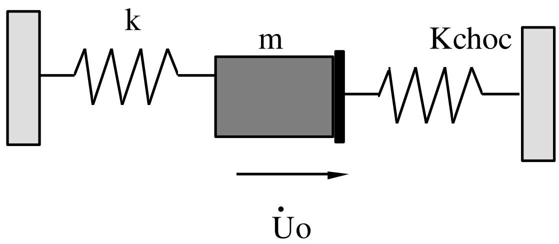
SDND101 - To release of a system masses spring with shock

Summary

This problem corresponds to a transitory analysis by modal recombination of a nonlinear discrete system to a degree of freedom. Non-linearity consists of a contact with shock on a rigid level. The mass is launched with a nonworthless initial speed against the obstacle. The initial game between the material point and the obstacle is null. This problem makes it possible to test the postprocessing of the forces of impact: velocity impact, duration of shock…

1 Problem of reference

1.1 Geometry



1.2 Properties of materials

The system consists of a mass m and a spring of stiffness k. The thrust of shock has a stiffness equalizes with K_{choc}.

Mass	$m=100\,kg$
Stiffness	$k=10^4\,N/m$
Normal rigidity of shock	$K_{choc}=10^6\,N/m$

1.3 Initial conditions

The system is initially in position at rest ($U_0=0$) and has an initial speed $\dot{U}_0>0$. One will choose for the application an initial speed $\dot{U}_0=1\,m/s$.

2 Reference solution

2.1 Method of calculating used for the reference solution

During the phase of impact, the system is solution of the differential equation:

$m.\ddot{u}+k.u+K_c<u>^+=0$ with $u_0=0$ and $\dot{u}_0=\dot{U}_0$.
$<x>^+$ indicate the positive value of x .

The analytical solution of this problem is:

$$u=\frac{\dot{U}_0}{\omega_c}\sin(\omega_c t) \text{ where } \omega_c=\sqrt{\frac{k+K_c}{m}} .$$

Speed is cancelled for $t_{\dot{u}=0}=\frac{\pi}{2\omega_c}$.

The force of shock is then maximum and is worth $F_{\max}=K_c u(t_{\dot{u}=0})=K_c\frac{\dot{U}_0}{\omega_c}$.

By construction, the duration of the shock is worth $T_{choc}=2t_{\dot{u}=0}$.

The system returns to the position $u=0$ with speed $-\dot{U}_0$.

In the field $u<0$ the system has as an equation $m.\ddot{u}+k.u=0$ with for initial conditions $u_1=0$ and $\dot{u}_1=-\dot{U}_0$.

Its solution is $u=-\frac{\dot{U}_0}{\omega_0}\sin(\omega_0 t')$ where $\omega_0=\sqrt{\frac{k}{m}}$.

Speed is cancelled for: $t'_{\dot{u}=0}=\frac{\pi}{2\omega_0}$.

By construction, the time of coasting flight is worth: $T_{vol}=2t'_{\dot{u}=0}$.

The system is thus periodic with alternatively a phase of time of shock of duration T_{choc} where the system describes an arch of sine in the field of $u>0$ and a phase of coasting flight of duration T_{vol} where the system describes an arch of sine in the field as of $u<0$.

The impulse with each impact is worth: $I=\int_0^{T_{choc}} K_c u(t)\,dt=2K_c\frac{\dot{U}_0}{\omega_c^2}=\frac{2m\dot{U}_0}{1+\frac{k}{K_c}}$.

2.2 Results of reference

The results taken for reference are the values of the moments of maximum force, the value of maximum force, the duration of the time of shock, the value of the impulse and impact speed as well as the elementary number of impact for the first two oscillations of the system.

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

- G.JACQUART: Postprocessing of calculations of heart and interns REFERENCE MARK under seismic request - HP-61/95/074/A.

3 Modeling A

3.1 Characteristics of modeling

The system mass-arises is modelled by an element of the type POI1 to the node $NO1$. It is fixed to move according to the axis x . The node $NO1$ is positioned in $O=(0.0.0.)$.

An obstacle of the type `PLAN_Z` (two parallel plans separated by a game) is used to simulate the possible shocks of the system mass-arises against a rigid plan. One chooses to take the axis Oy for normal with the plan of shock, that is to say `NORM_OBST: (0. , 1. , 0.)`. Not to be constrained by the rebound of the oscillator on the symmetrical level, one very pushes back this one far (cf [Figure. 3.1-a]). One thus chooses to locate the origin of the obstacle in `ORIG_OBS: (- 1. 0. 0.)`.

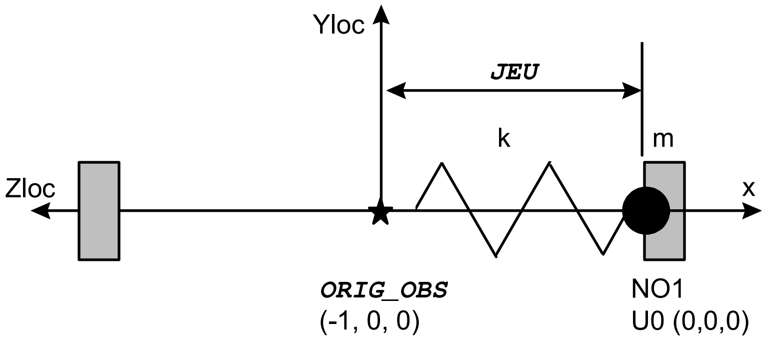


Figure 3.1-a: Modelled geometry

It remains to define the parameter JEU who gives the half-spacing between the plans in contact. One wishes here a game real no one, from where $JEU:1$. If one wishes a real game of j , it is necessary, in the case of figure presented, to impose $JEU:1+j$.

Temporal integration is carried out with the algorithm of Euler and a step of time of $5.10^{-4}s$. All the steps of calculation are filed. It is considered that reduced damping ξ_i for the whole of the calculated modes is null.

3.2 Characteristics of the grid

The grid consists of a node and a mesh of the type `POI1`.

3.3 Sizes tested and results

For the first two shocks, one compared to analytical values computed values of the moment when the impact occurs, of the maximum force of shock, the time of shock, the impulse and the impact speed. One also tests the value of the absolute extremum of the force of impact.

First shock:

Time (s)	Reference
INST	1,5630E-02
F_MAX	9,9500E+03
T_CHOC	3,1260E-02
IMPULSE	1,9805E+02
V_IMPACT	– 1.

Second shock:

Time (s)	Reference
INST	3,6100E-01
F_MAX	9,9500E+03
T_CHOC	3,1260E-02
IMPULSE	1,9805E+02
V_IMPACT	– 1,0000E+00

Time (s)	Reference
F_MAX_ABS	9,95E+03

4 Summary of the results

One notes, on the whole of the sizes, very a good agreement with the produced analytical solution. The sizes the least best represented are the duration of shock and the moment of shock (to better than 1% however). This problem is not related on the precision of calculation but to the only fact that a step of time of integration of $5.10^{-4} s$ was selected what over durations as short as $0{,}03\, s$ product already a temporal inaccuracy of $1{,}66\%$. To supplement this synthesis, one could carry out a test of convergence by decreasing the step of calculation.