TPLL101 - Joule effect heating of a cylinder hollow

Summary:

One imposes electric currents inside and outside a hollow roll finite length, then one calculates the temperature established under the effect of a source of heat produced by Joule effect. The reference solution is analytical.

The scope of application is it thermal linear stationary.

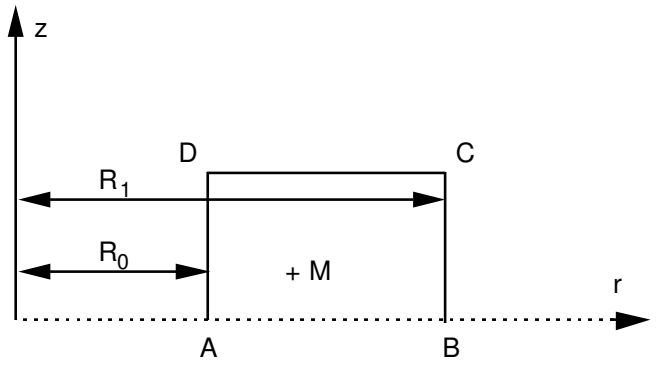
The model is axisymmetric.

The test is carried out on the elements QUAD8 and TRIA6 for the first modeling, axisymmetric (AXIS). With elements TETRA4 for the second modeling, 3D.

1 Problem of reference

1.1 Geometry

Axisymmetric



	With	B	C	D	M
R	1.	2.7182	2.7182	1.	1.8591
Z	0.	0.	0.1	0.1	0,025

1.2 Material properties

Electric characteristic: electric conductivity $\sigma = 1.\ \Omega^{-1} m^{-1}$

Thermal characteristics: $\lambda = 2.10^{-2}\ W/m°C$ $C_P = 0.$

1.3 Boundary conditions and loadings

Electric calculation:

$j.n = -10$ sur DA
$j.n = 3.6787944$ sur BC

Thermal calculation

$T = 0.$ on DA

$T = 0.$ on BC

$FLUX = 0.$ on AB

$FLUX = 0.$ on CD

1.4 Initial conditions

Stationary calculation.

2 Reference solution

2.1 Method of calculating used for the reference solution

- Problem elastostatic V electric potential

In volume $\Delta V=0.$

Boundary conditions NEUMANN $\begin{cases} j.n=0. & \text{sur } CD \text{ et } AB \\ j.n=-10. & \text{sur } AD \\ j.n=3.6787944 & \text{sur } BC \end{cases}$

electric conductivity $\sigma=1.$

$$j.n=-\sigma \nabla V$$

Axisymmetric solution

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)=0 \Rightarrow V=V_0 \log\frac{r}{A}$$

Boundary conditions on AD and BC impose:

$$V_0=10.$$

Note:

The knowledge of A is not necessary for thermal calculation.

- Thermal problem T the temperature

$-\lambda \Delta T=s$ with a voluminal source $s=\sigma (\nabla V)^2$

Boundary conditions: $T=0.$ on DA and BC

$$-\lambda \nabla T.n=0 \text{ on } DC \text{ and } AB$$

Axisymmetric solution:

$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=-\frac{\sigma}{1}\frac{v_0^2}{r^2}$ ⇒ taking into account the boundary conditions

$$T(r)=-\frac{1}{2}\sigma\frac{v_0^2}{\lambda}\log\left(\frac{r}{R_0}\right)\log\left(\frac{r}{R_1}\right)$$

2.2 Results of reference

$T=588.9313\,°C$ (temperature at the point M).

2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling AXIS with éléments THAXSE2, THAXTR3 and THAXQU4.

3.2 Characteristics of the grid

The grid contains 72 meshs of the type TRIA3 and 44 meshs of the type QUAD4.

3.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
Temperature at the point M	'ANALYTICAL'	588.9313	0.1%

The source of heat is tested (starting from an electric potential via the law of Ohm) :

Identification	Type of reference	Value of reference
SOUR_ELGA at the 1st point of Gauss of the mesh M67	'NON_REGRESSION'	13.7076

3.4 Remarks

The boundary conditions of the electric problem are all of the conditions of NEUMANN. Nevertheless, the analytical solution is found.

4 Modeling B

4.1 Characteristics of modeling

It is about a modeling 3D with éléments THER_FACE3 and THER_TETRA4.

4.2 Characteristics of the grid

The grid contains 716 meshs of the type TRIA3 and 910 meshs of the type TETRA4.

4.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
Temperature at the point M	'ANALYTICAL'	588.9313	1.0%

One tests the source of heat (starting from an electric potential via the law of Ohm) at the first point of Gauss of the mesh M_{834} :

Identification	Type of reference	Value of reference
SOUR_ELGA at the 1st point of Gauss of the mesh M834	'NON_REGRESSION'	41.7093

5 Summary of the results

Besides the test presented, one carried out a calculation on structure (COTHAA). The got results were compared with those obtained using Code CASTEM 2000. Very close results are got.