SDLV302 – Modal analysis by under-structuring: bi--supported beam

Summary:

This test validates the modal analysis of a structure by the method of under-structuring dynamic. The studied structure is a deformable beam bi--supported on the shearing action.
It is modelled by hexahedral elements volumes with 20 nodes (modeling 3D).

The modal analysis is carried out by three methods:
- direct approach;
- approach by dynamic under-structuring of Craig-Bampton;
- direct approach with viscous damping proportional.

A private interest of this test is the presence of linear relations between several degrees of freedom.

The calculated Eigen frequencies are compared with values obtained analytically for a model of beam of deformable Timoshenko to the shearing action and taking account of the rotatory inertia of the sections.

1 Problem of reference

1.1 Geometry

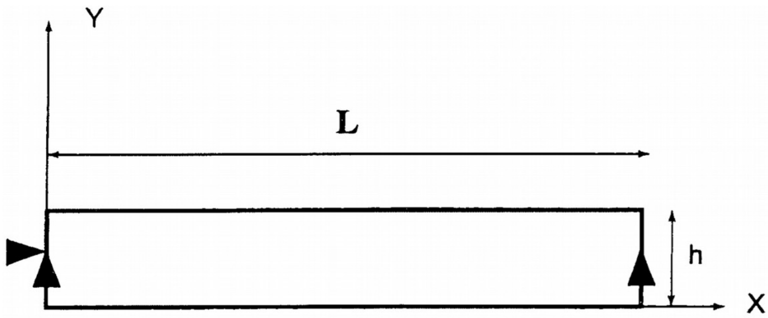

Figure 1.1-1 : Geometry of the problem

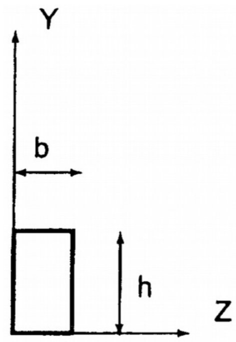

Figure 1.1-2 : Geometry of the problem

Height: $h=0.2\,\mathrm{m}$
Width: $b=0.1\,\mathrm{m}$
Length: $L=2\,\mathrm{m}$
Section: $A=b\times h=0.02\,\mathrm{m}$
Inertia: $I=\frac{b\times h^3}{12}=1.66\times 10^{-5}\,\mathrm{m}$
Coefficient of reduction of section $k'=\frac{5}{6}$

1.2 Properties of material

Young modulus	$E=2.1\times 10^{11}\,Pa$
Poisson's ratio	$\nu=0.3$
Density	$\rho=7800.0\,kg.m^{-3}$
Modulus of rigidity	$G=\frac{E}{2(1+\nu)}=8.076\times 10^{10}\,\mathrm{Pa}$

1.3 Boundary conditions and loadings

One authorizes that the inflection in the plan XY and extension along the axis X . The model being voluminal, the boundary conditions differ somewhat from those which one would impose on a model beam.

Imposed displacement:

In $X=0$, $Y=h/2$	$DX=0$, $DY=0$
In $X=L$, $Y=h/2$	$DY=0$

In $Z=b/2$	$DZ=0$

To the preceding conditions, one adds the constraint of flatness of the sections in $X=0$ and $X=L$. This constraint can be expressed as follows. Let us indicate by $x^T=(X, Y, Z)$ the vector of the coordinates and by $u^T=(DX, DY, DZ)$ the vector of displacements; the position of a point is located by the vector $x'^T=x^T+u^T=(X', Y', Z')$. Are A, B and C three non-aligned points of the section. An unspecified point P east compels in the condition:

$$\begin{vmatrix} X'_P & Y'_P & Z'_P & 1 \\ X'_A & Y'_A & Z'_A & 1 \\ X'_B & Y'_B & Z'_B & 1 \\ X'_C & Y'_C & Z'_C & 1 \end{vmatrix}=0$$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is obtained analytically for a beam of Timoshenko, fascinating of account the deformation with the shearing action and the rotatory inertia of the sections. The theoretical aspects are developed in the reference given in 2.4.

Let us define the following adimensional sizes:

$\Omega_n = \frac{\rho A L^4}{EI} \omega_n^2$ eigenvalues

$j = \frac{I}{A L^2}$ rotatory inertia

$g = \frac{EI}{k' A G L^2}$ coefficient of shearing

The Eigen frequencies of the first modes of inflections are given by the following expression:

$$\Omega_n = \frac{(g+j)\lambda_n^2 + 1 - \sqrt{(g-j)^2 \lambda_n^4 + 2(g+j)\lambda_n^2 + 1}}{2 g j}$$

with

$$\lambda_n = n\pi \,, \qquad n = 1, 2, 3, \ldots$$

The frequencies of the modes of extension are given by:

$$f_n = (2n-1)\frac{1}{4L}\sqrt{\frac{E}{\rho}} \,, \qquad n = 1, 2, 3, \ldots$$

2.2 Results of reference

Mode	Form	Frequency (Hz)
1	inflection	115.7
2	inflection	442.2
3	extension	648.6
4	inflection	931.6
5	inflection	1534.0

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

ROBERT G., Solutions analytical into dynamic of the structures, Report Samtech n° 121, Liege, 1996.

3 Modeling A

3.1 Characteristics of modeling A

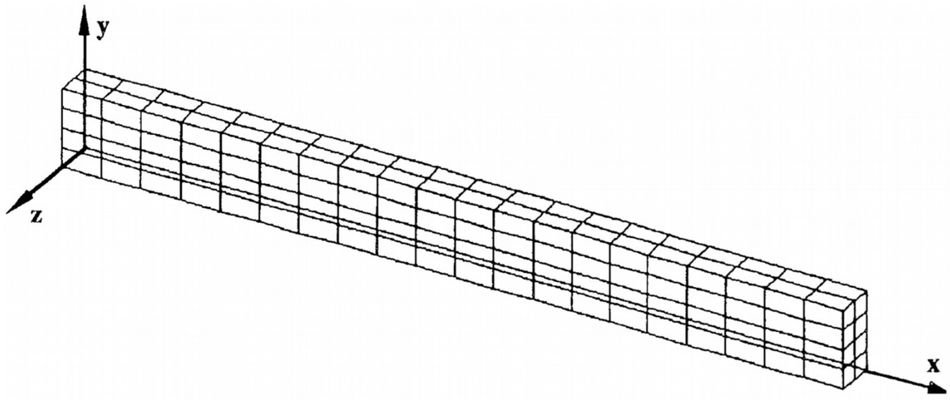

Figure 3.1-1 :Grid of the geometry of the problem

In this modeling, the complete structure is with a grid by means of voluminal elements 3D.

The constraint of flatness is expressed as follows. Are,

- point A $X=0$, $Y=h/2$, $Z=0$: $DX=DY=0$, $DZ\neq 0$;
- the point B $X=0$, $Y=h/2$, $Z=b$: $DX=DY=0$, $DZ\neq 0$;
- the point C $X=0$, $Y=h$, $Z=b/2$: $DX\neq DY\neq 0$, $DZ=0$

By neglecting the terms of the second order in displacements, the constraint of flatness results in a linear relation between displacements $DX=0$ points A , B , C and P , an unspecified point of the face:

$$\begin{vmatrix} DX_P & Y_P & Z_P & 1 \\ DX_A & h/2 & 0 & 1 \\ DX_B & h/2 & b & 1 \\ DX_C & h & b/2 & 1 \end{vmatrix}=0$$

The condition is written then for $DX_A=0$:

$$DX_P=DX_C\times(\frac{2Y_P}{h-1})$$

In $X=L$, the condition of flatness is obtained in a similar way but for DX_A unspecified. The relation is written then:

$$DX_P=DX_C\times(\frac{2Y_P}{h-1})+2\times(1-\frac{Y_P}{h})\times DX_A$$

3.2 Characteristics of the grid

Many nodes: 1077
Many meshs and types: 160 `HEXA20`

3.3 Sizes tested and results

Mode	Value of reference	Type of reference	Tolerance (%)
1	115.7	'ANALYTICAL'	1.0
2	442.2	'ANALYTICAL'	1.0
3	648.6	'ANALYTICAL'	1.0
4	931.6	'ANALYTICAL'	1.0
5	1534.0	'ANALYTICAL'	1.0

4 Modeling B

4.1 Characteristics of modeling B

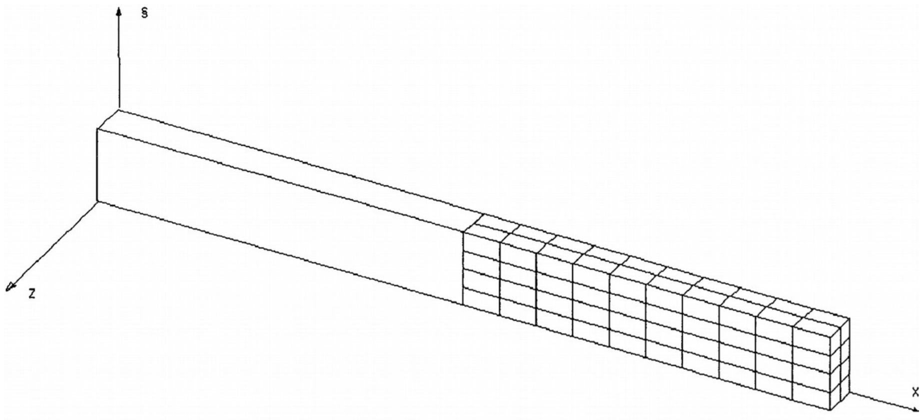

Figure 4.1-1 : Grid of the geometry of the problem.

The beam is divided into two equal parts. Each half is represented by a substructure. Those are generated by the method of Craig-Bampton. Its modal base is composed of normal with interface blocked, 10 and of the constrained static modes relating to the points such as $X=L$ and $Y=h/2$ (one considers in these points only the degrees of freedom not fixed by the boundary conditions).

4.2 Characteristics of the grid

Many nodes: 557
Many meshs and types: 80 HEXA20

4.3 Sizes tested and results

Mode	Value of reference	Type of reference	Tolerance (%)
1	115.7	'ANALYTICAL'	1.0
2	442.2	'ANALYTICAL'	1.0
3	648.6	'ANALYTICAL'	1.0
4	931.6	'ANALYTICAL'	1.0
5	1534.0	'ANALYTICAL'	1.0

5 Modeling C

5.1 Characteristics of modeling C

Identical to modeling A with addition of viscous damping proportional such as:

$$[C]=\alpha[K]+\beta[M]$$

with

$$\alpha=0.2852750549\times10^{-4} \text{ and } \beta=57.62031174$$

One uses the method of Lanczos (METHOD = 'TRI_DIAG') to calculate the clean modes.

5.2 Characteristics of the grid

Identical to modeling A.

5.3 Sizes tested and results

Tests of not-regression only except for the validation of the options MATE_ELGA and MATE_ELEM.

Identification	Value of reference	Type of reference	Tolerance
Field MATE_ELGA, Mesh MA000074, Point 1, comp. RHO	1E3	'ANALYTICAL'	0.1 %
Field MATE_ELEM, Mesh MA000074, Comp. E	1E3	'ANALYTICAL'	0.1 %
Field MATE_ELGA, Mesh MA000081, Point 1, comp. RHO	3E3	'ANALYTICAL'	0.1 %
Field MATE_ELEM, Mesh MA000081, Comp. E	3E3	'ANALYTICAL'	0.1 %

6 Modeling D

6.1 Characteristics of modeling D

Identical to modeling A with addition of viscous damping proportional such as:

$$[C]=\alpha[K]+\beta[M]$$

with,

$$\alpha=1.14110022\times10^{-4} \text{ and } \beta=230.4812469$$

One uses the method of Sorensen (`METHOD = 'SORENSEN'`) to calculate the clean modes.

6.2 Characteristics of the grid

Identical to modeling A.

6.3 Sizes tested and results

Tests of not-regression only.

7 Summary of the results

Modeling A makes it possible to validate the calculation of the clean modes compared to a reference solution obtained analytically; the maximum error is of less than 0.5 %.
Modeling B validates the dynamic under-structuring with interface of Craig-BamptoN. The relative error between modelings A and B is worthless.