TTNL02 - Thermal transient with change of phase

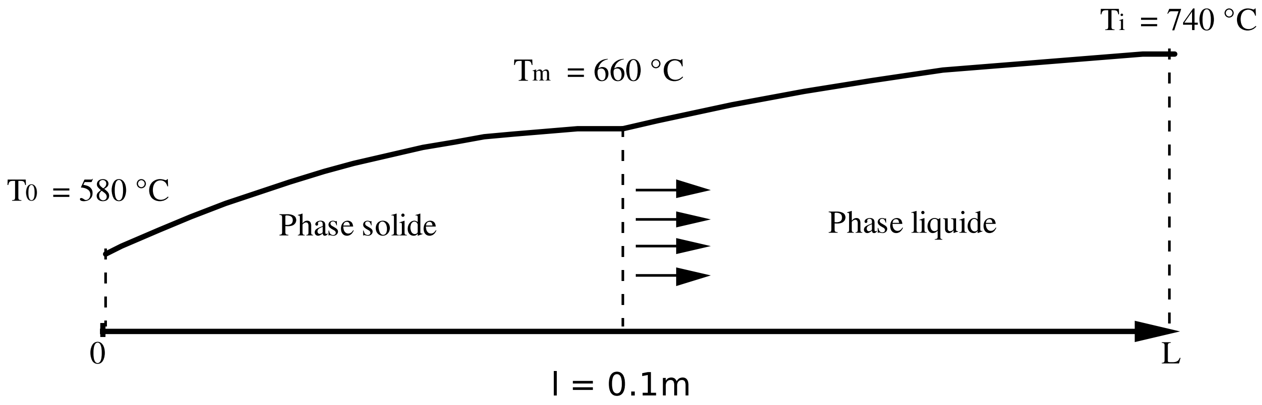
Summary:

This elementary test makes it possible to deal with an one-way problem in non-linear transitory thermics and to check the taking into account of a liquid/solid phase shift by *Code_Aster* by introducing via the voluminal enthalpy the latent heat of fusion. The solution is analytical and utilizes the functions of error erf and $erfc$. With the problem is dealt in the cases plan and voluminal.

For modelings presented here, the variations of the results got by *Code_Aster* range between 1 and 4% of the analytically calculated reference.

1 Problem of reference

1.1 Geometry



1.2 Material properties

They are the subscripted characteristics of aluminium by S for the solid phase and L for the liquid phase. They are supposed to be constant within each phase.

Density	$\rho_S = 2550\, kg/m^3$	$\rho_1 = 2390\, kg/m^3$
Voluminal heat	$c_S = 3.\,10^6\, J/m^3 {}^\circ C$	$c_1 = 2.58\,10^6\, J/m^3 {}^\circ C$
Thermal conductivity	$k_S = 210\, W/m {}^\circ C$	$k_1 = 95\, W/m {}^\circ C$
Latent heat of fusion	$L = 437.44\, 10^{-3}\, J.kg$	
Melting point	$T_m = 660.\, {}^\circ C$	
Voluminal variation of enthalpy	$\Delta H = 1.08048\, 10^9\, J/m^3$	

1.3 Boundary conditions and loadings

Temperature imposed at the ends.

$T_0 = 580\, {}^\circ C$ in $x = 0$
$T_i = 740\, {}^\circ C$ in $x = l$

1.4 Initial conditions

Uniform initial temperature

$$T_{init} = T_i = 740\, {}^\circ C$$

2 Reference solution

2.1 Method of calculating used for the reference solution

One has an semi-analytical solution utilizing the functions of errors:

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \text{ and } erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^{+\infty} e^{-t^2}\,dt$$

This solution is valid for a semi-infinite medium, it could thus be used only in one limited field of variation of the variable of time.

That is to say x_t the position of the solid interface/liquid. Are $s_t=\frac{L}{\sqrt{t_{total}}}$ and $\lambda=\frac{s_t}{2\sqrt{d_s}}$ where d_s and d_l indicate the diffusivity of the mediums solid and liquid $\left(d_s=\frac{k_s}{c_s}, d_l=\frac{k_l}{c_l}\right)$. The solution of the equation of heat is form:

$$T_s(x,t)=T_0+\frac{T_m-T_0}{erf(\lambda)}erf\left(\frac{x}{2\sqrt{d_s t}}\right) \text{ if } x\le x_t$$

$$T_l(x,t)=T_i+\frac{T_m-T_i}{erfc\left(\lambda\sqrt{\frac{d_s}{d_l}}\right)}erfc\left(\frac{x}{2\sqrt{d_l t}}\right) \text{ if } x\ge x_t$$

The data of t_{total} is enough to define the solution, one thus fixes $t_{total}=420.$

2.2 Results of reference

TIME: X-coordinate	0.5	1.0	1.5	2.0	2.5	3.0
.000	580.	580.	580.	580.	580.	580.
.005	682.43	661.33	647.50	638.74	632.69	628.20
.010	726.05	705.75	692.06	682.43	675.24	669.63
.015	738.11	728.70	718.44	709.60	702.23	696.06
.020	739.86	737.22	731.99	726.05	720.27	714.94
.025	740.	739.50	737.56	734.47	730.81	727.00
.030	740.	739.93	739.39	738.11	736.20	733.88
.035	740.	739.99	739.88	739.45	738.61	737.40
.040	740.	740.	739.98	739.86	739.55	739.00
.045	740.	740.	740.	739.97	739.87	739.65
.050	740.	740.	740.	740.	739.97	739.89
.055	740.	740.	740.	740.	740.	739.97
.060	740.	740.	740.	740.	740.	740.
.065	740.	740.	740.	740.	740.	740.
.070	740.	740.	740.	740.	740.	740.
.075	740.	740.	740.	740.	740.	740.
.080	740.	740.	740.	740.	740.	740.
.085	740.	740.	740.	740.	740.	740.
.090	740.	740.	740.	740.	740.	740.
.095	740.	740.	740.	740.	740.	740.
.100	740.	740.	740.	740.	740.	740.

TIME: X-coordinate	3.5	4.0	4.5	5.0	5.5	6.0
.000	580.	580.	580.	580.	580.	580.
.005	624.68	621.84	619.48	617.48	615.25	614.25
.010	665.09	661.33	657.43	653.65	650.37	647.49
.015	690.83	686.33	682.43	678.99	675.95	673.22
.020	710.11	705.75	701.81	698.25	709.92	692.06
.025	723.23	719.60	716.17	712.95	720.89	707.09
.030	731.34	728.70	726.05	723.43	728.48	718.44
.035	735.89	734.18	732.34	730.43	733.42	726.53
.040	738.21	737.22	736.07	734.79	736.44	731.99
.045	739.29	738.77	738.11	737.33	738.18	735.47
.050	739.74	739.50	739.15	738.71	739.12	737.56
.055	739.91	739.81	739.65	739.42	739.60	738.75
.060	739.97	739.93	739.86	739.75	739.83	739.39
.065	739.99	739.98	739.95	739.90	739.93	739.72
.070	740.	739.99	739.98	739.96	739.97	739.88
.075	740.	740.	740.	739.99	739.99	739.95
.080	740.	740.	740.	740.	740.	739.98
.085	740.	740.	740.	740.	740.	739.99
.090	740.	740.	740.	740.	740.	740.
.095	740.	740.	740.	740.	740.	740.
.100	740.	740.	740.	740.	740.	740.

(In $°C$, according to the X-coordinate in meter and of time in seconds).

Note:

One limits oneself to the variations during the 6 first second, beyond 10 seconds the boundary condition at the end $x=1$ is not assured any more.

2.3 Uncertainty on the solution

Unknown factor, due to the evaluation of the functions of error.

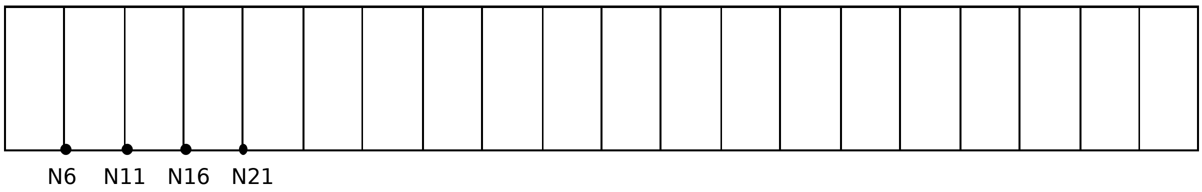
2.4 Bibliographical references

•Mr. Necati Özisik - Heat Conduction - Chapter 10: Phase-change problems example 10-3 - John Wiley & Sounds.

3 Modeling A

3.1 Characteristics of modeling

Modeling 2D:



3.2 Characteristics of the grid

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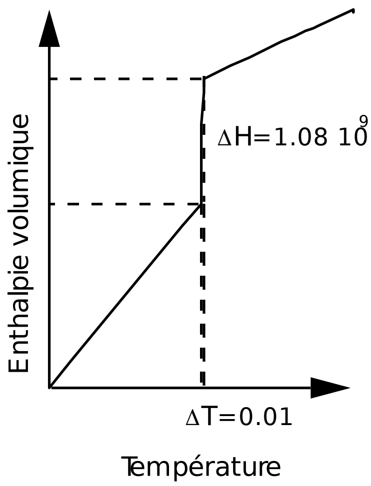
3.3 Notice

The latent heat of fusion is provided via the enthalpy on an interval of $0.01°C$.



3.4 Values tested

The nodes observed have as a coordinate $y=0.0$

Identification temperature	Reference
T = 0.5 S N6 (X = 0,005)	682.43
T = 1.0 S N6 (X = 0,005)	661.33
T = 3.0 S N6 (X = 0,005)	628.20
T = 6.0 S N6 (X = 0,005)	614.25
T = 0.5 S N11 (X = 0,010)	726.05
T = 1.0 S N11 (X = 0,010)	705.75
T = 3.0 S N11 (X = 0,010)	669.63

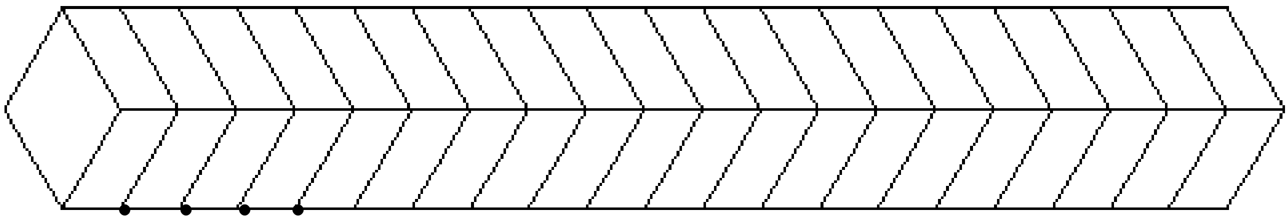
T = 6.0 S N11 (X = 0,010)	647.49
T = 0.5 S N16 (X = 0,015)	738.11
T = 1.0 S N16 (X = 0,015)	728.70
T = 3.0 S N16 (X = 0,015)	696.06
T = 6.0 S N16 (X = 0,015)	673.22
T = 0.5 S N21 (X = 0,020)	739.86
T = 1.0 S N21 (X = 0,020)	737.22
T = 3.0 S N21 (X = 0,020)	714.94
T = 6.0 S N21 (X = 0,020)	692.06

Calculation by finite elements requires a discretization in times of $\Delta t=5.10^{-4}s$ at least for the first steps. The boundary condition imposed at the origin making pass the temperature abruptly of $740.°C$ with $580.°C$. One observes on the level of the first steps of time some oscillations which are stabilized then rather quickly, despite everything the maximum temperature is exceeded, it does not have respect of the discrete maximum there. This phenomenon is observed at the time of the thermal shocks, only a particular digital processing on the level of the matrix of mass can cure this last.

4 Modeling B

4.1 Characteristics of modeling

Modeling 3D:



4.2 Characteristics of the grid

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4.3 Values tested

The nodes observed have as coordinates: $x=y=0.005$

Identification Temperature	Reference
T = 0.5 S No324 (Z = 0,005)	682.43
T = 1.0 S No324 (Z = 0,005)	661.33
T = 3.0 S No324 (Z = 0,005)	628.20
T = 6.0 S No324 (Z = 0,005)	614.25
T = 0.5 S No312 (Z = 0,010)	726.05
T = 1.0 S No312 (Z = 0,010)	705.75
T = 3.0 S No312 (Z = 0,010)	669.63
T = 6.0 S No312 (Z = 0,010)	647.49
T = 0.5 S No300 (Z = 0,015)	738.11
T = 1.0 S No300 (Z = 0,015)	728.70
T = 3.0 S No300 (Z = 0,015)	696.06
T = 6.0 S No300 (Z = 0,015)	673.22
T = 0.5 S No277 (Z = 0,020)	739.86
T = 1.0 S No277 (Z = 0,020)	737.22
T = 3.0 S No277 (Z = 0,020)	714.94
T = 6.0 S No277 (Z = 0,020)	692.06

5 Summaries of the results

The error obtained compared to the analytical solution remains reasonable for the points of observation listed in the tables. Let us announce however that the thermal shock imposed on the beginning of the transient cause oscillations (when one observes the variation in the temperature in a point in the course of time) which diminish quickly and which disappeared at time $t=0.5\,s$.