

## TTLP304 - Transfer of heat in a plate orthotropic: imposed flows

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### Summary:

This test is resulting from the validation independent of version 3 in linear transitory thermics.

It is about a problem 2D plan represented by a modeling (plane).

The features tested are the following ones:

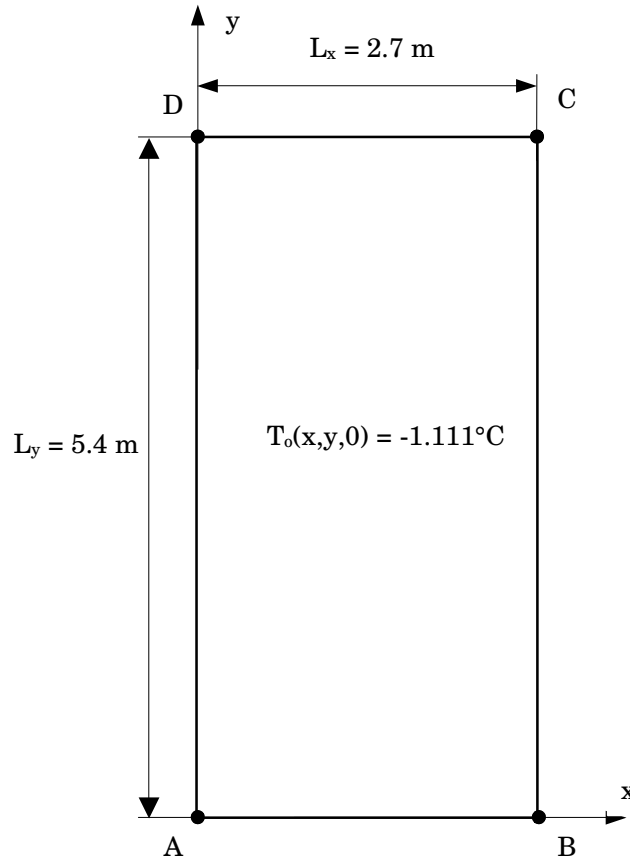
- thermal element plan,
- orthotropic material,
- transitory algorithm of thermics,
- limiting conditions: imposed flow.

The interest of the test lies in the taking into account of an orthotropic material.

The results are compared with an analytical solution.

## 1 Problem of reference

### 1.1 Geometry



### 1.2 Properties of material

$\lambda_x = 2.638 \text{ W/m}^\circ\text{C}$	thermal conductivity along the axis $x$
$\lambda_y = 0.633 \text{ W/m}^\circ\text{C}$	thermal conductivity along the axis $y$
$\rho C = 1899.1 \text{ J/m}^3^\circ\text{C}$	voluminal heat

### 1.3 Boundary conditions and loadings

Contour  $[AB]$ ,  $[BC]$ ,  $[CD]$  :  $T = -17.778^\circ\text{C}$   
 Side  $[AD]$  :  $\varphi = 0$

### 1.4 Initial conditions

$$T(t=0) = -1.111^\circ\text{C}$$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

$$T(x, y, t) = \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} B_n \cos \frac{(2n-1)\pi x}{2L_x} \sin \frac{j\pi y}{L_y} \exp \left[ - \left( \frac{\lambda_x (2n-1)^2 \pi^2}{4L_x^2} + \frac{\lambda_y j^2 \pi^2}{L_y^2} \right) t \right]$$

$$\text{where } B_n = \left[ \frac{8T_i}{\pi^2 j(2n-1)} (-1)^{n+2} [(-1)^j - 1] - 32 \right] \frac{5}{9} \quad T_i = \frac{5}{9} T_0 + 32$$

Temperature in °F with  $t = 1.2 \text{ hr} (4320\text{s})$

2.7	-	-15.6480	-15.7455	-15.9049	-16.1211	-16.3876	-16.6964	-17.0381	-17.4022	-
	15.6151									17.7778
2.4	-	-15.6786	-15.7748	-15.9318	-16.1449	-16.4076	-16.7120	-17.0487	-17.4076	-
	15.6462									17.7778
2.1	-	-15.7700	-15.8620	-16.0122	-16.2160	-16.4673	-16.7584	-17.0805	-17.4238	-
	15.7391									17.7778
1.8	-	-15.9208	-16.0058	-16.1447	-16.3333	-16.5657	-16.8349	-17.1328	-17.4503	-
	15.8921									17.7778
1.5	-	-16.1279	-16.2035	-16.3269	-16.4944	-16.7009	-16.9401	-17.2048	-17.4869	-
	16.1025									17.7778
1.2	-	-16.3869	-16.4506	-16.5547	-16.6959	-16.8700	-17.0716	-17.2947	-17.5325	-
	16.3655									17.7778
0.9	-	-16.6911	-16.7409	-16.8222	-16.9325	-17.0685	-17.2261	-17.4004	-17.5862	-
	16.6744									17.7778
0.6	-	-17.0318	-17.0660	-17.1218	-17.1975	-17.2909	-17.3991	-17.5187	-17.6462	-
	17.0203									17.7778
0.3	-	-17.3982	-17.4156	-17.4440	-17.4825	-17.5300	-17.5851	-17.6459	-17.7108	-
	17.3923									17.7778
0.0	-	-17.7778	-17.7778	-17.7778	-17.7778	-17.7778	-17.7778	-17.7778	-17.7778	-
	17.7778									17.7778
Y ↑										
X →	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7

The values of reference are obtained with  $n = j = 1000$

### 2.2 Results of reference

$t = 1.2 \text{ hr} (4320\text{s})$  : temperature at the following points:

- in  $x=0.0$  : for  $y=0.6, 1.5, 2.7$ ,
- in  $x=0.9$  : for  $y=0.6, 1.5, 2.7$ ,
- in  $x=1.8$  : for  $y=0.6, 1.5, 2.7$ .

### 2.3 Uncertainty on the solution

Analytical solution.

### 2.4 Bibliographical references

- J.C. Bruch Jr., G. Zyrolowski, 'Transient two-dimensional heat conduction problems solved by the finite element method', Int. J. num. Meth. Engng, flight 8, n°3, pp 481-494, 1974.

## 3 Modeling A

### 3.1 Characteristics of modeling

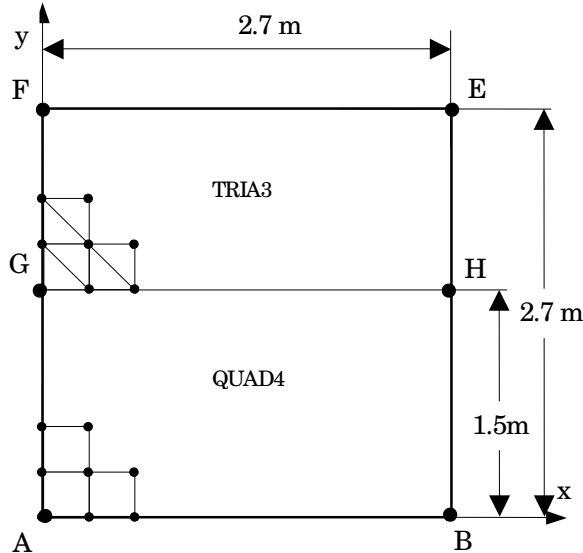
PLAN (QUAD4, TRIA3)

#### Conditions limites

- cotés AB, BH, HE:  $T = -1.111^\circ\text{C}$
- cotés EF, FG, GA:  $\phi = 0$

#### Découpage:

- AB, GH, FE 9 éléments
- AG, BH 5 éléments
- GF, HE 4 éléments



### 3.2 Characteristics of the grid

Many nodes: 100  
Many meshes and types: 117 (45 QUAD4, 72 TRIA3)

### 3.3 Remarks

The discretization in step of time is the following one:

10 pas for	[0., 5.00D0]	that is to say	$\Delta t = 0.5$
9 pas for	[5.00D0, 5.00D1]	that is to say	$\Delta t = 5.$
9 pas for	[5.00D1, 5.00D2]	that is to say	$\Delta t = 50.$
38 pas for	[5.00D2, 4.30D3]	that is to say	$\Delta t = 100.$
1 pas for	[4.30D3, 4.32D3]	that is to say	$\Delta t = 20.$

## 4 Results of modeling A

### 4.1 Values tested

Identification	Reference	Aster	Relative variation %		Absolute deviation	
			difference	tolerance	difference	tolerance
Temperature in °F						
<i>x</i> = 0.0						
<i>N3</i> ( <i>y</i> = 0.6)	17.0203	17.0146	0,033	1%	0,006	0.05
<i>N6</i> ( <i>y</i> = 1.5)	16.1025	16.0957	0,042	1%	0,007	0.05
<i>N10</i> ( <i>y</i> = 2.7)	15.6151	15.5784	0,235	1%	0,037	0.05
<i>x</i> = 0.9						
<i>N33</i> ( <i>y</i> = 0.6)	17.1218	17.1167	0,029	1%	0,005	0.05
<i>N36</i> ( <i>y</i> = 1.5)	16.3269	16.3127	-0,087	1%	0,014	0.05
<i>N40</i> ( <i>y</i> = 2.7)	15.9049	15.8905	0,091	1%	0,014	0.05
<i>x</i> = 1.8						
<i>N63</i> ( <i>y</i> = 0.6)	17.3991	17.3961	0,017	1%	0,003	0.05
<i>N66</i> ( <i>y</i> = 1.5)	16.9401	16.9297	0,061	1%	0,010	0.05
<i>N70</i> ( <i>y</i> = 2.7)	16.6964	16.6930	0,020	1%	0,003	0.05

## 5 Summary of the results

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The got results are satisfactory, the maximum change obtained is of 0,235%.

For the points of observation selected, the variations are more important with the nodes belonging to meshes TRIA3.