
SDND122 – Calculation of non-linear mode – system with 1 degree of freedom impacting an elastic thrust

Summary:

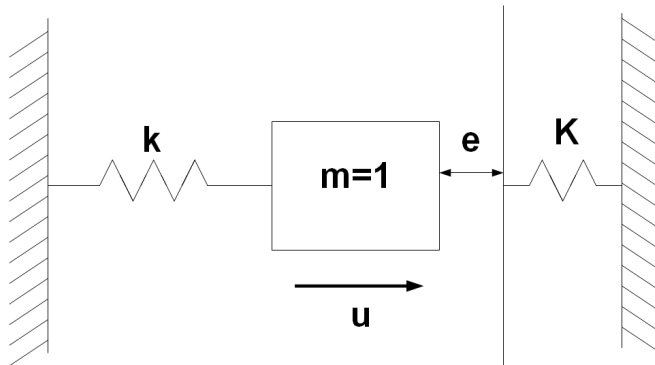
The objective of this test is to validate the calculation of non-linear mode with the operator `MODE_NON_LINE`. One also carries out postprocessings with `CALC_STABILITE` and `REST_MODE_NONL`.

An analytical solution connecting the frequency and energy is available in the case of a material point which comes to impact against an elastic thrust.

1 Problem of reference

1.1 Geometry

One considers a system made up of a spring fixed at an end and provided with a specific mass at the other end.



1.2 Properties of material

Stiffness of the spring: $k = 10 \text{ N/m}$

Mass: $m = 1 \text{ kg}$

Rigidity of the wall: $K = 50 \text{ N/m}$

1.3 Boundary conditions and loadings

The mass is fixed at an end of the spring and the other end is fixed. It moves according to the direction U and can be impacted against the wall which is with $e = 0.01 \text{ m}$ of its position of balance.

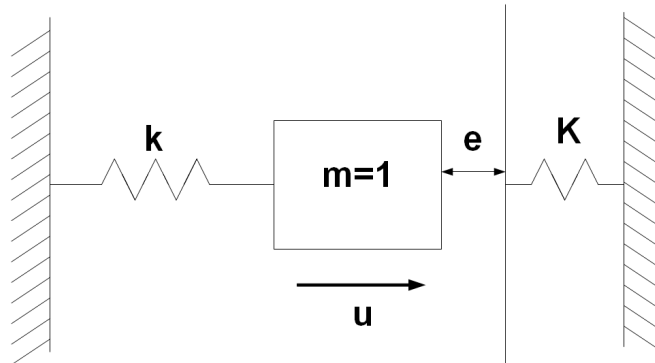
1.4 Initial conditions

For calculations of non-linear modes, the behavior depends on the energy stored in the system.

2 Reference solution

One is interested in calculation of periodic solutions of the system thus characterizing the non-linear mode [1].

2.1 Method of calculating



If one notes u the displacement of the mass and F the force of reaction, the equation of the system is written:

$$\ddot{u}(t) + ku(t) = -F(u(t))$$

Where:

$$\begin{aligned} F(u) &= 0 & \text{if } u \leq e \\ F(u) &= K(u - e) & \text{if } u > e \end{aligned}$$

The following initial conditions are supposed:

$$\dot{u}(0) = -\sqrt{\frac{2E}{m}} \quad \text{and} \quad u(0) = 0$$

Where E indicate stored mechanical energy.

One solves the problem by considering two linear problems. The first corresponds to the coasting flight (except impact), and the second corresponds to the system with the interdependent mass with the wall (during the impact).

After calculation, the duration is obtained T_1 phase of coasting flight:

$$T_1 = 2\sqrt{\frac{m}{k}} \arccos\left(-e\sqrt{\frac{k}{2E}}\right)$$

And duration of the impact T_2 is given by the following relation:

$$T_2 = 2\sqrt{\frac{m}{K+k}} \arccos\left(ek\sqrt{\frac{1}{2E(K+k)}}\right)$$

The frequency of the periodic solution is thus equal to: $N(E) = \frac{1}{T_1 + T_2}$

The stability of the periodic solution is calculated while being based on the theory of Floquet, by a diagram of Newmark and a calculation with the eigenvalues.

2.2 Sizes and results of reference

The selected reference variables are the couple frequency – energy and the stability of the periodic solution obtained.

The periodic solution is stable for the couple frequency – energy such as:
 $0.644 \text{ Hz} < f < 0.6475 \text{ Hz}$ and $6.2 \cdot 10^{-3} \text{ J} < E < 6.9 \cdot 10^{-3} \text{ J}$

2.3 Uncertainties on the solution

The relation between the frequency and energy is obtained analytically.

2.4 Bibliographical references

- 1 E.H. MOUSSI, Analyzes vibrating structures equipped with located non-linearities with game using the non-linear modes. Doctorate 2013.

3 Modeling A

3.1 Characteristics of modeling

A modeling is used `DIS_T`.

One defines the loading associated with the system with the operator `AFFE_CHAR_CINE`.

3.2 Characteristics of the grid

The grid contains 1 element of the type `POI1` and 1 element of the type `SEG2`.

3.3 Sizes tested and results

One tests the value of the frequency relative to a given energy. This frequency is obtained by interpolation starting from the table produced by `MODE_NON_LINE`.

Frequency (Hz)	Energy (J)	Stability of the periodic solution
0.64622	$6.50108331624 \cdot 10^{-3}$	STABLE

4 Modeling B

4.1 Characteristics of modeling

A modeling is used `DIS_T`.

One defines the loading associated with the system by using the operator `AFFE_CHAR_MECA`.

4.2 Characteristics of the grid

The grid contains 1 element of the type `POI1` and 1 element of the type `SEG2`.

4.3 Sizes tested and results

One tests the value of the frequency relative to a given energy. This frequency is obtained by interpolation starting from the table produced by `MODE_NON_LINE`.

Frequency (<i>Hz</i>)	Energy (<i>J</i>)	Stability of the periodic solution
0.64661	6.5812965423810^{-3}	STABLE

5 Modeling C

5.1 Characteristics of modeling

A modeling is used `DIS_T`.

One defines the loading associated with the system by using the operator `AFFE_CHAR_CINE`.

One carries out a continuation of calculation on the results got with the first calculation of non-linear mode.

Not to overload the case test, one does not carry out the calculation of the stability of the solution.

5.2 Characteristics of the grid

The grid contains 1 element of the type `POI1` and 1 element of the type `SEG2`.

5.3 Sizes tested and results

One tests the value of the frequency relative to a given energy. This frequency is obtained by interpolation starting from the table produced by `MODE_NON_LINE`.

Frequency (Hz)	Energy (J)
0.64610	6.47656819016 10 ⁻³

6 Summary of the results

This case test validates the operator of calculation of non-linear modes (`MODE_NON_LINE`), the operator of calculation of stability of the non-linear modes (`CALC_STABILITE`) and the operator of restitution of the periodic solution (`REST_MODE_NONL`).

The matrix of the system is quasi-singular, the value of the solution obtained can depend on the precision of the machine on which one launches calculation. One tests the value of the frequency relative to a given energy. This frequency is obtained by interpolation starting from the table produced by `MODE_NON_LINE`. It is noted that the relation between energy and the frequency is always checked on the got results.