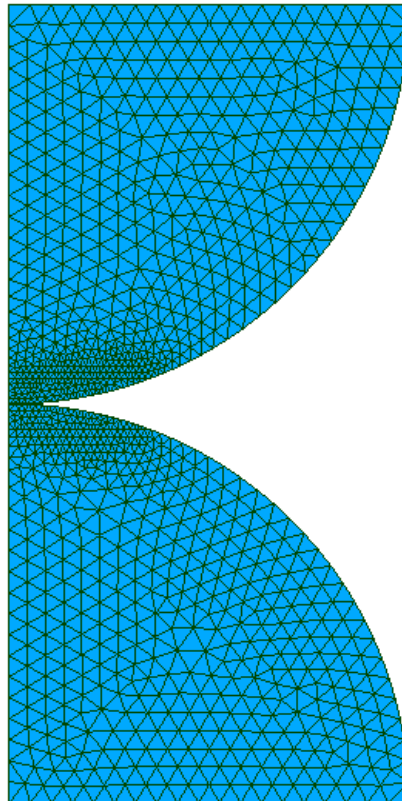
Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ at the point $G$ moment 1.0	'ANALYTICAL'	-2798.3 $N$	5, 0%
$\sigma_{yy}$ At the point $G'$ moment 1.0	'ANALYTICAL'	-2798.3 $N$	5, 0%

## 13 Modeling L

### 13.1 Characteristics of modeling

A modeling is used `AXIS`. Only one calculation is carried out with Mortar pairing and cutting `LAKE`.

### 13.2 Characteristics of the grid



Nodes: 4204.

Meshs: 2008 `SORTED6`.

The grids are incompatible on the level of the zone of contact.

### 13.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ at the point $G$ moment 1.0	'ANALYTICAL'	-2798.3 $N$	5, 0%
$\sigma_{yy}$ At the point $G'$ moment 1.0	'ANALYTICAL'	-2798.3 $N$	5, 0%

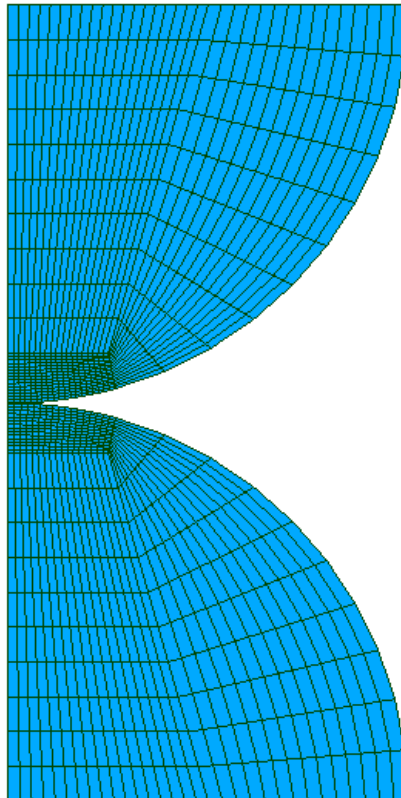


## 14 Modeling M

### 14.1 Characteristics of modeling

A modeling is used `AXIS`. Only one calculation is carried out with Mortar pairing and cutting LAKE.

### 14.2 Characteristics of the grid



Nodes: 1260.

Meshes: 1173 QUAD4.

The grids are incompatible on the level of the zone of contact.

### 14.3 Sizes tested and results

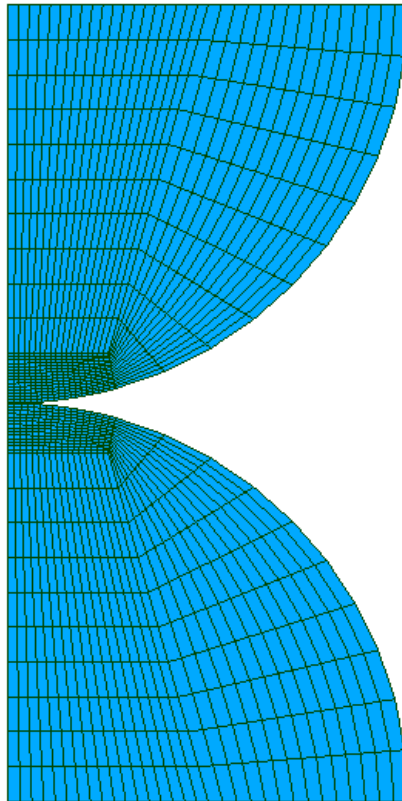
Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ at the point $G$ moment 1.0	'ANALYTICAL'	-2798.3 N	5, 0%
$\sigma_{yy}$ At the point $G'$ moment 1.0	'ANALYTICAL'	-2798.3 N	5, 0%

## 15 Modeling NR

### 15.1 Characteristics of modeling

A modeling is used `AXIS`. Only one calculation is carried out with Mortar pairing and cutting `LAKE`.

### 15.2 Characteristics of the grid



Nodes: 3691.

Meshs: 1173 `QUAD8`.

The grids are incompatible on the level of the zone of contact.

### 15.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ at the point $G$ moment 1.0	'ANALYTICAL'	-2798.3 $N$	5, 0%
$\sigma_{yy}$ At the point $G'$ moment 1.0	'ANALYTICAL'	-2798.3 $N$	5, 0%

## 16 Summary of the results

---

The comparison in each modeling to the analytical reference (limited to the point  $G$ ) is satisfactory.

The not-regression of the results is ensured by testing displacements in two nodes of the edge,  $H$  and  $I$ . These nodes occupying of the slightly different positions according to modelings, one should not seek to compare them between modelings.

This test in curved geometry highlights, the utility of certain parameters of pairing such as:

- smoothing: it is about a process of modifications of the normals which are used to write the conditions of contact. Although the grids of sphere are symmetrical one notes always in practice a light shift on the last point of the surface of contact. The smoothing of the normals makes it possible here to find symmetry. This parameter proves also useful to improve geometrical convergence of modelings using of the grids coarse (it is the case of modelings 2D this test).
- Nodal pairing or the average of normals (master-slave): little used in general, these two options of pairing, like smoothing, makes it possible to find the symmetry of the problem.

The modeling A where the contact is replaced by a unilateral connection puts also ahead the very important part played by the geometrical reactualization on the precision of the constraints. Indeed this modeling makes it possible to remove the geometrical non-linearity of this problem, it thus provides a **reference solution**. Thus when one compares the results got on the same problem 3D by modelings A, I and J, one notes that:

- the values of displacements obtained by modeling I present variations going up to 4 % compared to two other modelings
- modelings A and J find a pressure of Hertz identical, while modeling I finds a pressure 10% lower
- in modeling I the geometrical criterion is satisfied to 5% close whereas it is it to 1% in modeling J

It thus appears important to satisfy the geometrical criterion as well as possible to obtain the best possible precision on the values of constraints.